

## Spin observables of $pd$ scattering and null-test for T-invariance violation

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Time-reversal invariance violating but P-parity conserving (TVPC) effects will be tested at COSY in proton-deuteron scattering at 135 MeV. A null-test signal for these effects provides an integrated cross section for vector polarized proton and tensor polarized deuteron. We use the Glauber theory with accounting for full spin-dependence of the proton-nucleon scattering amplitudes to calculate spin observables of  $pd$  elastic scattering. The obtained results are in reasonable agreement with existing data at energies 100-250 MeV. This provides a theoretical basis for estimation of TVPC effects and possible false signals in the planned  $pd$  scattering experiment.

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## 1. Introduction

Symmetries associated with the inversion of the space (P) and time (T) are important for understanding the properties of fundamental interactions. T-odd, P-odd effects arise in the Standard Model via CP violating phase in Cabibbo-Kobayashi-Maskawa matrix and the QCD  $\theta$ -term. Fundamental time-reversal-noninvariant parity-conserving (T-odd P-even) flavor-conserving interactions do not arise in the Standard Model but can be generated by weak corrections to T-odd P-odd interactions. In such a case the relative strength of the T-odd P-even nuclear force with respect to the T-even P-even one is too small (the ratio of the corresponding matrix elements is  $\alpha_T \sim 2 \times 10^{-6}$  [1]) and cannot be detected in present experiments. However, much larger intensity of TVPC interaction is not excluded if this interaction is the low energy limit of some unknown interaction beyond the Standard Model [2].

T-violating P-conserving (TVPC) interactions between nucleons in terms of boson exchanges were considered in Ref. [3] where it was found that this interaction is restricted to partial waves with total angular momentum  $J \geq 1$ ; thus, the (pseudo)scalar  $\pi$ -,  $\sigma$ -exchanges do not contribute. The lowest mass meson allowed is the  $\rho$ -meson. The TVPC NN-interaction potential for the  $\rho$ -meson exchange

$$V_\rho^T = i \frac{\bar{g}_\rho g_\rho^2 \kappa}{2m_N^2} [\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]_z \frac{1}{m_\rho^2 + \mathbf{q}^2} ([(\mathbf{p} + \mathbf{p}') \times \mathbf{q}] \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)) \quad (1.1)$$

is the C-odd one. This interaction cannot contribute to the  $nn$  or  $pp$  systems because it contains the nucleon-nucleon isovector charge exchange operator  $[\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]_z = 2i(\tau_1^+ \tau_2^- - \tau_1^- \tau_2^+)$ . In Eq. (1.1)  $\mathbf{p}$  ( $\mathbf{p}'$ ) is the initial (final) cms momentum,  $\mathbf{q} = \mathbf{p} - \mathbf{p}'$ ,  $\boldsymbol{\sigma}_i$  ( $\boldsymbol{\tau}_i$ ) is the Pauli spin (isospin) matrix,  $m_N$  and  $m_\rho$  are the masses of the nucleon and  $\rho$ -meson, respectively,  $g_\rho$  ( $\bar{g}_\rho$ ) is the ordinary (TVPC) coupling constant of the  $\rho$ -meson and nucleon;  $\kappa$  is the anomalous magnetic moment of the nucleon. The  $\rho$ -meson is expected to dominate the TVPC NN interaction like the  $\pi$ -meson in the strong interaction. If the  $\rho$ -meson does not contribute for some reasons, then the axial  $h_1$  meson is expected to dominate. However, due to larger mass of the  $h_1$  meson ( $m_{h_1} = 1.17$  GeV) its contribution is of short-range type and, consequently, is less important due to NN-repulsive core at short distances between nucleons. One should note that the most general structure of the TVPC NN-potential contains 18 different terms [4].

