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Finite Unified Theories: A successful example of Reduction of Couplings

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We apply the method of reduction of couplings in a Finite Unified Theory (FUT). The method consists on searching for renormalization group invariant relations among couplings/parameters of a renormalizable theory holding to all orders in perturbation theory. It has a remarkable predictive power since, at the unification scale, it leads to relations between gauge and Yukawa couplings in the dimensionless sectors and relations involving the trilinear terms and the Yukawa couplings, as well as a sum rule among the scalar masses and the unified gaugino mass in the soft breaking sector. We predict, in a FUT model, the masses of the top and bottom quarks and the light Higgs in remarkable agreement with the experiment. Furthermore we also predict the masses of the other Higgses, as well as the supersymmetric spectrum, both being in very confortable agreement with the LHC bounds on Higgs and supersymmetric particles.

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1. Introduction

The Higgs boson was the last missing particle from the Standard Model (SM) to be discovered [1–4]. However, since no evidence of new physics has been found yet, the pressure to find theories that explain the number of open questions left by the SM, which moreover do not change the low energy observed picture, is growing.

One of the most celebrated strategies to reduce the number of free parameters is to consider that the world at high energies is more symmetric. Supersymmetric Grand Unified Theories (SUSY GUTs) are representative examples of this. An alternative, and possibly complementary way, is to search for valid functional all-loop renormalization group invariant (RGI) relations among couplings, achieving reduction of couplings in this way. In this approach, the RGIs which preserve perturbative renormalizability [5, 6] just below the Planck, are maintained down to the unification scale [7–15], making it possible to apply this method to conventional SUSY GUTs. In this way it is possible unify the gauge and Yukawa sectors of the theory [7–16].

2. The Method of Reduction of Couplings

In this section we will briefly outline the reduction of couplings method. Any RGI relation among couplings (i.e. which does not depend on the renormalization scale μ explicitly) can be expressed, in the implicit form $\Phi(g_1, \cdots, g_A) = \text{const.}$, which has to satisfy the partial differential equation (PDE)

$$\frac{d\Phi}{dt} = \sum_{a=1}^{A} \frac{\partial \Phi}{\partial g_a} \frac{dg_a}{dt} = \sum_{a=1}^{A} \frac{\partial \Phi}{\partial g_a} \beta_a = \vec{\nabla}\Phi \cdot \vec{\beta} = 0, \tag{2.1}$$

where $t = \ln \mu$ and β_a is the β-function of g_a. This PDE is equivalent to a set of ordinary differential equations, the so-called reduction equations (REs) [5, 6, 17],

$$\beta_g \frac{dg_a}{dg} = \beta_a \ , \ a = 1, \cdots, A \ , \tag{2.2}$$

where g and β_g are the primary coupling and its β-function, and the counting on a does not include g. Since maximally $(A-1)$ independent RGI "constraints" in the A-dimensional space of couplings can be imposed by the Φ_a's, one could in principle express all the couplings in terms of a single coupling g. The strongest requirement in the search for RGI relations is to demand power series solutions to the REs,

$$g_a = \sum_{n=0} \rho_a^{(n)} g^{2n+1} \ , \tag{2.3}$$

which formally preserve perturbative renormalizability. Remarkably, the uniqueness of such power series solutions can be decided already at the one-loop level [5, 6, 17].

Searching for a power series solution of the form (2.3) to the REs (2.2) is justified since various couplings in supersymmetric theories have the same asymptotic behaviour, thus one can rely that keeping only the first terms in the expansion is a good approximation in realistic applications.

3. Reduction of Couplings in Soft Breaking Terms

The method of reducing the dimensionless couplings was extended [13, 18, 19] to the soft supersymmetry breaking (SSB) dimensionful parameters of $N = 1$ supersymmetric theories. In addition it was found [20, 21] that RGI SSB scalar masses in Gauge-Yukawa unified models satisfy a universal sum rule.

