An Introduction to Supergravity

D.G. Cerdeño and C. Muñoz
Departamento de Física Teórica C–XI and Instituto de Física Teórica C–XVI
Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
E-mail: dgarcia@delta.ft.uam.es, carlos.munnoz@uam.es

Abstract: In these lectures we give an elementary introduction to supergravity. The role it plays in the context of superstring theory is emphasized. The level is suitable for postgraduate students and non–specialist researches in the subject. The outline is the following:

1. From general relativity to higher dimensional Supergravity

Experimental observations confirm that the standard model of particle physics works outstandingly well. However, if we believe that it should be embedded within a more fundamental theory, we are faced with grave theoretical problems. For example, in the context of a grand unified theory the characteristic scale is of order $M_{GUT} \approx 10^{15}$ GeV which gives rise to the *gauge hierarchy problem.* There is no symmetry protecting the masses of the scalar particles, Higgses, against quadratic divergences in perturbation theory. Therefore they will be proportional to the huge scale M_{GUT}. This problem of naturalness, to stabilize the electroweak scale M_W against quantum corrections $M_W << M_{GUT}$, may be solved postulating a new symmetry which relates bosons and fermions introducing new particles, the so–called *supersymmetry.* The scalar masses and the masses of their superpartners, the fermions, are related and as a consequence, only a logarithmic divergence in scalar masses is left. In diagrammatic language, the dangerous diagrams of standard model particles are canceled with new ones which are present due to the existence of the additional partners and couplings. This is shown schematically in Fig. 1.

On the other hand, gravity, the fourth interaction in nature, is not included in the standard model. Nowadays we know that the correct description of nature involves *quantum field theories.* This is of course the case of the standard model, whose finishing touches were put around 1974. It describes strong, weak and electromagnetic interactions using the internal (gauge) symmetry $SU(3) \times SU(2) \times U(1)$. However, the the-

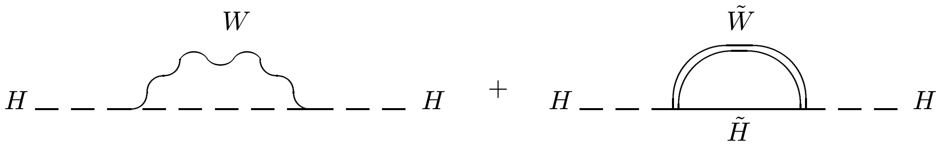

Figure 1: The quadratic divergence due to the loop of standard–model bosons is cancel with the loop of fermionic superpartners which has opposite sign.

ory describing the gravitational interaction, general relativity, which was completed in 1915, is a *classical theory.* In this sense both theories seem to be completely disconnected as is schematically shown in Fig. 2. The question then is to know if

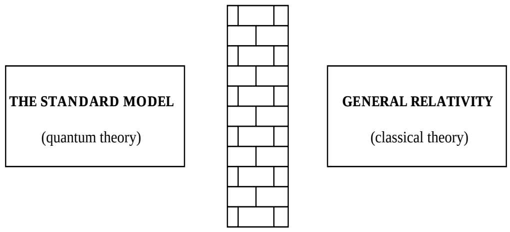

Figure 2: The standard model and general relativity are disconnected. The former is a quantum field theory whereas the latter is still a classical theory.

we will be able to quantize general relativity and to unify it with the standard model. As we will see in the rest of the paper, an important role in the answer to this question is played again by supersymmetry, in particular by its local version, *supergravity.*

To carry out this project a basic ingredient is the *graviton.* This is an elementary particle with spin 2 which is an excitation of the gravitational field similarly to the photon which is an excitation of the electromagnetic field. The charge associated with gravity is mass and therefore energy since mass and energy are equivalent in relativity. It is then natural to extend the approach of quantum field theory to gravity. In particular, the use of Feynman diagrams as in the example shown in Fig. 3. The problem with this approach is that the gravitational interaction is extremely weak since the coupling constant, the Newtonian gravitational constant G_N, is

$$G_N = \frac{1}{M_{Planck}^2} \approx 10^{-38}\ (GeV)^{-2}\ . \qquad (1.1)$$

Thus experimental verification of quantum gravity at low energies ($\approx M_W$) in accelerators seems to be unlikely. For example, the contribution of the diagram of Fig. 3 to electron–electron scattering is negligible with respect to the one of the diagram shown in Fig. 4, since the relation between the electromagnetic and effective gravitational coupling constants is

$$\alpha_{e.m.} = \frac{e^2}{4\pi} \approx 10^{-2} >> \alpha_G = \frac{E^2}{M_{Planck}^2} \approx 10^{-34}\ . \qquad (1.2)$$

Note that the energy E must be included to have a dimensionless scattering amplitude.

Another process that we might study with the hope of detecting quantum corrections experimentally is the gravitational light bending. The first diagram in Fig. 5 reproduces the classical result for the deflected photon. As mentioned above, mass and energy are equivalent in relativity and since all particles have energy, gravity couples with everything, and in particular with photons. Unfortunately, the lowest–order quantum correction with a loop of electrons shown in the second diagram, is too small to be detected in present solar light–bending experiments.

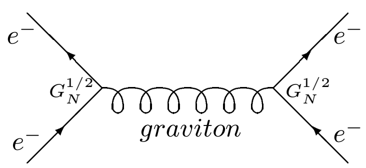

Figure 3: Electron–electron scattering through the exchange of a graviton.

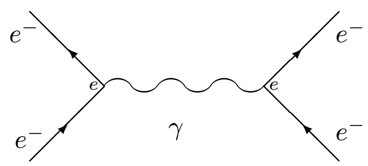

Figure 4: Electron–electron scattering through the exchange of a photon (γ).

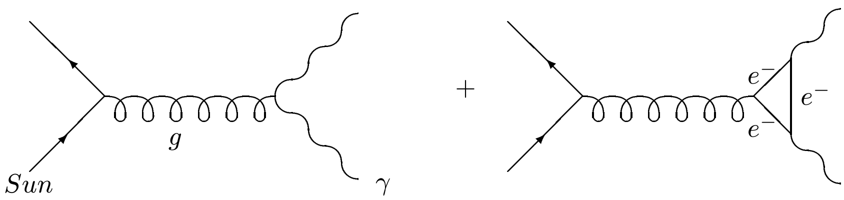

Figure 5: Lowest–order gravitational light bending due to the Sun and a quantum correction.

In conclusion, even if a consistent quantum theory of gravity is ever built, an issue that we will discuss in this section below, its experimental verification is apparently unlikely. Fortunately, a possible candidate to quantize gravity, *supergravity*, gives rise to low–energy signals which could be detected. This will be discussed in section 2.

Before entering into details, a few words about bibliography. Although we discussed quantum gravity above and we will continue discussing it in the next subsection, it is not the main topic of these lectures. A simple and interesting introduction can be found in refs. [1]–[3]. There are excellent books and reviews of supergravity. We quote several of them in the references. In particular, refs. [4]–[8] focus mainly on theory and refs. [9]–[11] focus mainly on phenomenology. Refs. [12] and [13] although are not so exhaustive as the above mentioned, are quite recent and introduce supergravity from a modern perspective. Other references interesting to understand specific issues will be mentioned in the text.

1.1 Quantum gravity

Let us recall first the relation between global and local symmetries in quantum field theory using the Noether procedure. Consider for instance a free massless spin 1/2 field. Its action

$$S = i \int d^4x \ \bar{\psi}\gamma^\mu \partial_\mu \psi \ , \qquad (1.3)$$

where $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$, is clearly invariant under the global transformation

$$\psi \to e^{-i\varepsilon} \ \psi \qquad (1.4)$$

with ε a constant phase. However when the transformation is local

$$\psi \to e^{-i\varepsilon(x)} \ \psi \ , \qquad (1.5)$$

where ε now depends on the space–time coordinates, the action is invariant only if we add a spin 1 gauge field A_μ. This gives rise precisely to Quantum electrodynamics (QED). Therefore, spin 1 fields correspond to generalizing internal (i.e. non–Lorentz) symmetries.

Similarly, a spin 2 appears when space–time symmetries, global Poincare invariance,

$$x^\mu \to \Lambda^\mu_\nu \ x^\nu + a^\mu \ , \qquad (1.6)$$

are made *local* in space–time, i.e. general coordinate transformations:

$$x^\mu \to x'^\mu(x) \ . \qquad (1.7)$$

Let us consider for example the action of a scalar field in flat space–time:

$$S = \int d^4x \left[\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \ , \qquad (1.8)$$

where $\eta_{\mu\nu}$ is the Minkowski metric. This action is invariant under (1.7) only if we add a spin 2 field, the graviton:

$$S = \int d^4x\sqrt{-detg_{\mu\nu}} \left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \ . \qquad (1.9)$$

Adding the usual Einstein piece,

$$S \ = \ -\frac{1}{2k^2}\int d^4x\sqrt{-detg_{\mu\nu}} \ R \ , \qquad (1.10)$$

one obtains the complete action. Here k is the gravitational coupling constant

$$k = \sqrt{8\pi G_N} = \frac{\sqrt{8\pi}}{M_{Planck}} \equiv \frac{1}{M_P} \qquad (1.11)$$

with $M_P = 2.4 \times 10^{18}$ GeV the so–called reduced Planck mass.

Once we know the action, we may proceed to compute various processes. However, as is well

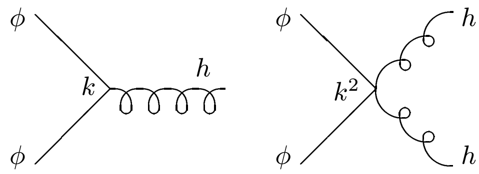

Figure 6: Graviton–scalar interactions

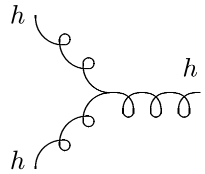

Figure 7: Graviton–graviton vertex.

known, field equations are *non* linear. One possible way to solve the problem consists of introducing the expansion

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + k\ h_{\mu\nu}(x)\ , \tag{1.12}$$

where $h_{\mu\nu}(x)$ measures the *deviation* of the space–time from flat Minkowski space. We have to introduce the constant k in front of it since its quanta should have mass dimensions 1 as appropriate to a bosonic field describing the graviton. Recalling that perturbation theory works outstandingly well in the standard model, and taking into account that gravity is much weaker, the use of *perturbations* should be appropriate. For example, using (1.12) the free part of the action (1.9) can be written as

$$\begin{aligned}&\int d^4x \sqrt{-det g_{\mu\nu}}\ \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = \\ &\int d^4x \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \\ &-k\int d^4x \frac{1}{2} h^{\mu\nu}\overline{\partial_\mu\phi\partial_\nu\phi} \\ &-k^2\int d^4x \left(\frac{1}{8}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi h^\sigma_\rho \bar{h}^\rho_\sigma \right. \\ &\left. -\frac{1}{2}h^{\mu\rho}\bar{h}^\nu_\rho\partial_\mu\phi\partial_\nu\phi\right) + \ldots\ ,\end{aligned} \tag{1.13}$$

where the "bar" operation on an arbitrary second rank tensor is defined by $\bar{X}_{\mu\nu} = X_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}X^\sigma_\sigma$. The first term in the right–hand side of (1.13) is the usual action for free scalar fields in flat space–time (see (1.8)). The second and third ones correspond to the diagrams such as are shown in Fig. 6. Besides, a graviton has energy and therefore interact with each other similarly to gluons in Quantum Chromodynamics (QCD). This interaction which arises from the Einsten piece (1.10) is shown in Fig. 7.

As a matter of fact, we are attacking the problem of quantizing gravity from the perspective of *particle physics*. We have reduce quantum gravity to another quantum field theory, i.e. gravitons and other quanta interact and propagate within a *fixed* space–time background. In this sense, the language of general relativity where the geometry is crucial tends to get *lost*. In any case, if using perturbation theory we are able to compute different gravitational processes, the analysis is worthwhile. Unfortunately, this is *not* what happens. For example, the computation of photon–photon scattering in the Maxwell–Einstein theory shown in Fig. 8, turns out to give a result which is *divergent*. Of course, this is not a problem if the theory is renormalizable. However, we know from quantum field theory that theories with negative coupling constant are non renormalizable, and this is precisely the case of gravity where

$$\left[k^2\right] = \left[M_P^{-2}\right] = -2\ . \tag{1.14}$$

So quantum gravity contains an infinite variety of infinities. One can use simple dimensional arguments to arrive to this conclusion. A dimensionless probability amplitude of order $(k^2)^n$ must diverge as

$$\frac{1}{M_P^{2n}}\int p^{2n-1}dp\ , \tag{1.15}$$

where p is the momentum. For instance for $n = 2$, which is the case of the diagram in Fig. 8, quartic divergences will appear.

Therefore a consistent theory of gravity must be *finite* order by order in perturbation theory.

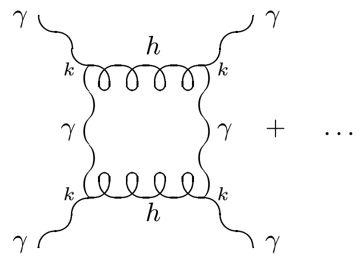

Figure 8: Contributions to the photon–photon scattering in the Maxwell–Einstein theory. The dots denote other one–loop diagrams involving only gravitons and photons.

We already know from the above discussion of supersymmetry that theories with more symmetry are more convergent. In supersymmetry quadratic divergences are canceled (see Fig. 1). Then, why not to apply the ideas of supersymmetry to gravity to solve the problem of divergences ?. This is what we will carry out in the next subsection.

1.2 Supergravity

We showed above that gravity is the 'gauge theory' of global space–time transformations. Likewise we will see in this section that supergravity is the *gauge theory* of global supersymmetry.