Direct measurements and indirect estimation of the upper limit on TVPC-effects give the following results. Test of the detailed balance, the direct reaction  $^{27}\text{Al}(p, \alpha)^{24}\text{Mg}$  and inverted reaction  $^{24}\text{Mg}(\alpha, p)^{27}\text{Al}$  gave the overall experimental agreement uncertainty  $\Delta = (\sigma_{dir} - \sigma_{inv}) / (\sigma_{dir} + \sigma_{inv}) \leq 5.1 \times 10^{-3}$  [5]. Numerous statistical analysis of corresponding data including nuclear energy-level fluctuations [6] gave  $\alpha_T < 2 \times 10^{-3}$ . Experimental study of polarized neutron  $\vec{n}$  transmission through aligned  $^{165}\text{Ho}$  nuclei [7] gave  $\alpha_T \leq 7.1 \times 10^{-4}$  (or  $\bar{g}_\rho \leq 5.9 \times 10^{-2}$ ). Elastic scattering of polarized protons on neutrons  $\vec{p}n$  and polarized neutrons on protons  $\vec{n}p$  gave  $\alpha_T \leq 8 \times 10^{-5}$  (or  $\bar{g}_\rho < 6.7 \times 10^{-3}$ ). Model-dependent analysis of T-odd P-even effects based on existing experimental estimations of the upper limit of the electric dipole moment (EDM) of the neutron and atoms gives  $\alpha_T \leq 1.1 \times 10^{-5}$  (or  $\bar{g}_\rho \leq 10^{-3}$ ) [8]. However, as shown in Ref. [9] for some scenarios of EDM there are no constraints for TVPC effects. The aim of the double polarized  $pd$ -transmission

experiment at COSY [10] is to improve the direct upper bound on TVPC effects obtained in [7] by one order of magnitude.

## 2. Formalism

The total number of transition amplitudes of  $pd$  elastic scattering is 36. The P-parity conservation constraint leads to 18 independent amplitudes. The T-invariance requirement for  $pd \rightarrow pd$  leads to 12 independent amplitudes. The transition matrix element  $M_{fi} = \langle \mu' \lambda' | M | \mu \lambda \rangle$  taken between definite initial  $|\mu \lambda \rangle$  and final  $|\mu' \lambda' \rangle$  spin states involves the following transition operator [11]

$$\begin{aligned}
M = & (A_1 + A_2 \boldsymbol{\sigma} \hat{\mathbf{n}}) + (A_3 + A_4 \boldsymbol{\sigma} \hat{\mathbf{n}})(\mathbf{S} \hat{\mathbf{q}})^2 + (A_5 + A_6 \boldsymbol{\sigma} \hat{\mathbf{n}})(\mathbf{S} \hat{\mathbf{n}})^2 + A_7(\boldsymbol{\sigma} \hat{\mathbf{k}})(\mathbf{S} \hat{\mathbf{k}}) + \\
& A_8(\boldsymbol{\sigma} \hat{\mathbf{q}})[(\mathbf{S} \hat{\mathbf{q}})(\mathbf{S} \hat{\mathbf{n}}) + (\mathbf{S} \hat{\mathbf{n}})(\mathbf{S} \hat{\mathbf{q}})] + (A_9 + A_{10} \boldsymbol{\sigma} \hat{\mathbf{n}})(\mathbf{S} \hat{\mathbf{n}}) + A_{11}(\boldsymbol{\sigma} \hat{\mathbf{q}})(\mathbf{S} \hat{\mathbf{q}}) + \\
& A_{12}(\boldsymbol{\sigma} \hat{\mathbf{k}})[(\mathbf{S} \hat{\mathbf{k}})(\mathbf{S} \hat{\mathbf{n}}) + (\mathbf{S} \hat{\mathbf{n}})(\mathbf{S} \hat{\mathbf{k}})] + \\
& (T_{13} + T_{14} \boldsymbol{\sigma} \hat{\mathbf{n}})[(\mathbf{S} \hat{\mathbf{k}})(\mathbf{S} \hat{\mathbf{q}}) + (\mathbf{S} \hat{\mathbf{q}})(\mathbf{S} \hat{\mathbf{k}})] + T_{15}(\boldsymbol{\sigma} \hat{\mathbf{q}})(\mathbf{S} \hat{\mathbf{k}}) + T_{16}(\boldsymbol{\sigma} \hat{\mathbf{k}})(\mathbf{S} \hat{\mathbf{q}}) + \\
& T_{17}(\boldsymbol{\sigma} \hat{\mathbf{k}})[(\mathbf{S} \hat{\mathbf{q}})(\mathbf{S} \hat{\mathbf{n}}) + (\mathbf{S} \hat{\mathbf{n}})(\mathbf{S} \hat{\mathbf{q}})] + T_{18}(\boldsymbol{\sigma} \hat{\mathbf{q}})[(\mathbf{S} \hat{\mathbf{k}})(\mathbf{S} \hat{\mathbf{n}}) + (\mathbf{S} \hat{\mathbf{n}})(\mathbf{S} \hat{\mathbf{k}})],
\end{aligned} \tag{2.1}$$