Consider the superpotential given by

$$W = \frac{1}{2}\,\mu^{ij}\,\Phi_i\,\Phi_j + \frac{1}{6}\,C^{ijk}\,\Phi_i\,\Phi_j\,\Phi_k\;, \tag{3.1}$$

where μ^{ij} (the mass terms) and C^{ijk} (the Yukawa couplings) are gauge invariant tensors and the matter field Φ_i transforms according to the irreducible representation R_i of the gauge group G. The Lagrangian for SSB terms is

$$-\mathscr{L}_{\rm SSB} = \frac{1}{6}\,h^{ijk}\,\phi_i\phi_j\phi_k + \frac{1}{2}\,b^{ij}\,\phi_i\phi_j + \frac{1}{2}\,(m^2)^{j}_{i}\,\phi^{*\,i}\phi_j + \frac{1}{2}\,M\,\lambda\lambda + \text{H.c.}, \tag{3.2}$$

where the ϕ_i are the scalar parts of the chiral superfields Φ_i, λ are the gauginos and M their unified mass, h^{ijk} and b^{ij} are the trilinear and bilinear dimensionful couplings respectively, and $(m^2)^{j}_{i}$ the soft scalars masses.

Let us recall that the one-loop β-function of the gauge coupling g is given by [22–26]

$$\beta^{(1)}_g = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\,[\sum_i T(R_i) - 3\,C_2(G)]\;, \tag{3.3}$$

where $C_2(G)$ is the quadratic Casimir of the adjoint representation of the associated gauge group G. $T(R)$ is given by the relation $\mathrm{Tr}[T^aT^b] = T(R)\delta^{ab}$, where T^a are the generators of the group in the appropriate representation. Similarly the β-functions of C_{ijk}, by virtue of the non-renormalization theorem, are related to the anomalous dimension matrix γ^i_j of the chiral superfields as:

$$\beta_C^{ijk} = \frac{dC_{ijk}}{dt} \;=\; C_{ijl}\,\gamma^l_k + C_{ikl}\,\gamma^l_j + C_{jkl}\,\gamma^l_i\;. \tag{3.4}$$

At one-loop level the anomalous dimension, $\gamma^{(1)}{}^{i}_{j}$ of the chiral superfield is [22–26]

$$\gamma^{(1)}{}^{i}_{j} = \frac{1}{32\pi^2}\,[C^{ikl}\,C_{jkl} - 2\,g^2\,C_2(R)\delta^i_j], \tag{3.5}$$

where $C_2(R)$ is the quadratic Casimir of the representation R_i, and $C^{ijk} = C^*_{ijk}$. Then, the $N = 1$ non-renormalization theorem [27–29] ensures there are no extra mass and cubic-interaction-term renormalizations, implying that the β-functions of C_{ijk} can be expressed as linear combinations of the anomalous dimensions γ^i_j.

Here we assume that the reduction equations admit power series solutions of the form

$$C^{ijk} = g\,\sum_{n=0}\rho^{ijk}_{(n)}g^{2n}\;. \tag{3.6}$$

In order to obtain higher-loop results instead of knowledge of explicit β-functions, which anyway are known only up to two-loops, relations among β-functions are required.

The progress made using the spurion technique, [29–33] leads to all-loop relations among SSB β-functions [34–39]. The assumption, following [35], that the relation among couplings

$$h^{ijk} = -M(C^{ijk})' \equiv -M\frac{dC^{ijk}(g)}{d\ln g} \,, \tag{3.7}$$

is RGI and furthermore, the use the all-loop gauge β-function of Novikov *et al.* [40,41]

$$\beta_g^{\rm NSVZ} = \frac{g^3}{16\pi^2}\left[\frac{\sum_l T(R_l)(1-\gamma_l/2) - 3C_2(G)}{1-g^2C_2(G)/8\pi^2}\right] \,, \tag{3.8}$$

lead to the all-loop RGI sum rule [42] (assuming $(m^2)^i{}_j = m_j^2\delta^i_j$),

$$\begin{aligned} m_i^2+m_j^2+m_k^2 = |M|^2\left\{\frac{1}{1-g^2C_2(G)/(8\pi^2)}\frac{d\ln C^{ijk}}{d\ln g} + \frac{1}{2}\frac{d^2\ln C^{ijk}}{d(\ln g)^2}\right\} \\ +\sum_l \frac{m_l^2 T(R_l)}{C_2(G)-8\pi^2/g^2}\frac{d\ln C^{ijk}}{d\ln g} \,. \end{aligned} \tag{3.9}$$

The all-loop results on the SSB β-functions lead to all-loop RGI relations (see e.g. [43]). If we assume:
(a) the existence of a RGI surfaces on which $C = C(g)$, or equivalently that

