Introduction to Cosmology

G. Lazarides
Physics Division , School of Technology,
Aristotle University of Thessaloniki,
Thessaloniki 540 06, Greece
E-mail: lazaride@eng.auth.gr

Abstract: The standard big bang cosmological model and the history of the early universe according to the grand unified theories of strong, weak and electromagnetic interactions are summarized. The shortcomings of big bang are discussed together with their resolution by inflationary cosmology. Inflation and the subsequent oscillation and decay of the inflaton field are studied. The density perturbations produced during inflation and their evolution during the matter dominated era are analyzed. The temperature fluctuations of the cosmic background radiation are summarized. Finally, the nonsupersymmetric as well as the supersymmetric hybrid inflationary model is described.

1. The Big Bang Model

The discovery of the cosmic background radiation (CBR) in 1964 together with the observed Hubble expansion of the universe had established hot big bang cosmology as a viable model of the universe. The success of the theory of nucleosynthesis in reproducing the observed abundance pattern of light elements together with the proof of the black body character of the CBR then established hot big bang as the standard cosmological model. This model combined with grand unified theories (GUTs) of strong, weak and electromagnetic interactions provides an appropriate framework for discussing the very early stages of the universe evolution. A brief introduction to hot big bang follows.

1.1 Hubble Expansion

For cosmic times $t \gtrsim t_P \equiv M_P^{-1} \sim 10^{-44}$ sec ($M_P = 1.22 \times 10^{19}$ GeV is the Planck scale) after the big bang, quantum fluctuations of gravity cease to exist. Gravitation can then be adequately described by classical relativity. Strong, weak and electromagnetic interactions, however, require relativistic quantum field theoretic treatment and are described by gauge theories.

An important principle, on which the standard big bang (SBB) cosmological model [1] is based, is that the universe is homogeneous and isotropic. The strongest evidence so far for this *cosmological principle* is the observed [2] isotropy of the CBR. Under this assumption, the four dimensional spacetime in the universe is described by the Robertson-Walker metric

$$ds^2 = -dt^2+$$
$$a^2(t)\left[\frac{dr^2}{1-kr^2}+r^2(d\theta^2+\sin^2\theta\, d\varphi^2)\right] , \quad (1.1)$$

where r, φ and θ are 'comoving' polar coordinates, which remain fixed for objects that have no other motion than the general expansion of the universe. The parameter k is the 'scalar curvature' of the 3-space and $k=0$, $k>0$ or $k<0$ correspond to flat, closed or open universe. The dimensionless parameter $a(t)$ is the 'scale factor' of the universe and describes cosmological expansion. We normalize it by taking $a_0 \equiv a(t_0) = 0$, where t_0 is the present cosmic time.

The 'instantaneous' radial physical distance is given by

$$R = a(t)\int_0^r \frac{dr}{(1-kr^2)^{1/2}} \cdot \quad (1.2)$$

For flat universe ($k=0$), $\bar{R} = a(t)\bar{r}$ ($\bar{r}$ is a 'comoving' and $\bar{R}$ a physical vector in 3-space) and the velocity of an object is

$$\bar{V} = \frac{d\bar{R}}{dt} = \frac{\dot{a}}{a}\bar{R} + a\frac{d\bar{r}}{dt} \ , \qquad (1.3)$$

where overdots denote derivation with respect to cosmic time. The first term in the right hand side (rhs) of this equation is the 'peculiar velocity', $\bar{v} = a(t)\dot{\bar{r}}$, of the object, i.e., its velocity with respect to the 'comoving' coordinate system. For $\bar{v} = 0$, Eq.(1.3) becomes

$$\bar{V} = \frac{\dot{a}}{a}\bar{R} \equiv H(t)\bar{R} \ , \qquad (1.4)$$

where $H(t) \equiv \dot{a}(t)/a(t)$ is the Hubble parameter. This is the well-known Hubble law asserting that all objects run away from each other with velocities proportional to their distances and is considered as the first success of SBB cosmology.