where  $\mathbf{S}$  is the spin-1 angular momentum operator,  $\boldsymbol{\sigma}$  is the Pauli matrix and  $\hat{\mathbf{q}} = (\mathbf{p} - \mathbf{p}')/|\mathbf{p} - \mathbf{p}'|$ ,  $\hat{\mathbf{k}} = (\mathbf{p} + \mathbf{p}')/|\mathbf{p} + \mathbf{p}'|$ ,  $\hat{\mathbf{n}} = [\mathbf{k} \times \mathbf{q}]$  are unit vectors representing the directions of coordinate frame axes:  $OX \uparrow \mathbf{q}$ ,  $OZ \uparrow \mathbf{k}$ ,  $OY \uparrow \mathbf{n}$ . In Eq. (2.1)  $A_1 \div A_{12}$  are T-even P-even amplitudes [12] and  $T_{13} \div T_{18}$  are TVPC amplitudes. Using those amplitudes we can obtain full set of spin observables. The polarized elastic differential  $pd$  cross section can be written as [13]

$$\left( \frac{d\sigma}{d\Omega} \right)_{pol} = \left( \frac{d\sigma}{d\Omega} \right)_0 \left[ 1 + \frac{3}{2} p_j^p p_i^d C_{j,i} + \frac{1}{3} P_{ij} A_{ij} + \dots \right], \tag{2.2}$$

where  $(d\sigma/d\Omega)_0$  is the unpolarized cross section,  $A_i = Tr M \boldsymbol{\sigma}_i M^+ / Tr M M^+$  and  $A_{ij}$  are analyzing powers,  $C_{i,j} = Tr M S_i \boldsymbol{\sigma}_j M^+ / Tr M M^+$ ,  $C_{ij,k} = Tr M S_i \boldsymbol{\sigma}_k M^+ / Tr M M^+$  are spin correlation parameters and  $K_j^i(d \rightarrow p) = Tr M S_j M^+ \boldsymbol{\sigma}_i / Tr M M^+$ ,  $K_j^i(p \rightarrow d) = Tr M \boldsymbol{\sigma}_j M^+ S_i / Tr M M^+$ ,  $K_j^i(p \rightarrow p) = Tr M \boldsymbol{\sigma}_j M^+ \boldsymbol{\sigma}_i / Tr M M^+$ ,  $K_j^i(d \rightarrow d) = Tr M S_j M^+ S_i / Tr M M^+$  are spin-transfer coefficients,  $p_j^p$  ( $p_j^d$ ) is the vector polarization of the proton (deuteron),  $P_{ij}$  is the tensor polarization of the deuteron;  $i, j = x, y, z$