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g} \tag{3.10}$$

holds, i.e. reduction of couplings is possible, and
(b) the existence of a RGI surface on which

$$h^{ijk} = -M\frac{dC(g)^{ijk}}{d\ln g} \tag{3.11}$$

holds too in all-orders, then one can prove that the following relations are RGI to all-loops [44, 45] (note that in the above assumptions (a) and (b) we do not rely on specific solutions to these equations)

$$M = M_0\,\frac{\beta_g}{g}, \tag{3.12}$$

$$h^{ijk} = -M_0\,\beta_C^{ijk}, \tag{3.13}$$

$$b^{ij} = -M_0\,\beta_\mu^{ij}, \tag{3.14}$$

$$(m^2)^i{}_j = \frac{1}{2}\,|M_0|^2\,\mu\frac{d\gamma^i{}_j}{d\mu}, \tag{3.15}$$

where M_0 is an arbitrary reference mass scale to be specified shortly.

Finally we would like to emphasize that under the same assumptions (a) and (b) the sum rule given in Eq.(3.9) has been proven [42] to be all-loop RGI, which gives us a generalization of Eq.(3.15) to be applied in considerations of non-universal soft scalar masses, which are necessary in many cases including the MSSM.

As it was emphasized in ref [44] the set of the all-loop RGI relations (3.12)-(3.15) is the one obtained in the *Anomaly Mediated SB Scenario* [46,47], by fixing the M_0 to be $m_{3/2}$, which is the natural scale in the supergravity framework. A final remark concerns the resolution of the fatal problem of the anomaly induced scenario in the supergravity framework, which is here solved thanks to the sum rule (3.9). Other solutions have been provided by introducing Fayet-Iliopoulos terms [48].

4. A Successful Application of the Reduction of Couplings Method: Finiteness

Field theories are said to be finite if their beta functions vanish. Consider a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g. Consider the superpotential Eq. (3.1) together with the soft supersymmetry breaking Lagrangian Eq. (3.2). All the one-loop β-functions of the theory vanish if the β-function of the gauge coupling $\beta_g^{(1)}$ and the anomalous dimensions of the Yukawa couplings $\gamma_i^{j(1)}$ vanish, giving the following relations

$$\sum_i T(R_i) = 3C_2(G)\,, \qquad \frac{1}{2}C_{ipq}C^{jpq} = 2\delta_i^j g^2 C_2(R)\,. \tag{4.1}$$

These two conditions are also enough to guarantee two-loop finiteness [49]. A striking fact is the existence of a theorem [50–52], that guarantees the vanishing of the β-functions to all-orders in perturbation theory. This requires that, in addition to the one-loop finiteness conditions (4.1), the Yukawa couplings are reduced in favour of the gauge coupling to all-orders (see [53] for details). Alternatively, similar results can be obtained [54–56] using an analysis of the all-loop NSVZ gauge beta-function [40,57].

Since we would like to consider only finite theories here, we assume that the gauge group is a simple group and the one-loop β-function of the gauge coupling g vanishes.[1] We also assume that the reduction equations admit power series solutions of the form Eq. (3.6). According to the finiteness theorem of ref. [50–52,60], the theory is then finite to all orders in perturbation theory, if, among others, the one-loop anomalous dimensions $\gamma_i^{j(1)}$ vanish. The one- and two-loop finiteness for h^{ijk} can be achieved through the relation [61]

$$h^{ijk} = -MC^{ijk} + \cdots = -M\rho^{ijk}_{(0)}\,g + O(g^5)\,, \tag{4.2}$$

where ... stand for higher order terms.

In addition it was found that the RGI SSB scalar masses in Gauge-Yukawa unified models satisfy a universal sum rule at one-loop [20]. This result was generalized to two-loops for finite theories [21], and then to all-loops for general Gauge-Yukawa and finite unified theories [42]. From these latter results, the following soft scalar-mass sum rule is found [21]

$$\frac{(\,m_i^2 + m_j^2 + m_k^2\,)}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2}\Delta^{(2)} + O(g^4)\,, \tag{4.3}$$

where $m^2_{i,j,k}$ are the scalar masses and $\Delta^{(2)}$ is the two-loop correction which vanishes for the universal choice, i.e. when all the soft scalar masses are the same at the unification point, as well as in the model considered here.

[1] Realistic Finite Unified Theories based on product gauge groups, where the finiteness implies three generations of matter, have also been studied [58,59].