1.2 Friedmann Equation

Homogeneity and isotropy of the universe imply that the energy momentum tensor takes the diagonal form $(T_\mu^\nu) = \mathrm{diag}(-\rho, p, p, p)$, where ρ is the energy density of the universe and p the pressure. Energy momentum conservation (${T_\mu}^\nu{}_{;\nu} = 0$) then takes the form of the continuity equation

$$\frac{d\rho}{dt} = -3H(t)(\rho + p) \ , \qquad (1.5)$$

where the first term in the rhs describes the dilution of the energy due to the expansion of the universe and the second term corresponds to the work done by pressure. Eq.(1.5) can be given the following more transparent form

$$d\left(\frac{4\pi}{3}a^3\rho\right) = -p \ 4\pi a^2 da \ , \qquad (1.6)$$

which indicates that the energy loss of a 'comoving' sphere of radius $\propto a(t)$ equals the work done by pressure on its boundary as it expands.

For a universe described by the Robertson-Walker metric in Eq.(1.1), Einstein's equations

$$R_\mu{}^\nu - \frac{1}{2}\ \delta_\mu{}^\nu R = 8\pi G \ T_\mu{}^\nu \ , \qquad (1.7)$$

where $R_\mu{}^\nu$ and R are the Ricci tensor and scalar curvature tensor and $G \equiv M_P^{-2}$ is the Newton's constant, lead to the Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \ . \qquad (1.8)$$

Averaging p, we write $\rho + p = \gamma\rho$. Eq.(1.5) then becomes $\dot{\rho} = -3H\gamma\rho$, which gives $d\rho/\rho = -3\gamma da/a$ and $\rho \propto a^{-3\gamma}$. For a universe dominated by pressureless matter, $p = 0$ and, thus, $\gamma = 1$, which gives $\rho \propto a^{-3}$. This is easily interpreted as mere dilution of a fixed number of particles in a 'comoving' volume due to the cosmological expansion. For a radiation dominated universe, $p = 1/3$ and, thus, $\gamma = 4/3$, which gives $\rho \propto a^{-4}$. In this case, we get an extra factor of $a(t)$ due to the red-shifting of all wave-lengths by the expansion. Substituting $\rho \propto a^{-3\gamma}$ in Friedmann equation with $k = 0$, we get $\dot{a}/a \propto a^{-3\gamma/2}$ and, thus, $a(t) \propto t^{2/3\gamma}$. Taking into account the normalization of $a(t)$ ($a(t_0) = 1$), this gives

$$a(t) = (t/t_0)^{2/3\gamma} \ . \qquad (1.9)$$

For a matter dominated universe, we get the expansion law $a(t) = (t/t_0)^{2/3}$. 'Radiation', however, expands as $a(t) = (t/t_0)^{1/2}$.

The universe in its early stages of evolution is radiation dominated and its energy density is

$$\rho = \frac{\pi^2}{30}\left(N_b + \frac{7}{8}N_f\right)T^4 \equiv c\ T^4 \ , \qquad (1.10)$$

where T is the cosmic temperature and N_b (N_f) is the number of massless bosonic (fermionic) degrees of freedom. The combination $g_* = N_b + (7/8)N_f$ is called effective number of massless degrees of freedom. The entropy density is

$$s = \frac{2\pi^2}{45}\ g_*\ T^3 \ . \qquad (1.11)$$

Assuming adiabatic universe evolution, i.e., constant entropy in a 'comoving' volume ($sa^3 =$ constant), we obtain the relation $aT =$ constant. The temperature-time relation during radiation dominance is then derived from Friedmann equation (with $k = 0$):

$$T^2 = \frac{M_P}{2(8\pi c/3)^{1/2}t} \ . \qquad (1.12)$$

We see that classically the expansion starts at $t = 0$ with $T = \infty$ and $a = 0$. This initial singularity is, however, not physical since general relativity fails at cosmic times smaller than about the Planck time t_P. The only meaningful statement is that the universe, after a yet unknown initial stage, emerges at a cosmic time $\sim t_P$ with temperature $T \sim M_P$.

1.3 Important Cosmological Parameters

The most important parameters describing the expanding universe are the following:

i. The present value of the Hubble parameter (known as Hubble constant) $H_0 \equiv H(t_0) = 100\ h$ km sec^{-1} Mpc^{-1} ($0.4 \lesssim h \lesssim 0.8$).

ii. The fraction $\Omega = \rho/\rho_c$, where ρ_c is the critical density corresponding to a flat universe ($k = 0$). From Friedmann equation, $\rho_c = 3H^2/8\pi G$ and, thus, $\Omega = 1 + k/a^2H^2$. $\Omega = 1$, $\Omega > 1$ or $\Omega < 1$ correspond to flat, closed or open universe. Assuming inflation (see below), the present value of Ω must be $\Omega_0 = 1$. However, the baryonic contribution to Ω is $\Omega_B \approx 0.05 - 0.1$ [3]. This indicates that most of the energy in the universe must be in nonbaryonic form.

iii. The deceleration parameter

$$q = -\frac{(\ddot{a}/\dot{a})}{(\dot{a}/a)} = \frac{\rho + 3p}{2\rho_c} \ . \qquad (1.13)$$

For 'matter', $q = \Omega/2$ and, thus, inflation implies that the present deceleration parameter is $q_0 = 1/2$.