In general case the forward elastic  $pd$  scattering amplitude contains four independent amplitudes if only T-even P-even interactions are included [14] and one more term, if the TVPC interaction occurs [15]. Using the generalized optical theorem one can find that the total  $pd$  cross section for the case of strong and TVPC interactions has a form [15, 16, 17]

$$\sigma = \sigma_0 + \sigma_1 \mathbf{p}^p \cdot \mathbf{p}^d + \sigma_2 (\mathbf{p}^p \cdot \hat{\mathbf{k}})(\mathbf{p}^d \cdot \hat{\mathbf{k}}) + \sigma_3 P_{zz} + \tilde{\sigma} p_y^p P_{xz}, \tag{2.3}$$

the first four terms are P-even T-even and the last one is T-odd P-even. The cross section  $\tilde{\sigma}$  is the null-test signal for TVPC effects [18]. This observable is zero if TVPC interaction is absent, and is non-zero only in opposite case.

The TVPC  $pd$  scattering amplitude studied in Ref. [19] is the following

$$\begin{aligned}
t_{pd} = & h_N [(\boldsymbol{\sigma} \cdot \mathbf{k})(\boldsymbol{\sigma}_N \cdot \mathbf{q}) + (\boldsymbol{\sigma}_N \cdot \mathbf{k})(\boldsymbol{\sigma} \cdot \mathbf{q}) - \frac{2}{3}(\boldsymbol{\sigma}_N \cdot \boldsymbol{\sigma})(\mathbf{k} \cdot \mathbf{q})] + \\
& + g_N [\boldsymbol{\sigma} \times \boldsymbol{\sigma}_N] \cdot [\mathbf{q} \times \mathbf{k}] + g'_N (\boldsymbol{\sigma} - \boldsymbol{\sigma}_N) \cdot i [\mathbf{q} \times \mathbf{k}] [\boldsymbol{\tau} \times \boldsymbol{\tau}_N]_z,
\end{aligned} \tag{2.4}$$

where  $\mathbf{k} = \mathbf{p} + \mathbf{p}'$ . Here  $g'$ - term corresponds to the  $\rho$ -meson exchange (1.1) and  $h$ -term presents the axial  $h_1$  meson exchange.

The  $pd$  elastic scattering within the Glauber model is described by the following transition operator

$$M(\mathbf{q}, \mathbf{s}) = \exp\left(\frac{1}{2}i\mathbf{q}\mathbf{s}\right)M_{pp}(\mathbf{q}) + \exp\left(-\frac{1}{2}i\mathbf{q}\mathbf{s}\right)M_{pn}(\mathbf{q}) + \frac{i}{2\pi^{3/2}} \int \exp(i\mathbf{q}'\mathbf{s}) [M_{pp}(\mathbf{q}_1)M_{pn}(\mathbf{q}_2) + p \leftrightarrow n] d^2\mathbf{q}' \quad (2.5)$$

with on-shell elastic T-even P-even pN scattering amplitudes [12]

$$M_{pN}(\mathbf{p}, \mathbf{q}; \boldsymbol{\sigma}, \boldsymbol{\sigma}_N) = A_N + C_N \boldsymbol{\sigma}\hat{\mathbf{n}} + C'_N \boldsymbol{\sigma}_N\hat{\mathbf{n}} + B_N(\boldsymbol{\sigma}\hat{\mathbf{k}})(\boldsymbol{\sigma}_N\hat{\mathbf{k}}) + (G_N + H_N)(\boldsymbol{\sigma}\hat{\mathbf{q}})(\boldsymbol{\sigma}_N\hat{\mathbf{q}}) + (G_N - H_N)(\boldsymbol{\sigma}\hat{\mathbf{n}})(\boldsymbol{\sigma}_N\hat{\mathbf{n}}). \quad (2.6)$$

If T-invariance takes the place ( $T_{13} = T_{14} = T_{15} = T_{16} = T_{17} = T_{18} = 0$ ), then the following relations are valid for  $pd$ -elastic spin observables [13]