Let us consider schematically two consecutive infinitesimal global supersymmetric transformations of a boson field B

$$\delta_1 B \sim \bar{\varepsilon}_1 F \ , \tag{1.16}$$

$$\delta_2 F \sim \varepsilon_2 \, \partial B \ , \tag{1.17}$$

where F denote a fermion field. Using dimensional arguments one can deduce from (1.16) that the dimension of the anticommuting fermionic parameter ε must be $[\varepsilon] = -1/2$ in mass unit since $[B] = 1$ and $[F] = 3/2$. Thus in the second transformation (1.17) we must include a derivative to obtain the correct dimension. This implies that two internal supersymmetric transformations have led us to a space–time translation,

$$\{\delta_1, \delta_2\} B \sim a^\mu \partial_\mu B \ \ ; \ \ a^\mu = \bar{\varepsilon}_2 \gamma^\mu \varepsilon_1 \tag{1.18}$$

and therefore supersymmetry is an extension of the Poincare space–time symmetry

$$\{Q, \bar{Q}\} = 2\gamma^\mu P_\mu \ . \tag{1.19}$$

Clearly, the generator Q is not an internal symmetry generator like the ones of the standard model symmetries, $SU(3) \times SU(2) \times U(1)$, since it is related to the generator of space–time translations P_μ.

Promoting global supersymmetry to *local*, $\varepsilon = \varepsilon(x)$, space–time dependent translations $a^\mu \partial_\mu$ that differ from point to point are generated, i.e. *general coordinate* transformations. Therefore local supersymmetry necessarily *implies* gravity as shown schematically in Fig. 9. This situation is to be compared with the one summarized in Fig. 2. By obvious reasons local supersymmetry is also called *supergravity*.

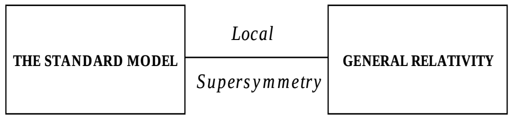

Figure 9: The standard model and general relativity are connected through local supersymmetry.

It is instructive to see explicitly the need for gravity in local supersymmmetry. Let us consider the simple case of a scalar field ϕ together with its supersymmetric partner the spin $1/2$ fermion ψ. The Lagrangian

$$\mathcal{L} = -(\partial^\mu \phi^*)(\partial_\mu \phi) - \frac{1}{2}\bar{\psi}\gamma^\mu \partial_\mu \psi \tag{1.20}$$

is invariant under global supersymmetry:

$$\delta\phi = \varepsilon\psi \tag{1.21}$$

$$\delta\psi = -i\sigma^\mu \bar{\varepsilon} \partial_\mu \phi \ . \tag{1.22}$$

However $\mathcal{L}$ is not invariant under local supersymmetry since $\varepsilon \to \varepsilon(x)$ implies

$$\delta\mathcal{L} = \partial_\mu \varepsilon^\alpha K^\mu_\alpha + h.c. \tag{1.23}$$

with

$$K^\alpha_\mu \equiv -\partial_\mu \phi^* \psi^\alpha - \frac{i}{2}\psi^\beta (\sigma_\mu \bar{\sigma}^\nu)^\alpha_\beta \partial_\nu \phi^* \tag{1.24}$$

To keep the action invariant, a gauge field has to be introduced (similarly to the case of an ordinary gauge symmetry where A_μ is introduced as we mentioned in the previous section) with the Noether coupling

$$\mathcal{L}_N = k \ K^\alpha_\mu \Psi^\mu_\alpha \ \ , \tag{1.25}$$

where k is introduced to give $\mathcal{L}_N$ the correct dimension, $[\mathcal{L}_N] = 4$, and Ψ is a Majorana vector spinor field with spin $3/2$, the so–called gravitino, transforming as

$$\Psi^\mu_\alpha \to \Psi^\mu_\alpha + \ \frac{1}{k}\partial^\mu \varepsilon_\alpha \ . \tag{1.26}$$

However, $\mathcal{L} + \mathcal{L}_N$ is not still invariant since

$$\delta(\mathcal{L} + \mathcal{L}_N) = k \ \bar{\Psi}_\mu \gamma_\nu \varepsilon T^{\mu\nu} \ , \tag{1.27}$$

where $T^{\mu\nu}$ is the energy–momentum tensor. This contribution can only be canceled adding a new term

$$\mathcal{L}_g = -g_{\mu\nu} T^{\mu\nu} \tag{1.28}$$

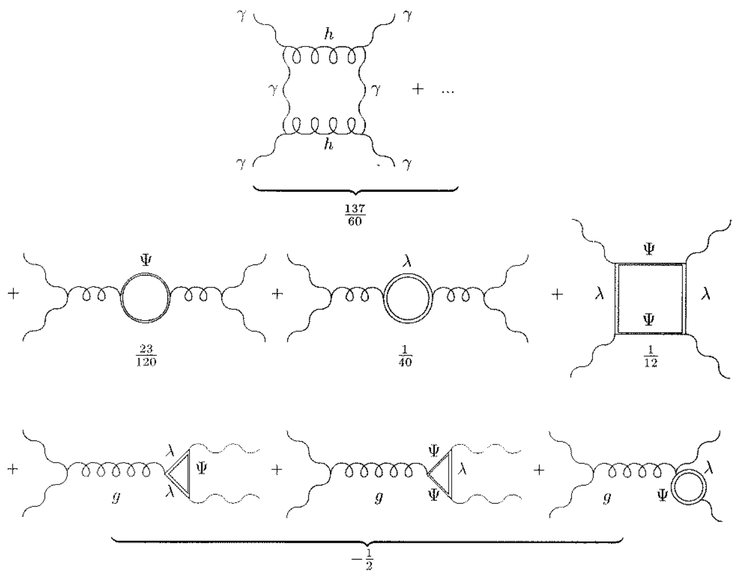

Figure 10: Contributions to the photon–photon scattering in the supersymmetric Maxwell–Einstein theory. Ψ and λ denote gravitino and photino respectively.

provided the tensor field $g_{\mu\nu}$ transforms as

$$\delta g_{\mu\nu} = k\bar{\Psi}_\mu \gamma_\nu \varepsilon \ . \tag{1.29}$$

Thus any locally supersymmetric theory has to include *gravity.*

In particular, the standard model which global supersymmetry contains the chiral supermultiplets (ψ, ϕ) studied above with ψ denoting quarks, leptons, Higgsinos and ϕ denoting squarks, sleptons, Higgses, plus vector supermultiplets (V, λ) with gauge bosons and gauginos (spin 1/2 Majorana fermions) respectively. In the presence of local supersymmetry we must include also the gravity supermultiplet ($g_{\mu\nu}$, Ψ^α_μ) with graviton and gravitino respectively. The gravitino plays the role of the gauge field of local supersymmetry.

In conclusion we can say that supergravity is a quantum theory of gravity. Since we have now more symmetry than in pure gravity we can expect that the high–energy (short distance) behavior will improve. Although this is basically true, still the (super)symmetry is not enough to cancel all divergences in the theory. To see this diagrammatically we have to include the supersymmetric partners in the graphs. For example, in the Maxwell–Einstein theory discussed in the previous subsection, we have to add to graphs in Fig. 8 those with supersymmetric particles, gravitino and photino, shown in Fig. 10. The contribution from each diagram is equal to an infinite quantity multiplied by the coefficient written below the figure. The infinite quantity is the same for all figures. Unfortunately, adding the coefficients shows that the sum is 25/12, i.e. *non* zero, and therefore the divergence is not canceled.

We have shown then that simple supersymmetry is not enough to solve the problem of infinities in quantum gravity. In the next subsection we will try to answer the following question: Is it possible to extended supersymmetry to a bigger symmetry solving the problem?.

1.3 Extended supergravity

It is natural to wonder what would be the consequences of the introduction of more than one supersymmetry generator, i.e.

$$Q_A \ \ (A = 1, 2, \ \dots \ , N) \ , \tag{1.30}$$

where

$$Q_A|\lambda\rangle = |\lambda - 1/2\rangle \tag{1.31}$$

with λ the helicity of a massless state. In this case the gravity supermultiplet will contain N gravitinos since

$$Q_1|\lambda = 2\rangle = |\lambda = 3/2\rangle, \ \dots$$

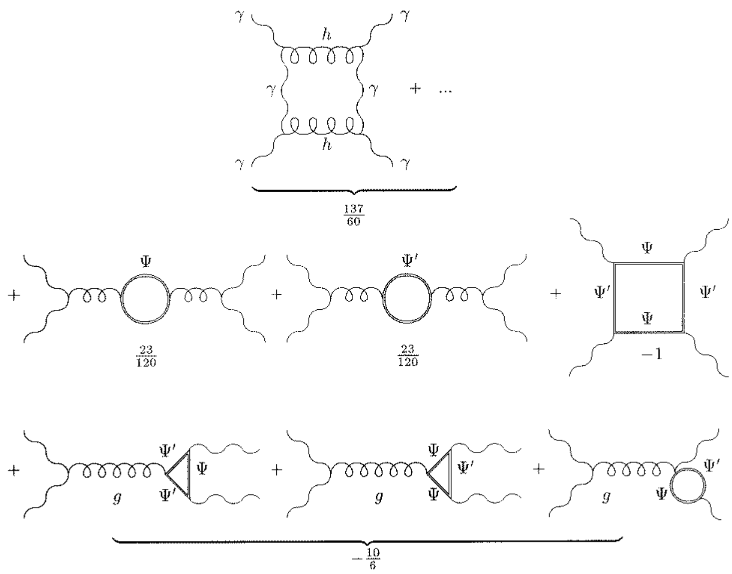

Figure 11: Contributions to the photon–photon scattering in the Maxwell–Einstein theory with $N = 2$ supersymmetry. Ψ and Ψ' denote the two gravitinos.

$$\ldots\ , Q_N|\lambda = 2\rangle = |\lambda = 3/2\rangle \ . \tag{1.32}$$

On the other hand, since

$$Q_1Q_2\ldots Q_N|\lambda = 2\rangle = |\lambda = 2 - N/2\rangle \tag{1.33}$$

we have the bound

$$N \leq 8 \tag{1.34}$$

to avoid massless particles with $\lambda > 2$. Recall that not consistent coupling between massless spin 2 particles and spin $5/2, 3, \ldots$ seems to exist.

The simplest extension of $N = 1$ supersymmetry discussed in the previous section is $N = 2$ supersymmetry. Precisely this case realizes Einstein's dream of unifying electromagnetism and gravity since

$$\begin{aligned} Q_{1,2}|\lambda = 2\rangle &= |\lambda = 3/2\rangle \\ Q_1Q_2|\lambda = 2\rangle &= |\lambda = 1\rangle \end{aligned} \tag{1.35}$$

giving rise to only one supermultiplet with graviton, two gravitinos and one photon $(2, 3/2, 1)$. This model was the first one where *finite* quantum corrections were found. This is shown in Fig. 11 where the photino of Fig. 10 corresponding to $N = 1$ supersymmetry is substituted by a second gravitino. Now adding the coefficients the sum is *zero*.

Although this result seems very promising, actually beyond one loop photon–photon scattering is *not* finite. Including matter the situation is worse, e.g. scalar–scalar scattering, etc. So again we have come back to our original problem in quantum gravity. Is there any way out? In principle we have not yet exhausted all possibilities. $N = 8$ is the maximum number of supersymmetries we can use. Since theories with more symmetries are more convergent we should analyze that case. In fact $N = 8$ case is also interesting because gravity, Yang–Mills, matter multiplets cannot exist in isolation. They belong to the only supermultiplet of the theory

$$\lambda = (2, \frac{3}{2}, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1, -\frac{3}{2}, -2) \ . \tag{1.36}$$

For example the multiplicity of states with helicity 1/2 is 56 since we can do the following combinations:

$$\begin{aligned} Q_1Q_2Q_3|\lambda = 2\rangle &= |\lambda = 1/2\rangle \\ Q_1Q_2Q_4|\lambda = 2\rangle &= |\lambda = 1/2\rangle \\ &\ldots \end{aligned} \tag{1.37}$$

In general, multiplicity of states with helicity $\lambda - m/2$ is

$$\binom{N}{m} = \frac{N!}{m!(N-m)!} \ . \tag{1.38}$$

Therefore $N = 8$ supergravity has enough degrees of freedom to unify all interactions as well as constituents. But again *divergences* are present. For instance starting at seven loops in graviton–graviton scattering.

As a matter of fact this is not the only problem of extended supergravities. All of them are *non* chiral. E.g. in the case of $N = 2$

$$Q_{1,2}|\lambda = 1/2\rangle = |\lambda = 0\rangle \ , \qquad Q_1Q_2|\lambda = 1/2\rangle = |\lambda = -1/2\rangle$$

and therefore we have in the same supermultiplet

$$\lambda = (-\frac{1}{2} \ \ 0 \ + \frac{1}{2}) \tag{1.39}$$

left and right handed fields. Since in the case of the standard model e.g. $e_L \in SU(2)$ but e_R is a singlet, necessarily unobserved fields E_R must belong to the same supermultiplet as e_L. These are the so–called *mirror* partners. They must have a mass beyond the current experimental bounds. On the one hand, it is extremely involved to build realistic models which generate masses dynamically to the mirrors. On the other hand, since chiral anomaly cancels within each generation, why such mirrors should exist?. All these arguments make unlikely that extended supergravities be realistic four–dimensional theories.

1.4 Discussion: supergravity as the low–energy limit of superstring theory

We can summarize the analyses of previous sections in the following way. $N = 0$ quantum gravity, i.e. withouth supersymmetry, is non renormalizable. $N = 1$ supergravity includes gravity in a *natural* way but it is also non renormalizable. $N > 1$ supergravity is not only non renormalizable but also non interesting from phenomenological viewpoint (at least in four dimensions).

Given these pessimistic conclusions, one wonders whether to work at low energies with the physically relevant $N = 1$ supergravity is consistent. The answer is *yes* if we are considering the supergravity Lagrangian as an effective phenomenological Lagrangian which comes from a (finite) *bigger structure*. This is a situation

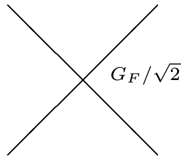

Figure 12: Weak interaction in Fermi theory.

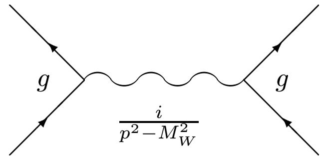

Figure 13: Weak interaction in Weinberg–Salam theory.

similar to the one of the old Fermi theory. The weak interaction was described as an interaction of four fermions with a dimensionfull coupling constant $[G_F] = -2$. This is shown in Fig. 12. Now we know that the correct theory involves in fact the graph shown in Fig. 13. For $p \approx M_W$ this is the correct way to carry out any computation. However, for $p << M_W$ this diagram is effectively like the one of Fig. 12 with $G_F/\sqrt{2} = g^2/M_W^2$. Thus although the Fermi theory is non renormalizable, in some particular energy regime one can trust the results.