1.4 Particle Horizon

Light travels only a finite distance from the time of big bang ($t = 0$) till some cosmic time t. From the Robertson-Walker metric in Eq.(1.1), we find that the propagation of light along the radial direction is described by the equation $a(t)dr = dt$. The particle horizon, which is the 'instantaneous' distance at time t travelled by light since the beginning of time, is then given by

$$d_H(t) = a(t) \int_0^t \frac{dt'}{a(t')} \ . \qquad (1.14)$$

The particle horizon is a very important notion since it coincides with the size of the universe already seen at time t or, equivalently, with the distance at which causal contact has been established at t. Eqs.(1.9) and (1.14) give

$$d_H(t) = \frac{3\gamma}{3\gamma - 2} t \ , \ \gamma \neq 2/3 \ . \qquad (1.15)$$

Also,

$$H(t) = \frac{2}{3\gamma} t^{-1} \ , \ d_H(t) = \frac{2}{3\gamma - 2} H^{-1}(t) \ . \qquad (1.16)$$

For 'matter' ('radiation'), this becomes $d_H(t) = 3t = 2H^{-1}(t)$ ($d_H(t) = 2t = H^{-1}(t)$). The present particle horizon is $d_H(t_0) = 2H_0^{-1} \approx 6,000\ h^{-1}$ Mpc, the present cosmic time is $t_0 = 2H_0^{-1}/3 \approx 6.7 \times 10^9\ h^{-1}$ years and the present value of the critical density is $\rho_c = 3H_0^2/8\pi G \approx 1.9 \times 10^{-29}\ h^{-2}$ gm/cm^3.

1.5 Brief History of the Early Universe

We will now briefly describe the early stages of the universe evolution according to GUTs [4]. We will take a GUT based on the gauge group G ($= SU(5)$, $SO(10)$, $SU(3)^3$, ...) with or without supersymmetry. At a superheavy scale $M_X \sim 10^{16}$ GeV (the GUT mass scale), G breaks to the standard model gauge group $G_S = SU(3)_c \times SU(2)_L \times U(1)_Y$ by the vacuum expectation value (vev) of an appropriate higgs field ϕ. (For simplicity, we will consider that this breaking occurs in just one step.) G_S is, subsequently, broken to $SU(3)_c \times U(1)_{em}$ at the electroweak scale M_W.

GUTs together with the SBB cosmological model (based on classical gravitation) provide a suitable framework for discussing the early history of the universe for cosmic times $\gtrsim 10^{-44}$ sec. They predict that the universe, as it expands and cools down after the big bang, undergoes [5] a series of phase transitions during which the initial gauge symmetry is gradually reduced and several important phenomena take place.

After the big bang, the GUT gauge group G was unbroken and the universe was filled with a hot 'soup' of massless particles which included not only photons, quarks, leptons and gluons but also the weak gauge boson $W^{\pm}$, Z^0, the GUT gauge bosons X, Y, ... as well as several higgs bosons. (In the supersymmetric case, all the supersymmetric partners of these particles were also present.) At cosmic time $t \sim 10^{-37}$ sec corresponding to temperature $T \sim 10^{16}$ GeV, G broke down to G_S and the X, Y, ... gauge bosons together with some higgs bosons acquired superheavy masses of order M_X. The out-of-equilibrium decay of these superheavy particles can produce [6] the observed baryon asymmetry of the universe (BAU). Important ingredients for this mechanism to work are the violation of baryon number, which is inherent in GUTs, and

C and CP violation. This is the second important success of the SBB model.

During the GUT phase transition, topologically stable extended objects [7] such as magnetic monopoles [8], cosmic strings [9] or domain walls [10] can also be produced. Monopoles, which exist in all GUTs, can lead into cosmological problems [11] which are, however, avoided by inflation [12, 13] (see Secs.2.3 and 3.3). This is a period of an exponentially fast expansion of the universe which can occur during some GUT phase transition. Strings can contribute [14] to the primordial density fluctuations necessary for structure formation [15] in the universe whereas domain walls are [10] absolutely catastrophic and GUTs predicting them should be avoided or inflation should be used to remove them from the scene.