PoS(CORFU2014)092

4.1 An $SU(5)$ Finite Unified Theory

In this section we show how to apply the reduction of couplings method in Finite Unified Theories. We will apply it only to the third generation of fermions and in the soft supersymmetry breaking terms. After the reduction of couplings takes place, we are left with relations at the unification scale for the Yukawa couplings of the quarks in terms of the gauge coupling according to Eq. (3.6), for the trlininear terms in terms of the Yukawa couplings and the unified gaugino mass Eq. (3.11), and a sum rule for the soft scalar masses also proportional to the unified gaugino mass Eq. (3.9), as applied in each model.

We examine an all-loop Finite Unified theory with $SU(5)$ as gauge group, where the reduction of couplings has been applied to the third generation of quarks and leptons. The particle content of the model we will study, which we denote **FUT** consists of the following supermultiplets: three $(\overline{\mathbf{5}}+\mathbf{10})$, needed for each of the three generations of quarks and leptons, four $(\overline{\mathbf{5}}+\mathbf{5})$ and one $\mathbf{24}$ considered as Higgs supermultiplets. When the gauge group of the finite GUT is broken the theory is no longer finite, and we will assume that we are left with the MSSM [7, 9–11, 14].

A predictive Gauge-Yukawa unified $SU(5)$ model which is finite to all orders, in addition to the requirements mentioned already, should also have the following properties:

1. One-loop anomalous dimensions are diagonal, i.e., $\gamma_i^{(1)\,j} \propto \delta_i^j$.
2. Three fermion generations, in the irreducible representations $\overline{\mathbf{5}}_i, \mathbf{10}_i$ $(i=1,2,3)$, which obviously should not couple to the adjoint $\mathbf{24}$.
3. The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and anti-quintet, which couple to the third generation.

After the reduction of couplings the symmetry is enhanced, leading to the following superpotential [62]

$$\begin{aligned} W \;=\; & \sum_{i=1}^{3}\Big[\,\frac{1}{2}g_i^u\,\mathbf{10}_i\mathbf{10}_i H_i + g_i^d\,\mathbf{10}_i\overline{\mathbf{5}}_i\overline{H}_i\,\Big] + g_{23}^u\,\mathbf{10}_2\mathbf{10}_3 H_4 \\ & + g_{23}^d\,\mathbf{10}_2\overline{\mathbf{5}}_3\,\overline{H}_4 + g_{32}^d\,\mathbf{10}_3\overline{\mathbf{5}}_2\,\overline{H}_4 + g_2^f\,H_2\,\mathbf{24}\,\overline{H}_2 + g_3^f\,H_3\,\mathbf{24}\,\overline{H}_3 + \frac{g^\lambda}{3}\,(\mathbf{24})^3\;. \end{aligned} \tag{4.4}$$

The non-degenerate and isolated solutions to $\gamma_i^{(1)}=0$ give us:

$$\begin{aligned} & (g_1^u)^2=\frac{8}{5}\,g^2\;,\;(g_1^d)^2=\frac{6}{5}\,g^2\;,\;(g_2^u)^2=(g_3^u)^2=\frac{4}{5}\,g^2\;, \\ & (g_2^d)^2=(g_3^d)^2=\frac{3}{5}\,g^2\;,\;(g_{23}^u)^2=\frac{4}{5}\,g^2\;,\;(g_{23}^d)^2=(g_{32}^d)^2=\frac{3}{5}\,g^2\;, \\ & (g^\lambda)^2=\frac{15}{7}g^2\;,\;(g_2^f)^2=(g_3^f)^2=\frac{1}{2}\,g^2\;,\;(g_1^f)^2=0\;,\;(g_4^f)^2=0\;, \end{aligned} \tag{4.5}$$

and from the sum rule we obtain:

$$m_{H_u}^2+2m_{\mathbf{10}}^2=M^2\;,\qquad m_{H_d}^2-2m_{\mathbf{10}}^2=-\frac{M^2}{3}\;,\qquad m_{\overline{\mathbf{5}}}^2+3m_{\mathbf{10}}^2=\frac{4M^2}{3}\;, \tag{4.6}$$

i.e., in this case we have only two free parameters $m_{\mathbf{10}}$ and M for the dimensionful sector.