Although for $p \approx M_P$ one must use the theory behind supergravity, for $p << M_P$ is a good approximation to work with supergravity. We will see in section 2 that in fact below M_P one is left with a global supersymetric Lagrangian plus supersymmetry–breaking terms. This effective Lagrangian is renormalizable and in order to study phenomenology we are interested only in this region.

There is, at the time of writing this lectures, only one consistent theory of quantum gravity with matter: string theory. There the solution to the problem of divergences in quantum field theory consists of considering the elementary particles to be not points but one-dimensional extended objects, strings, as shown in Fig. 14.

In fact, the consistency of the theory need the presence of supersymmetry and that is the reason why it is called superstring theory. Remarkably, the *low–energy* limit (massless modes)

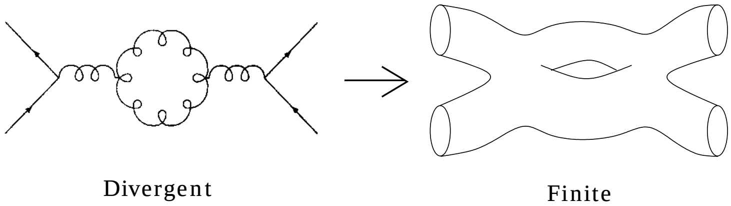

Figure 14: Exchange of a graviton between two elementary particles in quantum field theory and superstring theory.

of superstring theory is *supergravity*. The picture is then the following. Around the Planck scale,

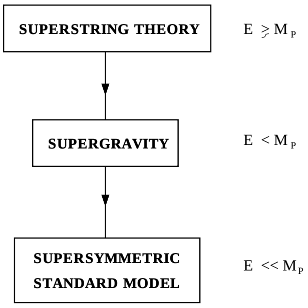

Figure 15: Supergravity is the intermediate step between the possible final theory of elementary particles and the supersymmetric standard model observable at low energy.

superstring theory might be the correct theory of elementary particles, but below that scale supergravity can be used as an effective theory. This is schematically shown in Fig. 15. The last step, around the electroweak scale, corresponds to the supersymmetric standard model arising from the spontaneous breaking of supergravity as mentioned above.

From the above arguments we conclude that the study of supergravity is crucial. It is the connection between the possible final theory of elementary particles and the low–energy effective theory which might be tested experimentally.

1.5 Higher dimensional supergravity

Since superstring theory is only consistent in a ten–dimensional ($D = 10$) space–time, to build supergravities in extra space–time dimensions is important. For example, the coupled $D = 10$, $N = 1$ supergravity super Yang–Mills system is the massless sector of the type I superstring theory and heterotic string theory.

To build the pure $N = 1$ supergravity one has to realize that the number of bosonic and fermionic degrees of freedom must be equal. These are shown in Table 1.

For example, to deduce the dimension of Dirac spinors in D space–time dimensions one can construct the Dirac gamma matrices obeying the Clifford algebra $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$. The result is

$$\begin{aligned} D_\Gamma &= 2^{D/2} \quad ; \; D \; even \; , \\ D_\Gamma &= 2^{(D-1)/2} \quad ; \; D \; odd \; . \end{aligned} \tag{1.40}$$

In our case $D = 10$ implies that λ has 32 complex components. Taking into account Majorana condition we reduce this number to 16. With the Weyl condition it is further reduced to 8 and finally field equations reduce it to 8 real components.

The Lagrangian can be explicitly obtained by the Noether's method or by a formal dimensional reduction from higher dimensional theories, in particular from $D = 11$, $N = 1$ supergravity. The result is

$$\begin{aligned} e^{-1}\mathcal{L} = & -\frac{1}{2k^2}R - \frac{3}{4}\phi^{-3/2}H_{\mu\nu\rho}H^{\mu\nu\rho} \\ & -\frac{9}{16k^2}\frac{\partial_\mu\phi\partial^\mu\phi}{\phi^2} - \frac{1}{2}\bar{\Psi}_\mu\Gamma^{\mu\nu\rho}D_\nu\Psi_\rho \\ & -\frac{1}{2}\bar{\lambda}\Gamma^\mu D_\mu\lambda - \frac{3\sqrt{2}}{8}\frac{\partial_\nu\phi}{\phi}\bar{\Psi}_\mu\Gamma^\nu\Gamma^\mu\lambda \\ & +\frac{\sqrt{2}k}{16}\phi^{-3/4}H_{\nu\rho\sigma}[\bar{\Psi}_\mu\Gamma^{\mu\nu\rho\sigma\tau}\Psi_\tau \\ & +6\bar{\Psi}^\nu\Gamma^\rho\Psi^\sigma - \sqrt{2}\bar{\Psi}_\mu\Gamma^{\nu\rho\sigma}\Gamma^\mu\lambda] \\ & + four - fermion \; terms \; , \end{aligned} \tag{1.41}$$

where $\Gamma^{\mu_1\cdots\mu_n}$ stands for the completely antisymmetrized product of Γ matrices and $H_{\mu\nu\rho}$ is the field strength of the antisymmetric tensor $B_{\mu\nu}$. The vierbein e_μ^m with m a local Lorentz index

FIELD CONTENT IN $D = 10$, $N = 1$ SUPERGRAVITY			
graviton ($g_{\mu\nu}$)	=	35	components
dilaton (ϕ)	=	1	"
antisymmetric tensor ($B_{\mu\nu}$)	=	28	"
		64	real field components
Majorana–Weyl gravitino (Ψ^{μ}_{α})	=	56	components
Majorana–Weyl fermion (λ_{α})	=	8	"
		64	real field components

Table 1: Bosonic and fermionic degrees of freedom in ten–dimensional pure supergravity

FIELD CONTENT IN $D = 11$, $N = 1$ SUPERGRAVITY			
graviton ($g_{\mu\nu}$)	=	44	components
3rd. rank antisymmetric tensor ($A_{\mu\nu\rho}$)	=	84	"
		128	real field components
Majorana gravitino (Ψ^{μ}_{α})	=	128	components
		128	real field components

Table 2: Bosonic and fermionic degrees of freedom in eleven–dimensional pure supergravity

must be used instead of the metric $g_{\mu\nu}$ when fermions are present. Their relation is $g_{\mu\nu} = e^m_\mu e^n_\nu \eta_{mn}$ and therefore $e \equiv det\, e^m_\mu = \sqrt{-det\ g_{\mu\nu}}$.

This Lagrangian is invariant under the local supertransformations

$$\delta e^m_\mu = \frac{k}{2}\bar{\varepsilon}\Gamma^m\Psi_\mu \ , \qquad (1.42)$$

$$\delta\phi = -\frac{\sqrt{2}\,k}{3}\phi\bar{\varepsilon}\lambda \ , \qquad (1.43)$$

$$\delta B_{\mu\nu} = \frac{\sqrt{2}}{4}\phi^{3/4}(\bar{\varepsilon}\Gamma_\mu\Psi_\nu - \bar{\varepsilon}\Gamma_\nu\Psi_\mu - \frac{\sqrt{2}}{2}\bar{\varepsilon}\Gamma_{\mu\nu}\lambda) \ , \qquad (1.44)$$

$$\delta\lambda = -\frac{3\sqrt{2}}{8}\frac{1}{\phi}(\Gamma_\mu\partial^\mu\phi)\varepsilon + \frac{1}{8}\phi^{-3/4}\Gamma^{\mu\nu\rho}\varepsilon H_{\mu\nu\rho} + two - fermion\ terms \ , \qquad (1.45)$$

$$\delta\Psi_\mu = \frac{1}{k}D_\mu\varepsilon + \frac{\sqrt{2}}{32}\phi^{-3/4}(\Gamma_\mu^{\nu\rho\sigma} - 9\delta^\nu_\mu\Gamma^{\rho\sigma})\varepsilon H_{\nu\rho\sigma} + two - fermion\ terms \ . \qquad (1.46)$$

On the other hand, the dimension of space–time in supersymmetry is constrained to be

$$D \leq 11 \ . \qquad (1.47)$$

Otherwise, counting the number of degrees of freedom as above, massless particles with spin higher than 2 would appear. This is extremely interesting since $D = 11$, $N = 1$ supergravity is the low–energy limit of so–called *M–theory* [14]. There is the *proposal* that M–theory, from which the five existent superstring theories can be derived, is a consistent quantum theory containing extended objects. In this sense the study of $D = 11$ supergravity may be important.

In fact, $D = 11$ supergravity is a very attractive theory by its own since supergravity equations look very simple and natural. Besides, this theory is unique. The field content of the theory together with their degrees of freedom are shown in Table 2. By brute force the Lagrangian was built [15] with the following relatively simple result:

$$\mathcal{L} = -\frac{1}{2k^2}eR - \frac{1}{48}eF_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} - \frac{1}{2}e\bar{\Psi}_\mu\Gamma^{\mu\nu\rho}D_\nu\Psi_\rho - \frac{\sqrt{2}k}{384}e(\bar{\Psi}_\mu\Gamma^{\mu\nu\rho\sigma\delta\lambda}\Psi_\lambda$$

$$+12\bar{\Psi}^{\nu}\Gamma^{\rho\sigma}\Psi^{\delta})(F+\hat{F})_{\nu\rho\sigma\delta}$$
$$-\frac{\sqrt{2}k}{3456}\varepsilon^{\mu_1\ldots\mu_4\nu_1\ldots\nu_4\rho\sigma\delta}F_{\mu_1\ldots\mu_4}F_{\nu_1\ldots\nu_4}A_{\rho\sigma\delta}\ . \tag{1.48}$$

It is invariant under

$$\delta e^m_\mu = \frac{k}{2}\bar{\varepsilon}\Gamma^m\Psi_\mu\ , \tag{1.49}$$

$$\delta A_{\mu\nu\sigma} = -\frac{\sqrt{2}}{8}\bar{\varepsilon}\Gamma_{[\mu\nu}\Psi_{\sigma]}\ , \tag{1.50}$$

$$\delta\Psi_\mu = \frac{1}{k}D_\mu\varepsilon + \frac{\sqrt{2}}{288}(\Gamma_\mu^{\nu\rho\sigma\lambda} \tag{1.51}$$
$$-8\delta^\nu_\mu\Gamma^{\rho\sigma\lambda})\varepsilon\hat{F}_{\nu\rho\sigma\lambda}\ . \tag{1.52}$$

2. $D=4$, $N=1$ Supergravity

Although some higher dimensional theory will probably be the unified theory of particle physics as discussed above, needless to say to connect it with the observable world one has to compactify the extra dimensions. At the end of the day, the theory which is left in four dimensions is $N=1$ supergravity. Thus the study of $D=4$, $N=1$ supergravity is *crucial*. We will carry it out in this section in a completely general way without assuming any particular underlying higher dimensional theory. After analyzing the Lagrangian, we will study the spontaneous breaking of supergravity. This gives rise to the so–called soft supersymmetry–breaking terms which determine the spectrum of supersymmetric particles. The theory of soft terms provided by this computation enable us to interpret the (future) experimental results on supersymmetric spectra. This would be an (indirect) test of supergravity.

In section 3 we will apply these general results to the particular case of $D=4$ supergravity arising from compactifications of $D=10$ superstrings.

2.1 The Lagrangian

In section 1.2, starting with the global supersymmetry Lagrangian of free chiral supermultiplets, we were able to obtain their couplings to supergravity using the *Noether* procedure. In the general case, with chiral supermultiplets in interaction, we can also follow the same approach. However, it is worth noticing that other formulations of $D=4$, $N=1$ supergravity are also available. In fact, although various matter couplings had previously been constructed using the Noether procedure, the complete Lagrangian including vector supermultiplets [16] was constructed using the more efficient local *tensor calculus* method. For a review of the latter see [6], where also the references of the authors who have contributed to the subject can be found. In Appendix A we sketch another formulation, the *superspace* formalism which is the most elegant one.

In what follows we present the final result (with up to two derivatives), obtained using any of the available formulations. Let us concentrate first on the chiral supergravity Lagrangian. It turns out to depend only on a single *arbitrary* real function of the scalar fields ϕ_i^* and ϕ_j with $i,j=1,...,n$, the Kähler function

$$G(\phi^*,\phi)=K(\phi^*,\phi)+\ln|W(\phi)|^2\ , \tag{2.1}$$

which is a combination of a real function, the so called Kähler potential K, and an analytic function, the so called superpotential W. This expresses the fact that the scalar–field space in supersymmetry is a *Kähler manifold*, i.e. a special type of analytic Riemann manifold (see above (A.15) in Appendix A for more details). The scalar fields should be thought of as the coordinates of the Kähler manifold and, in particular, the metric K_{ij^*} is given by

$$K_{ij^*}=\frac{\partial^2K}{\partial\phi_i\partial\phi_j^*}\ . \tag{2.2}$$

An important property of G (and therefore of the Lagrangian) is its invariance under the transformations

$$K\to K+F(\phi)+F^*(\phi^*)\ ,$$
$$W\to e^{-F(\phi)}W\ . \tag{2.3}$$

where F is an arbitrary function. This property is known as Kähler invariance.