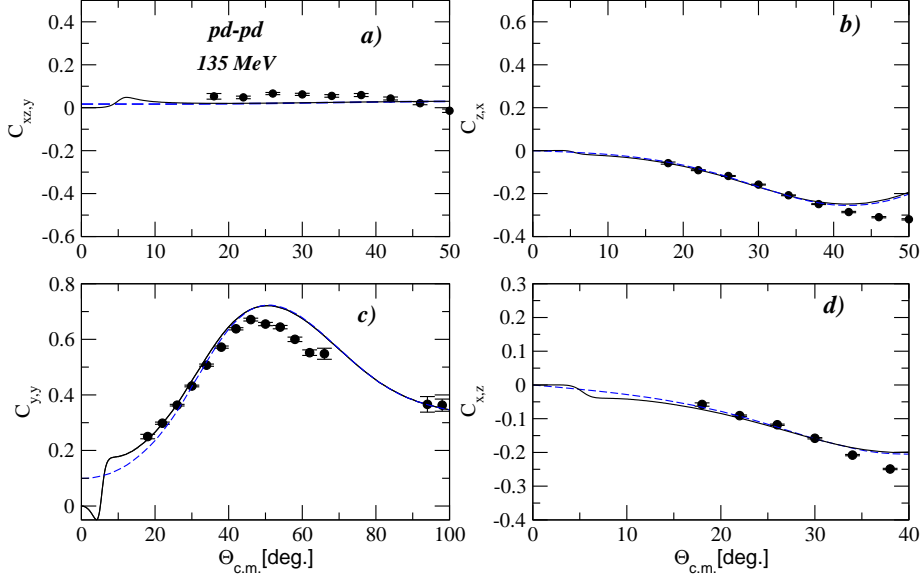
$$\begin{aligned} A_y^p &= P_y^p, \quad A_y^d = P_d, \\ K_x^z(\vec{p} \rightarrow \vec{p}) &= -K_z^x(\vec{p} \rightarrow \vec{p}), \\ K_x^z(\vec{p} \rightarrow \vec{d}) &= -K_z^x(\vec{d} \rightarrow \vec{p}), \\ K_x^z(\vec{d} \rightarrow \vec{p}) &= -K_z^x(\vec{p} \rightarrow \vec{d}), \\ K_x^z(\vec{d} \rightarrow \vec{d}) &= -K_z^x(\vec{d} \rightarrow \vec{d}), \end{aligned} \quad (2.7)$$

where  $A_y^p$  ( $A_y^d$ ) is the vector analyzing power for the proton (deuteron) and  $P_y^p$  ( $P_d$ ) is the polarization of the final proton (deuteron) in the case of unpolarized all initial particles. The relations (2.7) are violated if the T-odd interaction is included. We estimated the magnitudes of violation of these relations for the case of TVPC interaction given in Eq. (2.4) in the single scattering approximation, i.e. for the first two terms in the  $pd$  interaction operator (2.5). Within this approximation the  $g'$ -term gives zero contribution, because the matrix elements of isospin operator for this non-charge-exchange transitions are zero. The  $h$ - and  $g$ -terms give non-zero contribution due to non-zero amplitudes  $T_{15}$  and  $T_{16}$  given in Ref. [11]