At $t \sim 10^{-10}$ sec or $T \sim 100$ GeV, the electroweak transition takes place and G_S breaks to $SU(3)_c \times U(1)_{em}$. The $W^{\pm}$, Z^0 gauge bosons together with the electroweak higgs fields acquire masses $\sim M_W$. Subsequently, at $t \sim 10^{-4}$ sec or $T \sim 1$ GeV, color confinement sets in and the quarks get bounded forming hadrons.

The direct involvement of particle physics essentially ends here since most of the subsequent phenomena fall into the realm of other branches. We will, however, sketch some of them since they are crucial for understanding the earlier stages of the universe evolution where their origin lies.

At $t \approx 180$ sec ($T \approx 1$ MeV), nucleosynthesis takes place, i.e., protons and neutrons form nuclei. The abundance of light elements (D, ^3He, ^4He and ^7Li) depends [16] crucially on the number of light particles (with mass $\lesssim 1$ MeV), i.e., the number of light neutrinos, N_ν, and $\Omega_B h^2$. Agreement with observations [3] is achieved for $N_\nu = 3$ and $\Omega_B h^2 \approx 0.019$. This is the third success of SBB cosmology. Much later, at the so called 'equidensity' point, $t_{\rm eq} \approx 3,000$ years, 'matter' dominates over 'radiation'.

At cosmic time $t \approx 200,000\ h^{-1}$years ($T \approx 3,000$ K), we have the 'decoupling' of 'matter' and 'radiation' and the 'recombination' of atoms. After this, 'radiation' evolves as an independent (not interacting) component of the universe and is detected today as CBR with temperature $T_0 \approx 2.73$ K. The existence of this radiation is the fourth important success of the theory of big bang. Finally, structure formation [15] in the universe starts at $t \approx 2 \times 10^8$ years.

2. Shortcomings of Big Bang

The SBB cosmological model has been very successful in explaining, among other things, the Hubble expansion of the universe, the existence of the CBR and the abundances of the light elements which were formed during primordial nucleosynthesis. Despite its great successes, this model had a number of long-standing shortcomings which we will now summarize:

2.1 Horizon Problem

The CBR, which we receive now, was emitted at the time of 'decoupling' of matter and radiation when the cosmic temperature was $T_d \approx 3,000$ K. The decoupling time, t_d, can be calculated from

$$\frac{T_0}{T_d} = \frac{2.73\ \mathrm{K}}{3,000\ \mathrm{K}} = \frac{a(t_d)}{a(t_0)} = \left(\frac{t_d}{t_0}\right)^{2/3} \cdot \qquad (2.1)$$

It turns out that $t_d \approx 200,000\ h^{-1}$ years.

The distance over which the photons of the CBR have travelled since their emission is

$$a(t_0)\int_{t_d}^{t_0} \frac{dt'}{a(t')} = 3t_0 \left[1 - \left(\frac{t_d}{t_0}\right)^{2/3}\right]$$
$$\approx 3t_0 \approx 6,000\ h^{-1}\ \mathrm{Mpc}\ , \qquad (2.2)$$

which essentially coincides with the present particle horizon size. A sphere around us with radius equal to this distance is called the 'last scattering surface' since the CBR observed now has been emitted from it. The particle horizon size at t_d was $2H^{-1}(t_d) = 3t_d \approx 0.168\ h^{-1}$ Mpc and expanded till the present time to become equal to $0.168\ h^{-1}(a(t_0)/a(t_d))$ Mpc $\approx 184\ h^{-1}$ Mpc. The angle subtended by this 'decoupling' horizon at present is $\theta_d \approx 184/6,000 \approx 0.03$ rads $\approx 2\ ^{\circ}$. Thus, the sky splits into $4\pi/(0.03)^2 \approx 14,000$ patches that never communicated causally before sending light to us. The question then arises how come the temperature of the black body radiation from all these patches is so accurately tuned as the measurements of the cosmic background explorer [2] (COBE) require ($\delta T/T \approx 6.6 \times 10^{-6}$).