We split the (tree–level) Lagrangian into terms as

$$\mathcal{L}^C=\mathcal{L}^C_B+\mathcal{L}^C_{FK}+\mathcal{L}^C_F\ , \tag{2.4}$$

where $\mathcal{L}^C_B$ contains only bosonic fields, $\mathcal{L}^C_{FK}$ contains fermionic fields and covariant derivatives with respect to gravity (i.e. including the supersymmetric spin connection) and $\mathcal{L}^C_F$ contains fermionic fields but no covariant derivatives. Then,

$$e^{-1}\mathcal{L}_B^C = -\frac{1}{2}R - G_{ij^*}\partial_\mu\phi^i\partial^\mu\phi^{*j} - e^G(G_i(G^{-1})^{ij^*}G_{j^*} - 3) \ , \quad (2.5)$$

where repeated indices are summed in our notation and e was already defined below (1.41). It is worth recalling that e_μ^m gives rise to interactions of the graviton with all other particles using the expansion studied in section 1.2. Note that we have set the reduced Planck mass M_P defined in (1.11) equal to 1 for convenience (see e.g. the usual Hilbert–Einstein piece in (2.5)). It can easily be inserted using dimensional arguments as we will do below in some examples. We also follow the notation

$$G_i \equiv \frac{\partial G}{\partial\phi_i} \quad (2.6)$$

and

$$G_{ij^*} \equiv \frac{\partial^2 G}{\partial\phi_i\partial\phi_j^*} = G_{j^*i} \ , \quad (2.7)$$

with the matrix $(G^{-1})^{ij^*}$ the inverse of the G_{j^*k}

$$(G^{-1})^{ij^*}G_{kj^*} = \delta_k^i \ . \quad (2.8)$$

From (2.1) and (2.2) we deduce

$$G_{ij^*} = K_{ij^*} \quad (2.9)$$

and therefore the Kähler metric K_{ij^*} determines the kinetic terms for the scalars ϕ_i (see the second term in (2.5)). This is also the case for the spin 1/2 fermions ψ_i in (2.12) below since both belong to the same chiral multiplets. Thus in general we will have *non–renormalizable* kinetic terms as a consequence of the non renormalizability of supergravity (see (A.13) in Appendix A for more details). As we will see in the next subsection, some scalar fields ϕ_i may acquire dynamically vacuum expectation values implying $K_{ij^*} \neq 0$. This will give rise in general to non–canonical kinetic terms and therefore we will have to normalize the fields to obtain, at the end of the day, canonical kinetic terms. A simple form of K leading to canonical kinetic terms $K_{ij^*} = \delta_{ij}$ (the so called minimal supergravity model), which will be used in the next subsection as a toy model, is given by

$$K = \phi_i\phi^{*i} \ . \quad (2.10)$$

Finally, the third term in (2.5), which arises when the auxiliary fields F_i appearing in the chiral supermultiplets are eliminated by their equations of motion (an extra gaugino bilinear piece given by $-\frac{1}{4}\frac{\partial f_{ab}}{\partial\phi_k}(G^{-1})^{ik^*}\lambda^a\lambda^b$ should be added when vector supermultiplets be considered below)

$$F_i = e^{G/2}(G^{-1})^{ij^*}G_{j^*} - (G^{-1})^{ik^*}G_{jlk^*}\psi^j\psi^l + \frac{1}{2}\psi_i(G_{j^*}\psi^j) \ , \quad (2.11)$$

contributes to the (tree–level) scalar potential. It is a fundamental piece for *model building*. In particular, as we will discuss in the next subsections, it is crucial to analyze the breaking of supersymmetry as well as the so called soft terms which determine the supersymmetric spectrum. Note the exponential factor e^G which obviously indicates the non renormalizability of the theory. The piece

$$\begin{aligned} e^{-1}\mathcal{L}_{FK}^C = & -\frac{1}{2}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu\gamma_5\gamma_\nu D_\rho\Psi_\sigma \\ & +\frac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu\gamma_\nu\Psi_\rho(G_iD_\sigma\phi^i \\ & -G_iD_\sigma\phi^{*i}) + \Big[\frac{i}{2}G_{ij^*}\bar{\psi}_L^i \not{D}\psi_L^j \\ & +\frac{i}{2}\bar{\psi}_L^i\not{D}\phi^j\psi_L^k(-G_{ijk^*} + \frac{1}{2}G_{ik^*}G_j) \\ & +\frac{1}{\sqrt{2}}G_{ij^*}\bar{\Psi}_L^\mu\not{D}\phi^{i^*}\gamma_\mu\psi_R^j + h.c.\Big] \end{aligned} \quad (2.12)$$

contains the kinetic terms for the fermions (i.e. for the spin 3/2 gravitino Ψ and the spin 1/2 fermions ψ_i) and some *non–renormalizable* interaction terms. For example, even assuming canonical kinetic terms $G_{ij^*} = \delta_{ij}$, the last term in (2.12) has at least mass dimension 5 and therefore must be suppressed by a power of $1/M_P$. This interaction term is shown in Fig. 16. Finally,

$$\begin{aligned} e^{-1}\mathcal{L}_F^C = & \ e^{G/2}\bar{\Psi}^\mu\sigma_{\mu\nu}\Psi^\nu + \Big[e^{G/2}G_i\bar{\Psi}_L^\mu\gamma_\mu\psi_L^i \\ & +\frac{1}{2}e^{G/2}\big(-G_{ij} - G_iG_j \\ & +G_{ijk^*}(G^{-1})^{k^*l}G_l\big)\bar{\psi}_L^i\psi_R^j + h.c.\Big] \\ & +\left(\frac{1}{2}G_{ijk^*l^*} - \frac{1}{2}G_{ijm^*}(G^{-1})_{nm^*}G_{nk^*l^*}\right. \\ & \left. -\frac{1}{4}G_{ik^*}G_{jl^*}\right)\bar{\psi}_{Li}\psi_{Lj}\bar{\psi}_R^k\psi_R^l \end{aligned}$$

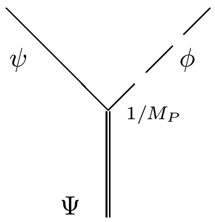

Figure 16: gravitino–fermion–scalar non–renormalizable interaction

$$
\begin{aligned}
& - \frac{1}{8} G_{i^*j} \bar{\psi}^i_R \gamma_d \psi_{Lj} \left(\varepsilon^{abcd} \bar{\Psi}_a \gamma_b \Psi_c \right. \\
& \left. - \bar{\Psi}^a \gamma_5 \gamma^d \Psi_a \right) \ , \qquad (2.13)
\end{aligned}
$$

where $\sigma_{\mu\nu}$ stands for the antisymmetrized product of γ matrices, contains the fermion Yukawa couplings and several non–renormalizable terms. The former are due to the third piece in (2.13) since G_{ij} is proportional to $\frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$, and therefore for a trilinear superpotential, $W = Y_{ijk}\ \phi_i \phi_j \phi_k$, terms of the type shown in Fig. 17 will arise. It is worth noticing that the first term in (2.13) is a potential mass for the gravitino (local supersymmetry breaking) if some of the scalar fields ϕ_i develop expectation values in such a way that $e^{G/2} \neq 0$. We will discuss this possibility in detail in the next subsection.

To obtain the complete supergravity Lagrangian which couples pure supergravity to supersymmetric chiral matter and Yang–Mills we still have to include the vector supermultiplets in the formulation. The result is:

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}^C_B + \mathcal{L}^C_{FK} + \mathcal{L}^C_F \\
&+ \mathcal{L}^V_B + \mathcal{L}^V_{FK} + \mathcal{L}^V_F \ , \qquad (2.14)
\end{aligned}
$$

where $\mathcal{L}^C_B$, $\mathcal{L}^C_{FK}$ and $\mathcal{L}^C_F$ are as in (2.5), (2.12) and (2.13), but with the derivatives covariantized also with respect to the gauge group in the usual way. For example, the term which gives rise to the kinetic energies for the fermions ψ_i in (2.12) gives

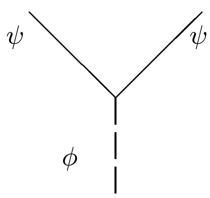

Figure 17: scalar–fermion–fermion Yukawa coupling

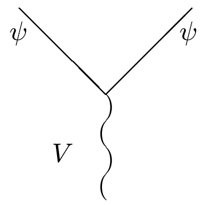

Figure 18: Gauge boson–fermion-fermion interaction

also rise to interactions between gauge bosons and fermions. This is shown in Fig. 18 for the minimal supergravity model. The other pieces in (2.14) are written below.

The piece $\mathcal{L}^V_B$ is given by

$$
\begin{aligned}
e^{-1}\mathcal{L}^V_B &= -\frac{1}{4}(Re f_{ab})(F^a)_{\mu\nu}(F^b)^{\mu\nu} \\
&+ \frac{i}{4}(Im\ f_{ab})(F^a)_{\mu\nu}(\tilde{F}^b)^{\mu\nu} \\
&- \frac{1}{2} g^2 \left[(Re\ f)^{-1} \right]^{ab} G_i (T_a)^{ij} \phi_j G_k (T_b)^{kl} \phi_l \qquad (2.15)
\end{aligned}
$$

where g denotes the gauge coupling constant, a denotes the gauge group index, T^a the group generators in the same representation as the chiral matter and $(F^a)_{\mu\nu}$ the gauge field strength.

Note that the supergravity Lagrangian depends not only on G but also on an *arbitrary* analytic function of the scalar fields ϕ_i,

$$
f_{ab}(\phi) \ . \qquad (2.16)
$$

It must transform as a symmetric product of adjoint representations of the gauge group G to render the Lagrangian invariant. The function f, which appears due to the non renormalizability of the theory (see (A.24) in Appendix A for more details), is called the gauge kinetic function since it is multiplying the usual gauge kinetic terms.

It is remarkable that this fact provide us with a mechanism to determine dynamically the gauge coupling constant. By defining $gV_\mu = V'_\mu$ in quantum field theory, g is removed from the field strength covariant derivative and appears only in an overall $1/g^2$ in the kinetic terms. Therefore if some scalar fields ϕ_i acquire dynamically vacuum expectation values we may obtain expectation values for f_{ab} and this may play the part of the coupling constant. In particular,

$$
Re\ f_{ab} = \frac{1}{g^2_{ab}} \ . \qquad (2.17)
$$

This is in fact the case of supergravity models deriving from superstring theory as we will see in section 3. In the formulae of this section we keep for completeness both g and f.

The third term in (2.15), which arises when the auxiliary fields D_a appearing in the vector supermultiplets are eliminated by their equations of motion,

$$D_a = i\ [(Ref)^{-1}]^{ab}\Big(gG_i(T_b)^{ij}\phi_j + \frac{1}{2}i\frac{\partial f_{bc}}{\partial \phi_i}\psi_i\lambda^c - \frac{1}{2}i\frac{\partial f^*_{bc}}{\partial \phi^{*i}}\psi^i\lambda_c\Big) + \frac{1}{2}\lambda_a(G_{i^*}\psi_i)\,, \qquad (2.18)$$

contributes to the (tree–level) scalar potential together with the third term in (2.5) which was studied above. The complete scalar potential is then

$$V = e^G\left(G_i(G^{-1})^{ij^*}G_{j^*} - 3\right) + \frac{1}{2}g^2\left[(Re\ f)^{-1}\right]^{ab}G_i(T_a)^{ij}\phi_j G_k(T_b)^{kl}\phi_l\ , \qquad (2.19)$$

where obviously the first piece is due to the F–term contribution and the second piece is due to the D–term contribution. Note that F and D terms in (2.11) and (2.18) are supergravity generalizations of the ones in global supersymmetry, where

$$F_i = -\frac{\partial W}{\partial \phi_i}\ ,$$
$$D_a = -g\phi^*_i(T_a)^{ij}\phi_j\ , \qquad (2.20)$$

and the scalar potential $V_{global} = F_iF^{i^*} + \frac{1}{2}D_aD^a$ is given by

$$V_{global} = \left|\frac{\partial W}{\partial \phi_i}\right|^2 + \frac{1}{2}g^2\phi^*_i(T_a)^{ij}\phi_j\phi^*_k(T_a)^{kl}\phi_l\ . \qquad (2.21)$$

It is worth mentioning the striking difference between the scalar potentials (2.21) and (2.19). Whereas the global one (2.21) is positive semi–definite, the local one (2.19) may be negative (see e.g. the piece $-3e^G$). This fact will be crucial to break supergravity being able to fine tune the vacuum energy density (cosmological constant) to zero, as we will discuss in the next subsection.

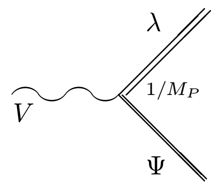

Figure 19: Gravitino–gauge boson–gaugino non–renormalizable interaction

The piece $\mathcal{L}^V_{FK}$ is given by

$$e^{-1}\mathcal{L}^V_{FK} = \frac{1}{2}(Re\ f_{ab})\Big(\frac{1}{2}\bar{\lambda}^a \not{D}\lambda^b + \frac{1}{2}\bar{\lambda}^a\gamma^\mu\sigma^{\nu\rho}\Psi_\mu(F^b)_{\nu\rho} + \frac{1}{2}G_iD^\mu\phi^i\bar{\lambda}^a_L\gamma_\mu\lambda^b_L\Big) - \frac{i}{8}(Im\ f_{ab})D_\mu(e\bar{\lambda}^a\gamma_5\gamma^\mu\lambda^b) - \frac{1}{2}\frac{\partial f_{ab}}{\partial \phi^i}\bar{\psi}_{iR}\sigma^{\mu\nu}(F^a)_{\mu\nu}\lambda^b_L + h.c.\ . \qquad (2.22)$$

The first term determines the kinetic energies for gauginos λ^a. They will be non canonical in general due to the contribution of the gauge kinetic function f. A simple form of f leading to canonical kinetic terms is

$$f_{ab} = \delta_{ab}\ . \qquad (2.23)$$

The other terms in (2.22) are in general non renormalizable. For example, the second one, even assuming f as in (2.23), has at least mass dimension 5. This interaction is shown in Fig. 19.