As already mentioned, after the $SU(5)$ gauge symmetry breaking we assume we have the MSSM, i.e. only two Higgs doublets. This can be achieved by introducing appropriate mass terms that allow to perform a rotation of the Higgs sector [7–11, 63–65], in such a way that only one pair of Higgs doublets, coupled mostly to the third family, remains light and acquire vacuum expectation values. To avoid fast proton decay the usual fine tuning to achieve doublet-triplet splitting is performed, although the mechanism is not identical to minimal $SU(5)$, since we have an extended Higgs sector.

Thus, after the gauge symmetry of the GUT theory is broken we are left with the MSSM, with the boundary conditions for the third family given by the finiteness conditions, while the other two families are not restricted.

4.2 Predictions of the Finite Model

Since the gauge symmetry is spontaneously broken below $M_{\rm GUT}$, the finiteness conditions do not restrict the renormalization properties at low energies, and all it remains are boundary conditions on the gauge and Yukawa couplings (4.5), the $h = -MC$ (3.7) relation, and the soft scalar-mass sum rule at $M_{\rm GUT}$. With these boundary conditions we run the MSSM RGEs down to the SUSY scale, which we take to be the geometrical average of the stop masses, and then run the SM RGEs down to the electroweak scale (M_Z), where we compare with the experimental values of the third generation quark masses. The RGEs are taken at two-loops for the gauge and Yukawa couplings and at one-loop for the soft breaking parameters. We let M_U and $|\mu|$ at the unification scale to vary between ~ 1 TeV ~ 11 TeV, for the two possible signs of μ. In evaluating the τ and bottom masses we have taken into account the one-loop radiative corrections that come from the SUSY breaking [66, 67]; in particular for large $\tan\beta$ they can give sizeable contributions to the bottom quark mass.

We use the experimental value of the top quark pole mass as [68][2]

$$m_t^{\rm exp} = (173.2 \pm 0.9)\ \text{GeV}\ . \tag{4.7}$$

The bottom mass is calculated at M_Z to avoid uncertainties that come from running down to the pole mass and, as previously mentioned, the SUSY radiative corrections both to the tau and the bottom quark masses have been taken into account [70]

$$m_b(M_Z) = (2.83 \pm 0.10)\ \text{GeV}. \tag{4.8}$$

In Fig.1 we show the **FUT** predictions for m_t and $m_b(M_Z)$ as a function of the unified gaugino mass M, for the two cases $\mu < 0$ and $\mu > 0$. The bounds on the $m_b(M_Z)$ and the m_t mass clearly single out $\mu < 0$, as the solution most compatible with these experimental constraints [71, 72].

We now analyze the impact of further low-energy observables on the model **FUT** with $\mu < 0$. For the lightest Higgs mass prediction we used the code `FeynHiggs` [73–77] where the prediction for M_h of **FUT** with $\mu < 0$ is shown in Fig. 2. The lightest Higgs mass ranges in

$$M_h \sim 124 - 132\ \text{GeV}\ , \tag{4.9}$$

[2] We did not include the latest LHC/Tevatron combination, leading to $m_t^{\rm exp} = (173.34 \pm 0.76)$ GeV [69], which would have a negligible impact on our analysis.

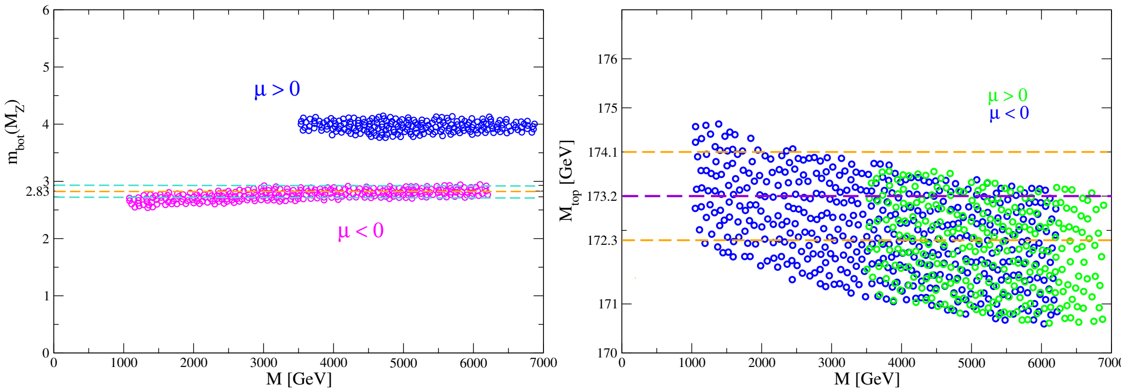

Figure 1: The bottom quark mass at the Z boson scale (left) and top quark pole mass (right) are shown as function of M, the unified gaugino mass.