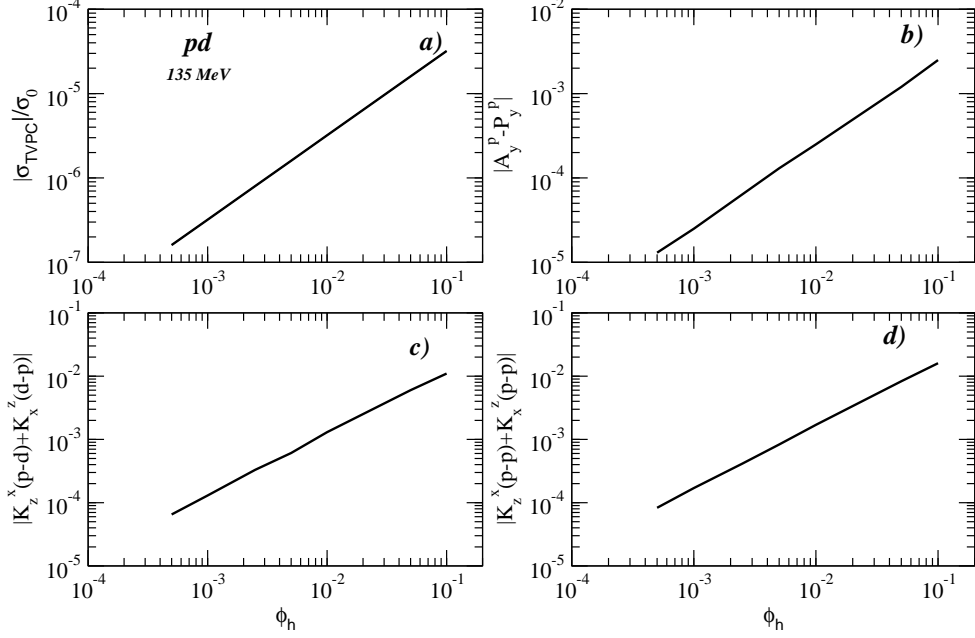
### 3. Results

The numerical results are obtained for the Cd Bonn deuteron wave function [20]. The  $pn$  scattering amplitudes (2.6) are taken from the SAID data base [21]. The coupling constants entering the  $h$ - and  $g$ - factors are unknown. Therefore, the estimations of TVPC effects were done in terms of this unknown constants. Thus, for the axial meson  $h_1$  this constant is the following dimensionless ratio  $\phi_h = \bar{g}_h/g_h$ , where  $\bar{g}$  ( $g_h$ ) is the TVPS (ordinary) coupling constant of the  $h_1$  meson and nucleon. Since the energy dependence of all TVPC effects for  $h$  term is similar to that for  $g$ -term we consider here only the  $h$ -term.

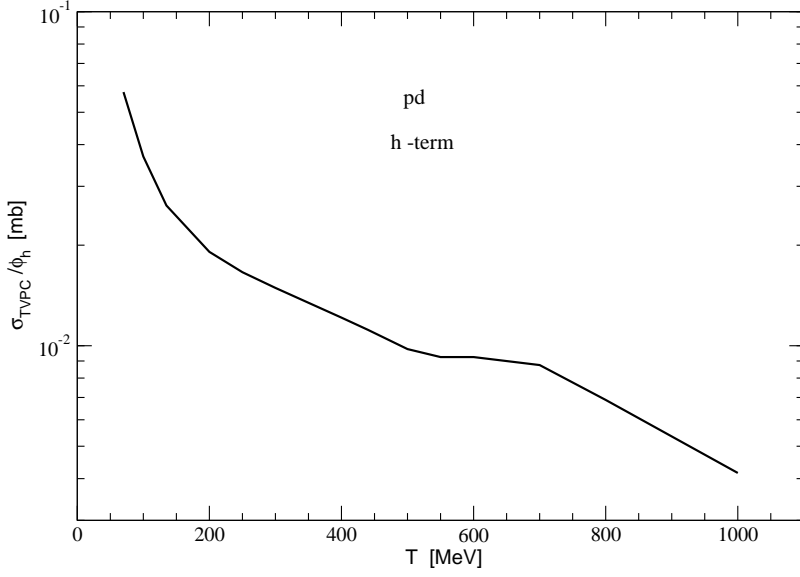
The results of our calculations for  $pd \rightarrow pd$  spin correlation coefficients at 135 MeV are shown in Fig. 1. Some other results for differential cross section and analyzing powers at 135 Mev and 250 MeV can be found in Ref. [15]. These results show the capability of the Glauber model at



**Figure 1:** Spin correlation coefficients  $C_{xz,y}$  (a),  $C_{z,x}$  (b),  $C_{y,y}$  (c),  $C_{x,z}$  (d) at 135 MeV versus the c.m.s. scattering angle calculated within the modified Glauber model [15] without (dashed lines) and with (full) Coulomb included in comparison with the data from [22].