Finally, the piece $\mathcal{L}^V_F$ is given by

$$e^{-1}\mathcal{L}^V_F = \frac{1}{4}e^{G/2}\frac{\partial f^*_{ab}}{\partial \phi^{*j}}(G^{-1})^{j^*k}G_k\lambda^a\lambda^b - \frac{i}{2}g\ G_i(T^a)^{ij}\phi_j\bar{\Psi}_{\mu L}\gamma^\mu\lambda_{aL} + 2ig\ G_{ij^*}(T^a)^{ik}\phi_k\bar{\lambda}_{aR}\psi_{iL} - \frac{i}{2}g\left[(Re\ f)^{-1}\right]^{ab}\frac{\partial f_{bc}}{\partial \phi_k}G_i(T_a)^{ij}\phi_j\bar{\psi}_{kR}\lambda_{cL} - \frac{1}{32}(G^{-1})^{lk^*}\frac{\partial f_{ab}}{\partial \phi_l}\frac{\partial f^*_{cd}}{\partial \phi^{*k}}\bar{\lambda}^a_L\lambda^b_L\bar{\lambda}^c_R\lambda^d_R + \frac{3}{32}\left(Ref_{ab}\bar{\lambda}^a_L\gamma_m\lambda^b_R\right)^2 + \frac{1}{8}Ref_{ab}\bar{\lambda}^a\gamma^\mu\sigma^{\rho\sigma}\Psi_\mu\bar{\Psi}_\rho\gamma_\sigma\lambda^b + \frac{1}{2}\frac{\partial f_{ab}}{\partial \phi_i}\Big(\bar{\psi}_{Li}\sigma^{\mu\nu}\lambda^a_L\bar{\Psi}_{\nu L}\gamma_\mu\lambda^b_R + \frac{1}{4}\bar{\Psi}_{\mu R}\gamma^\mu\psi_{Li}\bar{\lambda}^a_L\lambda^b_L\Big)$$

$$
\begin{aligned}
&+ \frac{1}{16}\bar{\psi}_{Li}\gamma^\mu\psi^j_R\bar{\lambda}^d_R\gamma_\mu\lambda^c_L\Big(2G_{ij^*}Ref_{cd} \\
&+ \left[(Re\ f)^{-1}\right]^{ab}\frac{\partial f_{ac}}{\partial\phi_i}\frac{\partial f^*_{bd}}{\partial\phi^{*j}}\Big) \\
&+ \frac{1}{16}\bar{\psi}_{Li}\psi_{Lj}\bar{\lambda}^c_L\lambda^d_L\Big(-4G_{ijk^*}(G^{-1})^{lk^*}\frac{\partial f_{cd}}{\partial\phi_l} \\
&+ 4\frac{\partial^2 f_{cd}}{\partial\phi_i\partial\phi_j} - \left[(Re\ f)^{-1}\right]^{ab}\frac{\partial f_{ac}}{\partial\phi_i}\frac{\partial f_{bd}}{\partial\phi_j}\Big) \\
&- \frac{1}{16}\bar{\psi}_{Li}\sigma_{\mu\nu}\psi_{Lj}\bar{\lambda}^c_L\sigma^{\mu\nu}\lambda^d_L\left[(Re\ f)^{-1}\right]^{ab} \\
&\times\frac{\partial f_{ac}}{\partial\phi_i}\frac{\partial f_{bd}}{\partial\phi_j} + h.c. \qquad (2.24)
\end{aligned}
$$

It is remarkable that if some of the scalar fields ϕ_i acquire vacuum expectation values, gaugino masses may appear through the first term in (2.24). This is an indication of supersymmetry breaking which will be discussed in detail in subsection 2.3. The third term is a typical supersymmetric interaction. It is shown in Fig. 20 for the minimal supergravity model. Note finally that (2.24) contains numerous four–fermion terms.

In summary, the $D = 4$, $N = 1$ supergravity Lagrangian (2.14) depends *only* on two functions of the scalar fields. the Kähler function and the gauge kinetic function

$$
\begin{aligned}
G(\phi^*, \phi) &= K(\phi^*, \phi) + \ln|W(\phi)|^2 \ , \\
&f_{ab}(\phi) \ . \qquad (2.25)
\end{aligned}
$$

The Kähler potential K is a real gauge–invariant function and f and the superpotential W are analytic functions. Once G and f are given, the full supergravity Lagrangian is specified. Unfortunately for the predictivity of the theory, both functions are *arbitrary*. However, as we will see in section 3, in supergravity models deriving from superstring theory they are more constrained and explicit computations can be carried out.

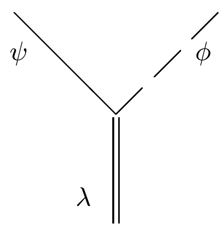

Figure 20: fermion–scalar–gaugino supersymmetric interaction

We finally give the explicit form of the local–supersymmetry transformations:

$$
\begin{aligned}
\delta e^m_\mu &= -ik\bar{\varepsilon}\gamma^m\Psi_\mu \ , \qquad (2.26) \\
\delta\Psi_\mu &= \frac{2}{k}D_\mu\varepsilon + k\varepsilon(G_iD_\mu\phi_i - G_{i^*}D_\mu\phi^{*i}) \\
&+ ie^{G/2}\gamma_\mu\varepsilon \\
&- \frac{1}{2}\sigma_{\mu\nu}\varepsilon_L G_{i^*j}\bar{\psi}^i_R\gamma^\nu\psi_{Lj} \\
&- \frac{1}{4}\Psi_{\mu L}\left(G_i\bar{\varepsilon}_L\psi_{Li} - G_{i^*}\bar{\varepsilon}_R\psi^i_R\right) \\
&+ \frac{1}{4}(g_{\mu\nu} - \sigma_{\mu\nu})\varepsilon_L\bar{\lambda}^a_L\gamma^\nu\lambda^b_R Ref_{ab} \ , \qquad (2.27) \\
\delta V^\mu_a &= -\bar{\varepsilon}_L\gamma^\mu\lambda_{aL} + h.c. \ , \qquad (2.28) \\
\delta\lambda_{aL} &= \sigma^{\mu\nu}(F_a)_{\mu\nu}\varepsilon_L \\
&+ \frac{i}{2}g[(Re\ f)^{-1}]^{ab}G_i(T_b)_{ij}\phi_j\varepsilon_L \\
&+ \frac{1}{4}i\varepsilon_R[(Re\ f)^{-1}]^{ab}\left(i\frac{\partial f_{bc}}{\partial\phi_i}\bar{\psi}_{Li}\lambda^c_L\right. \\
&- \left. i\frac{\partial f^*_{bc}}{\partial\phi^{*i}}\bar{\psi}^i_R\lambda^c_R\right) \\
&- \frac{1}{4}\lambda^\alpha_R\left(G_i\bar{\varepsilon}_L\psi_{Li} - G_{i^*}\bar{\varepsilon}_R\psi^i_R\right) \ , \qquad (2.29) \\
\delta\phi_i &= \sqrt{2}\bar{\varepsilon}\psi_i \ , \qquad (2.30) \\
\delta\psi_i &= \frac{1}{2}\not{D}\phi_i\varepsilon_R - \sqrt{2}e^{G/2}(G^{-1})^{ij^*}G_{j^*}\varepsilon \\
&+ \frac{1}{8}\varepsilon_L\bar{\lambda}^a_R\lambda^b_R(G^{-1})^{ik^*}\frac{\partial f^*_{ab}}{\partial\phi^{*k}} \\
&+ \frac{1}{2}\varepsilon_L(G^{-1})^{ik^*}G_{jlk^*}\bar{\psi}_{Lj}\psi_{Ll} \\
&- \frac{1}{4}\psi_{Li}\left(G_{j^*}\bar{\varepsilon}_R\psi^j_R - G_j\bar{\varepsilon}_L\psi_{Lj}\right) . \qquad (2.31)
\end{aligned}
$$

2.2 Spontaneous supersymmetry breaking

In section 1.2 we arrived to the conclusion that supergravity is the gauge theory of global supersymmetry with the gravitino as the gauge field. Now that we know the supergravity Lagrangian our next step is to ask whether the analog of the Higgs mechanism exists in this context. We will see in this subsection that this is indeed the case. The process is the following: scalar fields acquire dynamically vacuum expectation values giving rise to spontaneous breaking of supergravity. The goldstino, which is a combination of the fermionic partners of those fields, is swallowed by the massless gravitino to give a massive spin 3/2 particle. This is the so–called *super–Higgs effect.* Let us study it in detail.

As usual in gauge theories the condition for (super)symmetry breaking is $< \delta\chi_m > \neq 0$, where χ_m is at least one of the fields in the theory. The only local-supersymmetry transformations in (2.26–2.31) which may acquire non–vanishing expectation values without breaking Lorentz invariance are (2.29) and (2.31). Then, for non–spatially varying expectation values one obtains[1]

$$< \delta\lambda_a > = \frac{i}{2} g \left[(Re\ f)^{-1} \right]^{ab} G_i (T_b)_{ij} \phi_j \varepsilon_L \ , \quad (2.32)$$

$$< \delta\psi_i > = -\sqrt{2} e^{G/2} (G^{-1})^{ij^*} G_{j^*} \varepsilon \ , \quad (2.33)$$

where the scalar fields ϕ_i (present also through G and f in (2.32) and (2.33)) are being used to denote their vacuum expectation values. Note that (2.33) corresponds to vacuum expectation values for auxiliary fields F_i in (2.11)

$$F_i = e^{G/2} (G^{-1})^{ij^*} G_{j^*} \ . \quad (2.34)$$

Likewise, (2.32) corresponds to expectation values for auxiliary fields D_a in (2.18)

$$D_a = i \ [(Ref)^{-1}]^{ab} g G_i (T_b)^{ij} \phi_j \ . \quad (2.35)$$

There are then two ways of breaking supersymmetry, the so–called *F–term* and *D–term* supersymmetry breaking. For example, if gauge singlets scalar fields acquire expectation values, the right–hand side of (2.32) is zero and supersymmetry may be broken only by F terms (2.33). Clearly, in the case of gauge non–singlet scalar fields the two possibilities, F–term and D–term breaking, are allowed.

From the above discussion, we deduce that the relevant quantity for the study of supersymmetry breaking is G_i. We need

$$G_i \neq 0 \ , \quad (2.36)$$

for at least one value of i, if we want to break supersymmetry. For example, for the minimal supergravity model of (2.10) this means

$$\phi_i^* + \frac{1}{W} \frac{\partial W}{\partial \phi_i} \neq 0 \ . \quad (2.37)$$

[1] An expectation value of bilinear fermion–antifermion states may occur in presence of a strongly interacting gauge force. For example, the third piece in (2.31) must be taken into account if one wants to study supersymmetry breaking by the mechanism of a gaugino condensate [17].

Whether or not scalar fields acquire expectation values producing supersymmetry breaking by F and/or D terms is a dynamical question which must be answered minimizing the scalar potential (2.19). Note that using (2.34) and (2.35) this can also be written as

$$V = \left(F_{i^*} G_{i^* j} F_j - 3 e^G \right) + \frac{1}{2} (Re\ f_{ab})\ D_a D_b \ , \quad (2.38)$$

The form of the scalar potential allows in principle the possibility of local supersymmetry breaking with $V = 0$ (at tree level) unlike global supersymmetry breaking where the scalar potential (2.21) is always positive definite. The former possibility seems to be preferred experimentally since the upper bound on the cosmological constant is extremely close to zero $V \leq 10^{-45} (GeV)^4$. Of course this is not a solution to the cosmological constant problem. We do not know why the terms in the scalar potential conspire to produce $V = 0$, but at least we can fine tune them to obtain the value we want[2]. Otherwise, $V \approx m_{3/2}^2 M_P^2 \approx 10^{40} (GeV)^4$ as we will see below.

Let us now study a consequence of local supersymmetry breaking, the super–Higgs effect. Discussing first F–term supersymmetry breaking, we know that in global supersymmetry a linear combination of the spinors in the supermultiplets of the auxiliary fields F_i is the Goldstone fermion (Goldstino). In supergravity, where the mass terms from (2.13) are given by (assuming for simplicity the minimal model of (2.10))

$$e^{-1} \left(\mathcal{L}_F^C \right)_m = \frac{i}{2} e^{G/2} \bar{\Psi}_\mu \sigma^{\mu\nu} \Psi_\nu + \frac{i}{\sqrt{2}} e^{G/2} G_i \bar{\Psi}_\mu \gamma^\mu \psi^i - \frac{1}{2} e^{G/2} (G_{ij} + G_i G_j) \bar{\psi}^i \psi^j \ , \quad (2.39)$$

the Goldstino

$$\eta = G_i \psi^i \ , \quad (2.40)$$

as in ordinary gauge theory, gets mixed with the gravitino as shown in Fig. 21 due to the second term in (2.39). Its two degrees of freedom,

[2] In fact higher–order corrections to the scalar potential will spoil this cancellation.

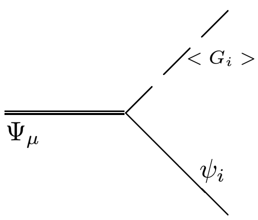

Figure 21: Gravitino–goldstino mixing interaction

helicity $\pm\frac{1}{2}$ states, are eaten by the gravitino which then gets helicity $\pm\frac{3}{2}, \pm\frac{1}{2}$ states. Redefining fields we are left with a gravitino

$$\Psi'_\mu = \Psi_\mu - \frac{i}{3\sqrt{2}}\gamma_\mu\eta - \frac{\sqrt{2}}{3}e^{-G/2}\partial_\mu\eta\,, \quad (2.41)$$

with mass

$$m_{3/2} = e^{G/2}M_P = e^{K/2}\frac{|W|}{M_P^2}\ , \quad (2.42)$$

where we have inserted the reduced Planck mass to obtain the correct mass unit, using G dimensionless and W with mass dimension 3. In particular,

$$e^{-1}\left(\mathcal{L}_F^C\right)_m = \frac{i}{2}e^{G/2}\bar{\Psi}'_\mu\sigma^{\mu\nu}\Psi'_\nu - \frac{1}{2}e^{G/2}\Bigg(G_{ij} + \frac{1}{3}G_iG_j\Bigg)\bar{\psi}^i\psi^j\,. \quad (2.43)$$

In the case of D–term supersymmetry breaking, the term in the Lagrangian mixing the gravitino with the Goldstino is not only given by the second term in (2.39) but also the second one in (2.24) contributes

$$e^{-1}\mathcal{L}_{mixing} = \frac{i}{\sqrt{2}}e^{G/2}G_i\bar{\Psi}_{\mu L}\gamma^\mu\psi_{iL} - \frac{i}{2}gG_i(T_a)_{ij}\phi_j\bar{\Psi}_{\mu L}\gamma^\mu\lambda_{aL} + h.c. \quad (2.44)$$

At the end of the day, with the Goldstino given by

$$\eta = G_i\psi^i - \frac{g}{\sqrt{2}}e^{-G/2}G_i(T_a)^{ij}\phi_j\lambda_a \quad (2.45)$$

one obtains similar results to the previous ones.