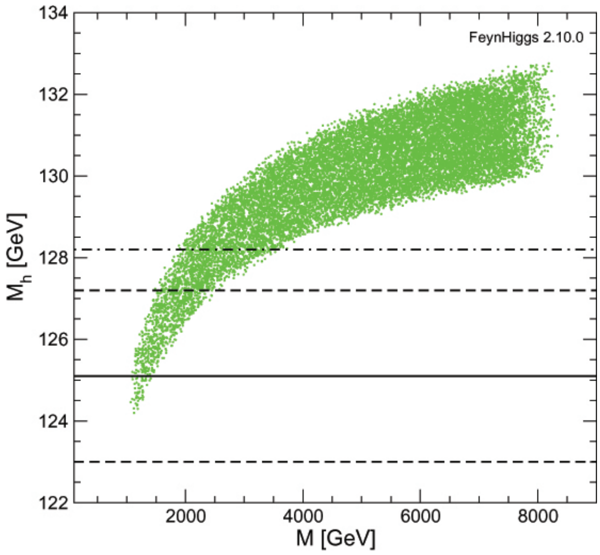

Figure 2: The lightest Higgs mass, M_h, as function of M for the model **FUT** with $\mu < 0$.

where the uncertainty comes from variations of the soft scalar masses. To this value one has to add at least ± 2 GeV coming from unkonwn higher order corrections [77]. We have also included a small variation, due to threshold corrections at the GUT scale, of up to 5% of the FUT boundary conditions.

We now impose the constraint of the lightest Higgs boson mass on our results, which is the value of the Higgs mass

$$M_h \sim 125.1 \pm 3.1 \pm 2.1 \text{ GeV} \,, \tag{4.10}$$

where ± 3.1 GeV corresponds to the current theory and experimental uncertainty, and ± 2.1 GeV to a reduced theory uncertainty in the future. We find that constraining the allowed values of the Higgs mass puts a limit on the allowed values of the unified gaugino mass, as can be seen from Fig. 2. The dashed-dotted lines indicate the current uncertainty, placing an upper bound of $M \lesssim 3.8$ TeV.

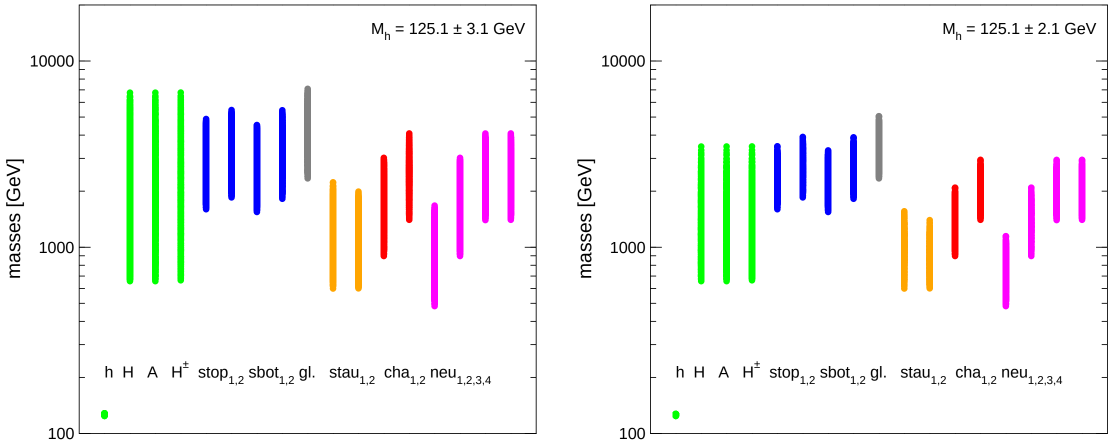

Figure 3: The left (right) plot shows the spectrum after imposing the constraint $M_h = 125.1 \pm 3.1\,(2.1)$ GeV. The light (green) points are the various Higgs boson masses, the dark (blue) points following are the two scalar top and bottom masses, the gray ones are the gluino masses, then come the scalar tau masses in orange (light gray), the darker (red) points to the right are the two chargino masses followed by the lighter shaded (pink) points indicating the neutralino masses.