**Figure 2:** The integrated TVPC cross section and maximal values of the differential TVPC signals calculated for the  $h$ -term versus the unknown coupling constant of the axial meson and nucleon  $\phi_h = \bar{g}_h/g_h$ : a) – the ratio of the TVPC integrated cross section  $\bar{\sigma}$  to the integrated unpolarized cross section  $\sigma_0$ , (b) –  $|A_y^p - P_y^p|$ , c) –  $|K_z^x(p \rightarrow d) + K_x^z(d \rightarrow p)|$ , (d) –  $|K_z^x(p \rightarrow p) + K_x^z(p \rightarrow p)|$ .



**Figure 3:** The TVPC cross section of the  $pd$  scattering for the  $h$ -term calculated in units of the unknown coupling constant of the axial meson and nucleon  $\phi_h = \bar{g}_h/g_h$  as a function of the beam energy.

100-250 MeV in case of pure strong interaction between nucleons. We find also that inclusion of Coulomb effects improves agreement with the data in forward hemisphere [15] as it was found in Faddeev calculations [23].

Therefore, the Glauber model with account of spin-dependence can be used as a theoretical basis for calculation of TVPC effects in  $pd$  scattering. In Fig. 2 we show the results of calculation of the ratio  $\tilde{\sigma}/\sigma_0$  as a function of the  $\phi_h$  constant. We show also the  $\phi_h$ -dependence of the values  $|A_y^p - P_y^p|$  and  $|K_z^x + K_x^z|$ , taken at their maximal values in forward hemisphere. This values demonstrate the deviation from the T-invariance relations given in Eqs. (2.7) as a function of the constant  $\phi_h$ . One can see that this dependence is linear for all observables under discussion.

The energy dependence of the null-test observable  $\tilde{\sigma}$  for the  $h$ -term is shown in Fig. 3. One can see that beam energy about 100 MeV is more preferable to search for TVPC effects in  $pd$  scattering than the  $\sim 1$  GeV region.

At the beam energy of 135MeV we obtain for the integrated hadronic cross sections defined by Eq.(2.3) the following results:  $\sigma_0 = 78.5\text{mb}$ ,  $\sigma_1 = 3.7\text{mb}$ , that gives the ratio  $r = \sigma_1/\sigma_0 = 0.047$ . The goal of TRIC experiment is to get the upper limit of TVPC effects at the level of  $R_T \leq 10^{-6}$ , where  $R_T = \tilde{\sigma}/\sigma_0$ . For non-zero vector polarization of the deuteron  $p_y^d$  the term connected with  $\sigma_1$  could lead to a false effect in measurement of  $\tilde{\sigma}$ . In order to suppress this effect one should assume  $p_y^d \sigma_1 / \tilde{\sigma} \sim 10^{-1}$  that at  $R_T \leq 10^{-6}$  and  $r = 0.047$  leads to restriction on the vector polarization of the deuteron  $p_y^d$  as  $p_y^d \leq 2 \times 10^{-6}$  [15].

#### 4. Conclusion

The null-test TVPC observable, which is not affected by final state interactions and does not depend on dynamics assumptions, will be measured in double-polarized  $pd$  scattering experiment

[10] at 135 MeV. We show that the Glauber model at these energies allows one to reasonably describe the differential cross section and spin observables of the  $pd$  elastic scattering in forward hemisphere. Therefore, using the generalized optical theorem one can reasonably estimate the integrated total cross section including the null-test observable (in units of unknown coupling constants of the TVPC NN interaction). The integrated  $pd$  cross sections  $\sigma_0, \sigma_1, \sigma_2, \sigma_3$  are calculated that gives a definite restriction on  $p_y^d$  in the planned experiment. Relations between differential observables caused by T-invariance are considered and their violation by TVPC effects is studied in comparison with the null-test observable.

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