Now we know that spontaneous supersymmetry breaking in the context of supergravity is possible, but still we have to specify the mechanism that generates it. Clearly, e.g. from (2.37) and (2.42), we can deduce that the crucial function regarding this issue is the superpotential of the theory. The most useful form for this consists of a sum of two functions

$$W(C_\alpha, h_m) = W_O(C_\alpha) + W_H(h_m)\ , \quad (2.46)$$

where C_α denote the usual scalar fields of the theory, squarks, sleptons, Higgses (and other possible particles in case we are working with a grand unified model), which constitute the so–called observable sector, and h_m denote additional scalar fields responsible for the spontaneous breaking of supersymmetry. The latter are assumed to have only gravitational interactions with the observable sector. This means that they are singlets under the observable gauge group, constituting the so–called *hidden sector*. Therefore the hidden–sector fields do not have neither gauge interactions nor Yukawa couplings with the observable sector. In principle, other very weak interactions between both sectors might be present without being in conflict with experimental observations, however we prefer not to consider this complication.

The simplest model, proposed in an unpublished paper by Polonyi [18], consists of a gauge–singlet scalar field h with the following superpotential:

$$W_H(h) = m^2(h+\beta)\,. \quad (2.47)$$

The superpotential must have mass dimension 3 and therefore m and β are parameters with dimension of mass. Since the hidden–sector field is a gauge singlet, the scalar potential that we have to minimize is given only by the F part of (2.19)

$$V = M_P^4 e^G\left(G_h(G^{-1})^{hh^*}G_{h^*} - 3\right)\ , \quad (2.48)$$

where G is dimensionless to obtain the correct mass unit for the potential. Note that if there is no any cancellation, at the minimum $V \approx m_{3/2}^2 M_P^2$ giving rise to a huge cosmological constant. Considering for simplicity the minimal supergravity model of (2.10)

$$G = \frac{h^*h}{M_P^2} + ln\frac{|W_H|^2}{M_P^6}\ , \quad (2.49)$$

one obtains

$$G_h = \frac{h^*}{M_P^2} + \frac{1}{W_H}\frac{\partial W_H}{\partial h}\ , \quad (2.50)$$

and

$$(G^{-1})^{hh^*} = M_P^2 \ , \qquad (2.51)$$

with the following result for the scalar potential (2.48):

$$V = m^4 e^{\frac{h^* h}{M_P^2}} \left(|\frac{h^*}{M_P^2}(h+\beta)+1|^2 - 3\frac{|h-\beta|^2}{M_P^2} \right) \ . \qquad (2.52)$$

Choosing

$$\beta = \left(2-\sqrt{3}\right) M_P \ , \qquad (2.53)$$

one can show that V has an absolute minimum at

$$< h >= \left(\sqrt{3}-1\right) M_P \ , \qquad (2.54)$$

with vanishing cosmological constant $V=0$. Note that at this minimum supersymmetry is broken since G_h in (2.50) is different from zero

$$G_h = \frac{\sqrt{3}}{M_P} \ . \qquad (2.55)$$

The gravitino acquires a mass eating the Goldstino which in this case is the fermionic partner of h. Using the result (2.42), this is given by

$$m_{3/2} = e^{\frac{1}{2}(\sqrt{3}-1)^2} \frac{m^2}{M_P^2} M_P \ . \qquad (2.56)$$

Note that the gravitino mass can be much smaller than the Planck mass if $\frac{m}{M_P}$ is small. For example, to obtain the gravitino mass of order the electroweak scale, $m_{3/2} \approx M_W$, we need $m \approx \sqrt{M_W M_P} \approx 10^{10}$ GeV. This implies $< W_H > \approx 10^{20} M_P \approx 10^{38} (GeV)^3$. We will discuss in the next subsection that this possibility is in fact crucial if we want to avoid a hierarchy problem. The soft parameters which determine the supersymmetric spectrum and in particular contribute to the Higgs potential are of order the gravitino mass. Therefore we need this mass to be of order M_W to obtain a correct electroweak symmetry breaking. A value of $m_{3/2}$ larger than 1 TeV would reintroduce the hierarchy problem solved by supersymmetry.

Defining the supersymmetry–breaking scale M_S by

$$M_S^2 =< F_i >= M_P < e^{G/2} (G^{-1})^{ij^*} G_{j^*} > \ , \qquad (2.57)$$

where we have inserted M_P in (2.34) to have the correct mass unit, one obtains using (2.42)

$$M_S^2 = m_{3/2} < (G^{-1})^{ij^*} G_{j^*} > \ . \qquad (2.58)$$

For example, in the Polonyi model studied above results (2.51) and (2.55) imply

$$M_S^2 = \sqrt{3} m_{3/2} M_P \ . \qquad (2.59)$$

If, as discussed above, $m_{3/2} \approx M_W$ one obtains

$$M_S \approx \sqrt{m_{3/2} M_P} \approx \sqrt{M_W M_P} \approx 10^{10} GeV \ . \qquad (2.60)$$

This is in fact a generic result that must be fulfilled by any supersymmetric model. Apart from that, let us finally remark that the Polonyi mechanism must be considered as a toy model. It is rather ad hoc. There is no a special reason why W should be given as in (2.47). For example, in a fundamental theory like superstring theory such kind of superpotentials are not present. A more realistic mechanism is gaugino condensation in some hidden–sector gauge group [17]. For example in superstring theory, besides the gauge group where the standard model is embedded, other extra gauge groups are usually present providing a hidden sector. Due to non–perturbative effects a superpotential breaking supersymmetry is generated dynamically.

2.3 Soft Supersymmetry–Breaking Terms

On experimental grounds supersymmetry cannot be an exact symmetry of Nature: supersymmetric partners of the known particles with the same masses (e.g. scalar electrons with masses of 0.5 MeV) has not been detected. Let us recall a mechanism to avoid this problem in the context of global supersymmetry. Simply one introduces terms in the Lagrangian which *explicitly* break supersymmetry. The only constraint they must fulfil is not to induce quadratic divergences in order not to spoil the supersymmetric solution to the gauge hierarchy problem. This is the reason why these terms are called soft supersymmetry–breaking terms, soft terms in short. The simplest supersymmetric model is the so–called minimal supersymmetric standard model (MSSM), where the matter consists of three generations of quark and lepton superfields plus two Higgs doublets superfields (supersymmetry demands the presence of two Higgs doublets unlike the standard model where only one is needed) of opposite hypercharge, and the gauge sector consists of $SU(3)_C \times SU(2)_L \times U(1)_Y$ vector superfields. The associated superpotential, in an obvious notation, is given by

$$W = \sum_{generations} \Big[Y_u \, \tilde{Q}_L \tilde{H}_2 \tilde{u}^c_L + \, Y_d \, \tilde{Q}_L \tilde{H}_1 \tilde{d}^c_L + \, Y_e \, \tilde{L}_L \tilde{H}_1 \tilde{e}^c_L \Big] + \mu \, H_1 H_2 \ , \quad (2.61)$$

where the first piece is related with the Yukawa couplings (we have taken for simplicity diagonal couplings), which eventually determine the fermion masses, and the second piece, the so–called μ term, is necessary in order to break the electroweak symmetry.

The soft terms can be parameterized by the following parameters: gaugino masses M_a, scalar masses m_α, trilinear parameters (associated with the Yukawa couplings) $A_{u,d,e}$ and a bilinear parameter (associated with the μ term) B. Thus the form of the soft Lagrangian is given by

$$\mathcal{L}_{soft} = \frac{1}{2} \left(M_a \ \hat{\lambda}_a \hat{\lambda}_a + h.c. \right) - m_\alpha^2 \ C^*_\alpha C_\alpha - \Bigg(\sum_{generations} \Big[A_u \ \hat{Y}_u \ \tilde{Q}_L \tilde{H}_2 \tilde{u}^c_L + \ A_d \ \hat{Y}_d \ \tilde{Q}_L \tilde{H}_1 \tilde{d}^c_L + A_e \ \hat{Y}_e \ \tilde{L}_L \tilde{H}_1 \tilde{e}^c_L \Big] + \ B \ \hat{\mu} \ H_1 H_2 + h.c. \Bigg) \ , \quad (2.62)$$

where C_α denote all the observable scalars, i.e. $\tilde{Q}_L, \tilde{u}^c_L, \tilde{d}^c_L, \tilde{L}_L, \tilde{e}^c_L, H_1, H_2$, and λ_a with a corresponding to $SU(3)_C, SU(2)_L, U(1)_Y$, denote all the observable gauginos, i.e. $\tilde{g}, \tilde{W}_{1,2,3}, \tilde{B}$. The hat on Yukawa couplings and μ parameter denote that they are rescaled. We will discuss these points in (2.66) and (2.73) below.

These soft terms are crucial not only because they determine the supersymmetric spectrum, like gaugino, Higgsino, squark and slepton masses, but also because they contribute to the Higgs potential (together with the quartic terms coming from D terms) generating the radiative breakdown of the electroweak symmetry. Let us recall that in the standard model the whole Higgs potential has to be postulated ad hoc.

Although in principle the breaking of supersymmetry explicitly may look arbitrary, remarkably in the context of local supersymmetry it happens in a *natural* way: when supergravity is spontaneously broken in a hidden sector, the soft terms for the observable fields are generated. Let us discuss this in some detail.

In the previous subsection we studied mechanisms in order to break local supersymmetry, where the presence of fields in a hidden sector was crucial to achieve it. Let us divide the superpotential as in (2.46) and consider for simplicity the form of the Kähler potential K that leads to canonical kinetic terms (2.10) for the chiral supermultiplets

$$G = \frac{1}{M_P^2} \left(\sum_\alpha C^*_\alpha C_\alpha + \sum_m h^*_m h_m \right) + ln \frac{|W|^2}{M_P^6} \ . \quad (2.63)$$

Then, assuming that supersymmetry is broken by F terms, only the first piece of the scalar potential in (2.19) will contribute

$$V = e^{\frac{1}{M_P^2}(\sum_\alpha C^*_\alpha C_\alpha + \sum_m h^*_m h_m)} \times \left(\sum_\alpha |\frac{\partial W_O}{\partial C_\alpha} + \frac{C^*_\alpha}{M_P^2}(W_H + W_O)|^2 + \sum_m |\frac{\partial W_H}{\partial h_m} + \frac{h^*_m}{M_P^2}(W_H + W_O)|^2 - 3 \frac{|W_H + W_O|^2}{M_P^2} \right) \ , \quad (2.64)$$

where we are using for the moment a generic hidden–sector superpotential $W_H(h_m)$. The observable sector superpotential $W_O(C_\alpha)$ might be for example the one of the MSSM in (2.61) or a GUT generalization of it. Note that non renormalizable terms can in principle be ignored for analyses far below the Planck scale, $M_P \to \infty$, since they are suppressed by powers of $\frac{1}{M_P^2}$. For example $\frac{-3|W_O|^2}{M_P^2} \to 0$. Thus apparently one is left in the observable sector with the usual global–supersymmetry scalar potential $|\frac{\partial W_O}{\partial C_\alpha}|^2$ and nothing new arises from the breaking of supergravity. However, if some fields acquire large vacuum expectation values, the new gravitationally induced terms may be important. We saw in the previous subsection that although the first term in (2.13) is non renormalizable it gives rise to a sizeable contribution to the gravitino mass $m_{3/2} \approx \frac{|W_H|}{M_P^2}$ (see (2.42)). For example, using the Polonyi mechanism we obtained $< h > \approx M_P$ implying $W_H \approx 10^{38} (GeV)^3$ and therefore $m_{3/2} \approx$

100 GeV. From this discussion we conclude that we must hold $m_{3/2} = e^{\frac{1}{2M_P^2}\left(\sum_m h_m^* h_m\right)} \frac{|W_H(h_m)|}{M_P^2}$ fixed when we take the limit $M_P \to \infty$. Here h_m are being used to denote their vacuum expectation values at the minimum of the potential (2.64). This limit where $M_P \to \infty$ but $m_{3/2}$ is held fixed is the so–called *flat limit*. Then, although the observable sector does not contribute to give mass to the gravitino and therefore does not feel directly the breaking of supersymmetry, it feels the breaking indirectly due to the appearance of soft terms through gravitational interactions in (2.64). For example, the term proportional to $\frac{-3W_H^*}{M_P^2} W_O$ will give rise to couplings $-3m_{3/2}W_O$. Also from the term proportional to $|\frac{W_H}{M_P^2} C_\alpha^*|^2$, masses $m_{3/2}^2 C_\alpha^* C_\alpha$ for the scalars will arise. In total, from (2.64) one obtains in the low–energy limit

$$
\begin{aligned}
V &= e^{\frac{1}{M_P^2}\sum_m h_m^* h_m} \left\{ \sum_\alpha \left|\frac{\partial W_O}{\partial C_\alpha}\right|^2 \right. \\
&+ \sum_\alpha \frac{|W_H|^2}{M_P^4} C_\alpha^* C_\alpha + \frac{W_H^*}{M_P^2} \left[\sum_\alpha C_\alpha \frac{\partial W_O}{\partial C_\alpha} \right. \\
&+ \left(\sum_m h_m^* \left(\frac{1}{W_H^*} \frac{\partial W_H^*}{\partial h_m^*} + \frac{h_m}{M_P^2} \right) - 3 \right) W_O \\
&+ c.c \Bigg] \Bigg\} . \qquad (2.65)
\end{aligned}
$$

Rescaling the observable superpotential

$$\hat{W}_O \equiv W_O \frac{W_H^*}{|W_H|} e^{\frac{1}{2M_P^2}\sum_m h_m^* h_m} , \qquad (2.66)$$

and taking into account that the expectation values F_m of the auxiliary fields associated with the scalars h_m are given by (see (2.57))

$$F_m = m_{3/2} M_P^2 \left(\frac{1}{W_H^*} \frac{\partial W_H^*}{\partial h_m^*} + \frac{h_m}{M_P^2} \right) , \qquad (2.67)$$

(2.65) may be rewritten as

$$
\begin{aligned}
V &= \sum_\alpha \left|\frac{\partial \hat{W}_O}{\partial C_\alpha}\right|^2 + \sum_\alpha m_{3/2}^2 C_\alpha^* C_\alpha \\
&+ m_{3/2} \left[\sum_\alpha C_\alpha \frac{\partial \hat{W}_O}{\partial C_\alpha} \right. \\
&+ \left. \left(\sum_m h_m^* \frac{F_m}{m_{3/2} M_P^2} - 3 \right) \hat{W}_O + c.c \right] . \qquad (2.68)
\end{aligned}
$$

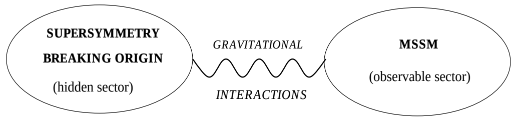

Figure 22: Supersymmetry breaking occurs in a hidden sector and is transmitted to the observable sector by gravitational interactions giving rise to soft terms.