The anticipated future uncertainty (keeping the current central value) would lower this bound to $M \lesssim 2.6$ TeV. These upper bounds yield restrictions to the corresponding SUSY spectrum.

The full particle spectrum of model **FUT** with $\mu < 0$, compliant with quark mass constraints and the lightest Higgs-boson mass is shown in Fig. 3. It can be seen from the figure that the lightest observable SUSY particle (LOSP) is the light scalar tau. In the left (right) plot we impose $M_h = 125.1 \pm 3.1\,(2.1)$ GeV. Including the Higgs mass constraints in general favours the lower part of the SUSY particle mass spectra [78–81], however in particular very heavy colored SUSY particles are favored, in agreement with the non-observation of those particles at the LHC [82–84]. Going to the anticipated future theory uncertainty of M_h (as shown in the right plot of Fig. 3) still permits SUSY masses which would remain unobservable at the LHC, the ILC or CLIC. On the other hand, large parts of the allowed spectrum of the lighter scalar tau or the lighter neutralinos might be accessible at CLIC with $\sqrt{s} = 3$ TeV.

5. Conclusions

One of the most interesting realisations of the idea that the physics beyond the SM can be understood through the addition of symmetries is provided by the MSSM, assuming further a GUT. However successful, it is not possible to reduce further the number of free parameters of the SM this way.

A new interesting possibility towards reducing the free parameters of a theory has been put forward in refs. [5,6], which consists on a systematic search on the RGI relations among couplings.

This method might lead to further symmetry, however its scope is much wider. After several trials it seems that the basic idea found very nice realisations in a Finite Unified Theory. In this case one is searching for RGI relations among couplings holding beyond the unification scale, which morever guarantee finiteness to all-orders in perturbation theory. Finiteness is related to some fundamental developments in Theoretical Particle Physics based on reconsiderations of the problem of divergencies and serious attempts to solve it. They include the motivation and construction of string and noncommutative theories, as well as $N = 4$ supersymmetric field theories [85, 86], $N = 8$ supergravity [87–91] and the AdS/CFT correspondence [92]. It is a thoroughly fascinating fact that many interesting ideas that have survived various theoretical and phenomenological tests, as well as the solution to the UV divergencies problem, find a common ground in the framework of $N = 1$ Finite Unified Theories, which we discussed here. From the theoretical side they solve the problem of UV divergencies in a minimal way. On the phenomenological, the celebrated success of predicting the top-quark mass [7, 9] is now extended to the predictions of the Higgs masses and the supersymmetric spectrum of the MSSM, which so far have been confronted very successfully with the discovery of the Higgs and and the bounds on the s-spectrum at the LHC.

Important improvements in the analysis are expected from progress on the theory side, in particular in an improved calculation of the light Higgs boson mass. The corrections considered in [73] not only introduce a shift in M_h (which should to some extent be covered by the estimate of theory uncertainties). They also reduce the theory uncertainties, see [73, 93], and in this way refine the selected model points, leading to a sharper prediction of the allowed spectrum. One can hope that with even more higher-order corrections included in the M_h calculation an uncertainty below the 0.5 GeV level can be reached.

The other important improvements in the future will be the continuing searches for SUSY particles at collider experiments. The LHC has restarted in spring 2015 with an increased center-of-mass enery of $\sqrt{s} \lesssim 13$ TeV, largely extending its SUSY search reach. The lower parts of the currently allowed/predicted colored SUSY spectra will be tested in this way. For the electroweak particles, on the other hand, e^+e^- colliders might be the better option. The ILC, operating at $\sqrt{s} \lesssim 1$ TeV, has only a limited potential for our model spectra. Going to higher energies, $\sqrt{s} \lesssim 3$ TeV, that might be realized at CLIC, large parts of the predicted electroweak model spectra can be covered.

All spectra, however, (at least with the current calculation of M_h and its corresponding uncertainty), contain parameter regions that will escape the searches at the LHC, the ILC and CLIC. In this case we would remain with a light Higgs boson in the decoupling limit, i.e. would be undistinguishable from a SM Higgs boson. The only hope to overcome this situation is that an improved M_h calculation would cut away such high spectra.

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