Therefore at the end of the day we are left with the usual global supersymmetric potential (with superpotential $\hat{W}_O$) plus soft terms. The procedure studied in this section and the previous one to break supersymmetry is schematically summarized in Fig. 22. We can apply this result to the case of the MSSM (2.62). For example, absorbing the above rescaling (2.66) in Yukawa couplings and μ parameter as usual,

$$
\begin{aligned}
\hat{Y}_{u,d,e} &= Y_{u,d,e} \frac{W_H^*}{|W_H|} e^{\frac{1}{2M_P^2}\sum_m h_m^* h_m} , \\
\hat{\mu} &= \mu \frac{W_H^*}{|W_H|} e^{\frac{1}{2M_P^2}\sum_m h_m^* h_m} , \qquad (2.69)
\end{aligned}
$$

one obtains the following soft parameters:

$$
\begin{aligned}
m_\alpha &= m_{3/2} , \\
A_{u,d,e} &= m_{3/2} \sum_m h_m^* \frac{F_m}{m_{3/2} M_P^2} , \\
B &= m_{3/2} \sum_m h_m^* \frac{F_m}{m_{3/2} M_P^2} - m_{3/2} (2.70)
\end{aligned}
$$

It is worth noticing that the gravitino mass set the overall scale of the soft parameters. This was expected since the F part of the scalar potential in (2.19) has an overall factor e^G and therefore it is a general result valid for any supergravity model. This implies that the effective supersymmetry–breaking scale in the observable sector is of order $m_{3/2}$ as already mentioned above and not the supersymmetry–breaking scale F_m which is of order 10^{10} GeV.

Note that the scalar masses are automatically *universal* $m_\alpha \equiv m$. In particular, in this model they are all equal to the gravitino mass, $m = m_{3/2}$. Actually, universality of scalar masses is a desirable property not only to reduce the number of independent parameters in supersymmetric models, but also for phenomenological reasons, particularly to avoid flavour changing neutral currents. Besides, the A parameters are also universal, $A_{u,d,e} \equiv A$, and they are related to the B parameter[3]

$$B = A - m_{3/2} \ . \tag{2.71}$$

These results show us that it is possible to learn things about soft terms without knowing the details of supersymmetry breaking, i.e. the explicit form of the function $W_H(h_m)$. We are left in fact with only two free parameters, $m_{3/2}$ and $A \equiv A_{u,d,e}$. Of course once this function is known we can compute these quantities since $m_{3/2}$ and F_m depend on $W_H(h_m)$. For example, for the Polonyi mechanism studied in the previous subsection one obtains straightforwardly

$$A = \left(3 - \sqrt{3}\right) \ m_{3/2} \ . \tag{2.72}$$

The above results (2.70) should be understood as being valid at some high scale $\mathcal{O}(M_P)$ and the standard renormalization group equations must be used to obtain the low–energy ($\approx M_W$) values.

On the other hand, from the fermionic part of the supergravity Lagrangian, (tree–level) soft gaugino masses may also be obtained. Let us assume for simplicity $f_{ab} = \delta_{ab} f_a$. After canonically normalizing the gaugino fields in the first term of (2.22),

$$\hat{\lambda}_a = (Ref_a)^{1/2} \lambda_a \ , \tag{2.73}$$

the first term in (2.24) gives rise to a mass term as in (2.62) with

$$M_a = \frac{1}{2} (Ref_a)^{-1} F_m \frac{\partial f_a}{\partial h_m} \ , \tag{2.74}$$

with F_m as in (2.57). Note that the gauge kinetic function f is dimensionless and therefore the mass unit above is correct. For this to be non–vanishing at tree level, which is phenomenologically interesting (experimental bounds on gluino masses imply $M_3 > 50$ GeV), it is necessary a non–canonical choice of the vector supermultiplets $f_a \neq const.$ For example, universal gaugino masses can be obtained if all the f_a have the same dependence on the hidden–sector fields, i.e. $f_a(h_m) = c_a f(h_m)$, for the different gauge group factors. This is in fact the case of supergravity models deriving from (tree–level) perturbative superstring theory as we will see in the next section. Let us use the general formula (2.74) to consider the simple case $f_a = h/M_P$ with the same Kähler function (2.63) studied above, where some $h_m \equiv h$. Then, one obtains the universal mass

$$M = \frac{1}{2} (Re \ h)^{-1} F_h \ , \tag{2.75}$$

where F_h is given by (2.67) with $h_m = h$. For example, for the Polonyi mechanism one obtains

$$M = m_{3/2} \ \frac{\sqrt{3}}{2(\sqrt{3}-1)} \ . \tag{2.76}$$

As in the case of scalar soft parameters (2.70), the gravitino mass set the overall scale of the soft gaugino masses. Note to this respect the overall factor $e^{G/2}$ in the first term of (2.24).

Let us finally remark that with the inclusion of the other unsuppressed terms in (2.14) we are left finally with the usual global–supersymmetry Lagrangian plus the soft terms. This effective Lagrangian is obviously renormalizable and therefore perfectly consistent in order to study phenomenology. For example, the third term in (2.13) gives rise to the usual Yukawa couplings and Higgsino masses. Note that G_{ij} give rises to the observable–sector piece $\frac{1}{W_H} \frac{\partial^2 W_O}{\partial C_\alpha \partial C_\beta}$ and therefore $e^{G/2} G_{ij}$ induces the contribution $\frac{\partial^2 \hat{W}_O}{\partial C_\alpha \partial C_\beta}$ with Yukawa couplings and the μ parameter rescaled as in (2.69), i.e.

$$-\frac{1}{2} \sum_{\alpha,\beta} \frac{\partial^2 \hat{W}_O}{\partial C_\alpha \partial C_\beta} \bar{\psi}_{\alpha L} \psi_{\beta R} + h.c. \tag{2.77}$$

The Higgsino masses given by $\hat{\mu}$ are obviously supersymmetric masses. The same contribution will appear in the Higgs masses through the first term in (2.70)

In conclusion, we have shown in this subsection that supergravity models are interesting and give rise to concrete predictions for the soft parameters. However, one can think of many possible supergravity models (with different K, W and

[3] In fact, this relation depends on the particular mechanism which is used to generate the μ parameter. Its generation is the so–called μ problem and several solutions have been proposed in the context of supergravity [20].

f) leading to *different* results for the soft terms[4] For example, if instead of assuming the form of K for the observable sector given by (2.63) we take $K = K_\alpha(h_m^*, h_m)C_\alpha^* C_\alpha$ it is straightforward to see that soft scalar masses are no longer universal (we also must canonically normalize the scalar fields in (2.62) $\hat{C}_\alpha = K_\alpha^{1/2} C_\alpha$ similarly to the case of gaugino fields (2.73)). Also, the Polonyi superpotential W_H in (2.47) give rise to soft terms which are different from those obtained using a gaugino–condensation mechanism. This arbitrariness, as we will see in the next section, can be ameliorated in supergravity models deriving from superstring theory, where K, f, and the hidden sector are more constrained. We can already anticipate, for example, that in such a context the kinetic terms are generically *not* canonical.

3. Supergravity from superstrings

Recently there have been studies of supergravity models obtained in particularly simple classes of superstring compactifications. Such models have a *natural* hidden sector built-in: the complex dilaton field S and the complex overall modulus field T. These gauge singlet fields are generically present in four-dimensional models: the dilaton arises from the gravitational sector of the theory and the modulus parameterizes the size of the compactified space $< T > \approx R^2 M_{string}^2$, where R denotes the overall radius. Both fields are taken dimensionless. Assuming that the auxiliary fields of those multiplets are the seed of supersymmetry breaking, interesting predictions for this simple class of models are obtained. Here we will analyze very briefly this issue. More details can be found in [20].

Once we choose the compact space, K and f are calculable. Starting with the $D = 10$ supergravity Lagrangian, obtained as the low–energy limit of superstring theory, and expanding in $D = 4$ fields, the $D = 4$, $N = 1$ supergravity Lagrangian can be computed. In particular, working with orbifold compactifications in the context of the perturbative heterotic string, one obtains

$$\begin{aligned} \mathcal{L} = &-\frac{1}{4}(Re\ S)\ F_a^{\mu\nu} F_{\mu\nu}^a \\ &+\frac{1}{(S+S^*)^2}\ M_P^2\ \partial_\mu S \partial^\mu S^* \\ &+\frac{3}{(T+T^*)^2}\ M_P^2\ \partial_\mu T \partial^\mu T^* \\ &+(T+T^*)^{n_\alpha} \partial_\mu C_\alpha \partial^\mu C_\alpha^* + \ldots\ , \end{aligned} \tag{3.1}$$

where n_α are negative integer numbers called modular weights of the matter fields C_α. A comparison of this result with the general supergravity Lagrangian (2.14), in particular with the second term in (2.5) and the first term in (2.15), allows us to deduce the Kähler potential and gauge kinetic function

$$\begin{aligned} K &= -\ln(S+S^*) - 3\ln(T+T^*) \\ &\quad + (T+T^*)^{n_\alpha}\ \frac{C_\alpha C_\alpha^*}{M_P^2}\ , \\ f_{ab} &= S\ \delta_{ab}\ . \end{aligned} \tag{3.2}$$

Note from our discussion in (2.17) that $< Re\ S >= 1/g_a^2$ and therefore the gauge coupling constants are unified even in the absence of a grand unified theory. Thus grand unification groups, as e.g. $SU(5)$ or $SO(10)$, are not mandatory in order to have unification in the context of superstrings. Recall that S and T fields are dimensionless unlike all other scalars which have dimension 1. This implies dimension 1 for the F terms of S and T whereas the F terms of all other scalars have dimension 2 as studied in section 2.2.

As we learnt in the previous section, we can compute with information (3.2) the soft terms (e.g. gaugino masses M_a are trivially obtained from (2.74)):

$$\begin{aligned} m_\alpha^2 &= m_{3/2}^2 + \frac{n_\alpha}{(T+T^*)^2}|F_T|^2\ , \\ M_a &= \frac{F_S}{(S+S^*)}\ , \\ A_{\alpha\beta\gamma} &= -\frac{F^S}{(S+S^*)} - \frac{F^T}{(T+T^*)}\Big(3 + n_\alpha + n_\beta \\ &\quad + n_\gamma - (T+T^*)\frac{1}{Y_{\alpha\beta\gamma}}\frac{\partial Y_{\alpha\beta\gamma}}{\partial T}\Big)\ . \end{aligned} \tag{3.3}$$

Note that to obtain this result we did not assume any specific supersymmetry–breaking mechanism, i.e. a particular value of $W_H(S,T)$. Due to

[4] General formulae for tree–level soft parameters were computed in [19]. See also [20] for a review with supergravity and superstring examples. One–loop corrections to the soft parameters were computed recently in [21].

the form of the gauge kinetic function gaugino masses turn out to be universal. However, due to the modular weight dependence the scalar masses (and A parameters) show a lack of universality. Universality can be obtained in the so–called dilaton dominated limit, i.e. if we assume that the mechanism of supersymmetry breaking is in such a way that only the F term associated with the dilaton acquires a vacuum expectation value, $F_T = 0$. Then,

$$\begin{aligned} m_\alpha &= m_{3/2} \ , \\ M_a &= \sqrt{3}\ m_{3/2} \ , \\ A_{\alpha\beta\gamma} &= -M_a \ , \end{aligned} \tag{3.4}$$

where to obtain the value of M_a we have assumed a vanishing cosmological constant, i.e. $V = 0$ in (2.38)

$$\frac{|F_S|^2}{(S+S^*)^2} + \frac{3|F_T|^2}{(T+T^*)^2} = 3m_{3/2}^2 \ . \tag{3.5}$$

Recall that only the F part of (2.38) contributes to supersymmetry breaking since the hidden sector fields, dilaton and modulus, are gauge singlets.

It is worth noticing that the above results (3.4) are independent of the compactification space since the dilaton couples in a universal manner to all particles, i.e. f and the dilaton part of K are not modified by the particular choice of compact space. Because of the simplicity of this scenario, predictions are quite precise. For example at high energies the relation between scalar masses and gaugino masses is $M_a = \sqrt{3}\ m_\alpha$. Using the renormalization group equations one obtains at low–energies

$$M_{\tilde{g}} \approx m_{\tilde{q}} >> m_{\tilde{l}} \ . \tag{3.6}$$

4. Conclusions

Supergravity *cannot* be the final theory of elementary particles since it is non renormalizable. However, it might be the *effective theory* of the final theory (perhaps superstrings). In that case, supergravity might be subject to experimental *test* through the prediction of the soft supersymmetry breaking terms which determine the supersymmetric spectrum. The study of supergravity is therefore worthwhile. Besides, there are still open problems whose solutions are crucial for the consistency of the theory. The cosmological constant problem and the mechanism of supersymmetry breaking are the most important ones.

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A. Appendix

Superspace formalism in supergravity

We sketch in this Appendix the derivation of the most general $D = 4$, $N = 1$ supersymmetric gauge theory coupled to supergravity, using the superspace formalism. For a review of this method see [5], where also the references of the authors who have contributed to the subject can be found.

Let us recall first how this approach works in the context of global supersymmetry. The superspace is defined as the space created by

$$x_\mu,\ \theta_\alpha,\ \bar{\theta}_{\dot{\alpha}} \ , \tag{A.1}$$

where x_μ are the usual four space–time dimensions and the anticommuting parameters θ_α, $\bar{\theta}_{\dot{\alpha}}$ ($\alpha = 1, 2$), which are elements of a Grassmann algebra, are introduced as supersymmetric partners of the x–coordinate. The components of the chiral supermultiplets (ϕ_i, ψ_i, F_i) with $i = 1, ..., n$, where ϕ_i are complex scalar fields, ψ_i are Weyl spinor fields and F_i are auxiliary complex scalar fields, arise as the coefficients in an expansion in powers of θ and $\bar{\theta}$ of the so called chiral *superfields* $\Phi_i(x, \theta, \bar{\theta})$, which are functions of the superspace coordinates. Then, working in the superspace, the most general renormalizable supersymmetric Lagrangian involving only chiral superfields (barring linear contributions which are forbidden, unless the superfields are neutral, once gauge invariance is included) is given by

$$\mathcal{L}_{global} = \int d^2\theta d^2\bar{\theta}\Phi_i^+\Phi_i + \left[\int d^2\theta \left(\frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}Y_{ijk}\Phi_i\Phi_j\Phi_k\right) + h.c.\right] , \qquad (A.2)$$

where repited indices are summed in our notation and the couplings m_{ij} and Y_{ijk} are symmetric in their indices.

Its generalization to the local–supersymmetry case parallels the construction of a Lagrangian including gravity in the non–supersymmetric case discussed in section 1.1. First, to obtain the supersymmetric gravitational action we need supersymmetric generalizations of the measure ed^4x, where $e = det\ e_\mu^m$, and Ricci scalar R. These are given by the chiral density superfield $\mathcal{E}$ and superspace curvature superfield $\mathcal{R}$. Their components, (e, Ψ_μ,...) and (R, Ψ_μ,...) respectively, where Ψ_μ is the gravitino and the dots denote auxiliary fields, arise as the coefficients in an expansion in powers of the superspace parameters Θ. The latter are generalized θ parameters which now carry local Lorentz indices. The pure supergravity Lagrangian is then

$$\mathcal{L}_{sg} = -\frac{6}{k^2}\int d^2\Theta\mathcal{E}\mathcal{R} , \qquad (A.3)$$

where $k^2 = 8\pi G_N$ is the gravitational coupling. The Lagrangian (A.2) is now easily extended to the local case. We first write it in chiral form

$$\mathcal{L}_{global} = \int d^2\theta \left[-\frac{1}{8}\bar{D}\bar{D}\Phi_i^+\Phi_i + \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}Y_{ijk}\Phi_i\Phi_j\Phi_k\right] + h.c. , \qquad (A.4)$$

where D and $\bar{D}$ are the usual supersymmetric derivatives and we are using the property

$$\int F\left(x,\theta,\bar{\theta}\right) d^2\theta d^2\bar{\theta} = \frac{-1}{4}\int \bar{D}\bar{D}F\left(x,\theta,\bar{\theta}\right) d^2\theta = \frac{-1}{4}\int DDF\left(x,\theta,\bar{\theta}\right) d^2\bar{\theta} . \qquad (A.5)$$

We then add the supergravity action (A.3) and replace $\theta \to \Theta$, $d^2\theta \to d^2\Theta 2\mathcal{E}$, and $-\frac{1}{4}\bar{D}\bar{D} \to -\frac{1}{4}(\bar{\mathcal{D}}\mathcal{D} - 8\mathcal{R})$, where $\mathcal{D}$ is the covariant derivative. This gives the Lagrangian (A.2) in local supersymmetry (setting k equal to one):

$$\mathcal{L}_{local} = \int d^2\Theta 2\mathcal{E}\left[-3\mathcal{R} - \frac{1}{8}\left(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\right)\Phi_i^+\Phi_i + \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}Y_{ijk}\Phi_i\Phi_j\Phi_k\right] + h.c. \qquad (A.6)$$

It is interesting to write this equation as

$$\mathcal{L}_{local} = \int d^2\Theta 2\mathcal{E}\left[-\frac{1}{8}\left(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\right)\Omega(\Phi_i, \Phi_i^+) + W(\Phi_i)\right] + h.c. , \qquad (A.7)$$

where

$$\Omega(\Phi_i, \Phi_i^+) = \Phi_i^+\Phi_i - 3 \qquad (A.8)$$

is the superspace kinetic energy, and

$$W(\Phi_i) = \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}Y_{ijk}\Phi_i\Phi_j\Phi_k \qquad (A.9)$$

is the superspace potential (also called superpotential).

After eliminating the auxiliary fields by their Euler equations, and rescaling and redefining the other fields, one arrives at the component Lagrangian in terms of the physical fields, ϕ, ψ, e_μ^m and Ψ_μ. Remarkably, its expression is written in terms of the real function

$$K(\phi_i, \phi_i^*) = -3\ln\left(-\frac{\Omega}{3}\right) , \qquad (A.10)$$

with

$$\Omega(\phi_i, \phi_i^*) = \phi_i^+\phi_i - 3 . \qquad (A.11)$$

For example, the kinetic terms of the scalars are proportional to

$$\frac{\partial^2 K}{\partial\phi_i\partial\phi_j^*}\partial_\mu\phi_i\partial^\mu\phi_j^* . \qquad (A.12)$$

They have properly normalized kinetic energies since $\frac{\partial^2 K}{\partial\phi_i\partial\phi_j^*} = \delta_{ij} + \ldots$. The Lagrangian also has higher–order interaction terms, such as non–renormalizable four–fermion couplings, which are suppressed by powers of Newton's constant. Below we shall see that K is the so called Kähler potential and that the component Lagrangian has a natural interpretation in the language of Kähler geometry.

Since supergravity is a non–renormalizable theory, we are in fact interested in the most general Lagrangian that can be built from chiral superfields. This is

$$\mathcal{L}_{global} = \int d^2\theta d^2\bar{\theta}\ K\left(\Phi_i^+, \Phi_j\right) + \left[\int d^2\theta\ W(\Phi_i) + h.c.\right] . \quad (A.13)$$

Here K and W are vector and chiral superfields respectively, with power series expansion in terms of the chiral superfields Φ_i,

$$\begin{aligned} K(\Phi_i, \Phi_j^+) &= \sum a_{i_1 \ldots i_N, j_1 \ldots j_M} \Phi_{i_1} \ldots \Phi_{i_N} \\ &\times \Phi_{j_1}^+ \ldots \Phi_{j_M}^+ , \\ W(\Phi_i) &= \sum b_{i_1 \ldots i_N} \Phi_{i_1} \ldots \Phi_{i_N} . \quad (A.14) \end{aligned}$$

The component expansion of K implies that kinetic terms are obviously non renormalizable. On the other hand, only kinetic terms with two space–time derivatives are present. A higher–derivative theory might be obtained if supersymmetric derivatives of superfields were allowed in K. We exclude this problematic possibility in what follows. To study the matter couplings of chiral multiplets it is convenient to use the language of Kähler geometry.

A *Kähler manifold* is a special type of analytic Riemann manifold, subject to certain conditions. These conditions imply that the metric and the connection are determined by the derivatives of a scalar function K called the Kähler potential. E.g. the metric g_{ij^*} is given by $g_{ij^*} = \frac{\partial^2 K(c,c^*)}{\partial c_i \partial c_j^*} \equiv K_{ij^*}$, where c_i and c_i^* are the complex coordinates parameterizing the Kähler manifold. Under an analytic coordinate transformation, $c_i \to c_i'(c)$ and $c_i^* \to c_i^{*\prime}(c^*)$. Note that the metric is invariant under analytic shifts of K, $K(c,c^*) \to K(c,c^*) + F(c) + F^*(c^*)$. They are called Kähler transformations of the Kähler potential.

Then, using the Kähler notation, it is possible to see that the component Lagrangian from (A.13) can be written in terms of the real function $K(\phi, \phi^*)$. For example, the scalar potential is given by

$$V = K_{ij^*} \frac{\partial W}{\partial \phi^i} \frac{\partial W^*}{\partial \phi^{*j}} . \quad (A.15)$$

The Lagrangian is invariant under coordinate transformations, where the scalar fields should be thought of as the coordinates of the Kähler manifold and the fermions as tensors in the tangent space. From the superspace viewpoint, the source of this Kähler geometry is the invariance of (A.13) under the superfield Kähler transformation:

$$K(\Phi, \Phi^+) \to K(\Phi, \Phi^+) + F(\Phi) + F^*(\Phi^+) . \quad (A.16)$$

Now, following the same steps as in the case of the renormalizable chiral Lagrangian (A.2), we generalize (A.13) to the local–supersymmetry case

$$\begin{aligned} \mathcal{L}_{local} &= \frac{1}{k^2} \int d^2\Theta\ 2\mathcal{E} \left[\frac{3}{8}\left(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\right) \right. \\ &\times \left. e^{-\frac{k^2}{3} K\left(\Phi_i, \Phi_j^+\right)} + k^2 W(\Phi_i)\right] \\ &+ h.c. , \end{aligned} \quad (A.17)$$

where $K(\Phi, \Phi^+)$ is a general hermitian function and $W(\Phi)$ is the superpotential. The exponential form is suggested by the relation (A.10). Note that expanding in k^2,

$$\begin{aligned} \mathcal{L}_{local} &= -\frac{6}{k^2} \int d^2\Theta\ \mathcal{E}\mathcal{R} + \int d^2\Theta 2\mathcal{E} \\ &\times \left[-\frac{1}{8}(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}) K\left(\Phi_i^+, \Phi_j\right) + W(\Phi_i)\right] \\ &+ \ldots + h.c. , \end{aligned} \quad (A.18)$$

we may recover the global–supersymmetry Lagrangian. After a straightforward computation, the component form of (A.17) turns out to be the same as that of (A.7), where now K is an arbitrary real function of the scalar fields, the lowest component of the superfield $K(\Phi, \Phi^+)$. Using a super–Weyl transformation one can simplify the component expression in such a way that it depends only on the real function

$$G(\phi^*, \phi) = K(\phi^*, \phi) + \ln |W(\phi)|^2 . \quad (A.19)$$

For example, the scalar potential is given by

$$V = e^G \left[\frac{\partial G}{\partial \phi_i} \left(\frac{\partial^2 G}{\partial \phi_i \phi_j^*}\right)^{-1} \frac{\partial G}{\partial \phi_j^*} - 3\right] . \quad (A.20)$$

This is different from the global–supersymmetry Lagrangian, where K and W enter in an independent way (see e.g. (A.15)). The final Lagrangian can be found in the text in eq. (2.4).

To obtain the complete supergravity Lagrangian which couples pure supergravity to supersymmetric chiral matter and Yang–Mills we need now the complete gauge–invariant global supersymmetry Lagrangian in superspace. Let us recall first how to obtain the renormalizable gauge–invariant Lagrangian. One imposes gauge invariance to the renormalizable chiral Lagrangian (A.2) with the result that a vector superfield must be introduced:

$$\begin{aligned}\mathcal{L}_{global} = & \int d^2\theta d^2\bar{\theta}\ \Phi^+ e^V \Phi \\ & + \left[\int d^2\theta \left(\frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}Y_{ijk}\Phi_i\Phi_j\Phi_k\right)\right. \\ & + \left. h.c.\right] \ , \end{aligned} \tag{A.21}$$

where now the chiral superfields Φ transforms as a representation of a gauge group G and $V \equiv 2gT^aV^a$ with T^a the group generators in that representation, V^a the vector superfields and g the gauge coupling constant. The components of V^a are the vector bosons belonging to the adjoint representation of G, their Majorana spinor partners λ^a and the auxiliary real scalar fields D^a. Adding the (gauge–invariant) kinetic term for the vector supermultiplet,

$$\frac{1}{16}\int d^2\theta\ W^{a\alpha}W^a_\alpha + h.c.\ , \tag{A.22}$$

we obtain the complete Lagrangian. Here W^a_α is the gauge field strength (chiral spinor) superfield with spinor index α. In particular, $W_\alpha = -\frac{1}{4}\bar{D}\bar{D}e^{-V}D_\alpha e^V$. We follow the same procedure to obtain the most general gauge–invariant Lagrangian. From (A.13) we deduce

$$\begin{aligned}\mathcal{L}_{global} = & \int d^2\theta d^2\bar{\theta}\left[K(\Phi_i^+,\Phi_j) + \Gamma(\Phi_i^+,\Phi_j,V)\right] \\ & + \left[\int d^2\theta\ W(\Phi_i) + h.c.\right] \ , \end{aligned} \tag{A.23}$$

where Γ is a counterterm which is necessary for gauge invariance. As above we have to add the kinetic term for the vector supermultiplet:

$$\frac{1}{16}\int d^2\theta f_{ab}(\Phi_i)W^aW^b \ , \tag{A.24}$$

but now an arbitrary analytic function of the chiral superfields $f_{ab}(\Phi_i)$, which would be just δ_{ab}. in the renormalizable case, may be included. Under supersymmetry it transforms as chiral superfield. Under a gauge transformation, it must transform as a symmetric product of adjoint representations of the gauge group.

Then, from (A.23) and (A.24), we deduce the local–supersymmetry Lagrangian:

$$\begin{aligned}\mathcal{L}_{local} = & \int d^2\Theta 2\mathcal{E}\left[\frac{3}{8}(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R})\right. \\ & \times\ exp\left\{-\frac{1}{3}\left[K(\Phi_i^+,\Phi_j) + \Gamma(\Phi_i^+,\Phi_j,V)\right]\right\} \\ & + \left.\frac{1}{16}f_{ab}(\Phi_i)W^aW^b + W(\Phi_i)\right] \\ & + h.c.\ , \end{aligned} \tag{A.25}$$

where $W_\alpha = -\frac{1}{4}(\bar{\mathcal{D}}\bar{\mathcal{D}}-8\mathcal{R})e^{-V}\mathcal{D}_\alpha e^V$ is the curved–space generalization of the Yang–Mills field strength superfield. After eliminating the auxiliary fields, and rescaling and redefining the other fields as in the case of chiral models, one arrives at the component Lagrangian in terms of the physical fields, $\phi, \psi, V^a, \lambda^a, e^m_\mu, \Psi_\mu$. It depends on the two *arbitrary* functions

$$\begin{aligned} G(\phi^*,\phi) = K(\phi^*,\phi) + \ln|W(\phi)|^2\ , \\ f_{ab}(\phi_i)\ , \end{aligned} \tag{A.26}$$

and can be found in the text in eq. (2.14).

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