2.2 Flatness Problem

The present energy density, ρ, of the universe has been observed to lie in the relatively narrow range $0.1\rho_c \lesssim \rho \lesssim 2\rho_c$, where ρ_c is the critical energy density corresponding to a flat universe. The lower bound has been derived from estimates of galactic masses using the virial theorem whereas the upper bound from the volume expansion rate implied by the behavior of galactic number density at large distances. Eq.(1.8) implies that $(\rho - \rho_c)/\rho_c = 3(8\pi G\rho_c)^{-1}(k/a^2)$ is proportional to a, for matter dominated universe. Consequently, in the early universe, we have $|(\rho - \rho_c)/\rho_c| \ll 1$ and the question arises why the initial energy density of the universe was so finely tuned to be equal to its critical value.

2.3 Magnetic Monopole Problem

This problem arises only if we combine the SBB model with GUTs [4] of strong, weak and electromagnetic interactions. As already indicated, according to GUTs, the universe underwent [5] a phase transition during which the GUT gauge symmetry group, G, broke to G_S. This breaking was due to the fact that, at a critical temperature T_c, an appropriate higgs field, ϕ, developed a nonzero vev. Assuming that this phase transition was a second order one, we have $\langle\phi\rangle(T) \approx \langle\phi\rangle(T=0)(1 - T^2/T_c^2)^{1/2}$, $m_H(T) \approx \lambda\langle\phi\rangle(T)$, for the temperature dependent vev and mass of the higgs field respectively at $T \leq T_c$ (λ is an appropriate higgs coupling constant).

The GUT phase transition produces magnetic monopoles [8] which are localized deviations from the vacuum with radius $\sim M_X^{-1}$, energy $\sim M_X/\alpha_G$ and $\phi = 0$ at their center ($\alpha_G = g_G^2/4\pi$ with g_G being the GUT gauge coupling constant). The vev of the higgs field on a sphere, S^2, with radius $\gg M_X^{-1}$ around the monopole lies on the vacuum manifold G/G_S and we, thus, obtain a mapping: $S^2 \longrightarrow G/G_S$. If this mapping is homotopically nontrivial the topological stability of the monopole is guaranteed.

Monopoles can be produced when the fluctuations of ϕ over $\phi = 0$ between the vacua at $\pm\langle\phi\rangle(T)$ cease to be frequent. This takes place when the free energy needed for ϕ to fluctuate from $\langle\phi\rangle(T)$ to zero in a region of radius equal to the higgs correlation length $\xi(T) = m_H^{-1}(T)$ exceeds T. This condition reads $(4\pi/3)\xi^3\Delta V \gtrsim T$, where $\Delta V \sim \lambda^2\langle\phi\rangle^4$ is the difference in free energy density between $\phi = 0$ and $\phi = \langle\phi\rangle(T)$. The Ginzburg temperature [17], T_G, corresponds to the saturation of this inequality. So, at $T \lesssim T_G$, the fluctuations over $\phi = 0$ stop and $\langle\phi\rangle$ settles on the vacuum manifold G/G_S. At T_G, the universe splits into regions of size $\xi_G \sim (\lambda^2 T_c)^{-1}$, the higgs correlation length at T_G, with the higgs field being more or less aligned in each region. Monopoles are produced at the corners where such regions meet (Kibble [7] mechanism) and their number density is estimated to be $n_M \sim \mathrm{p}\xi_G^{-3} \sim \mathrm{p}\lambda^6 T_c^3$, where $\mathrm{p} \sim 1/10$ is a geometric factor. The 'relative' monopole number density then turns out to be $r_M = n_M/T^3 \sim 10^{-6}$. We can derive a lower bound on r_M by employing causality. The higgs field ϕ cannot be correlated at distances bigger than the particle horizon size, $2t_G$, at T_G. This gives the causality bound

$$n_M \gtrsim \frac{\mathrm{p}}{\frac{4\pi}{3}(2t_G)^3} , \tag{2.3}$$

which implies that $r_M \gtrsim 10^{-10}$.

The subsequent evolution of monopoles, after T_G, is governed by the equation [11]

$$\frac{dn_M}{dt} = -Dn_M^2 - 3\frac{\dot{a}}{a}n_M , \tag{2.4}$$

where the first term in the rhs (with D being an appropriate constant) describes the dilution of monopoles due to their annihilation with antimonopoles while the second term corresponds to their dilution by the general cosmological expansion. The monopole-antimonopole annihilation proceeds as follows. Monopoles diffuse towards antimonopoles in the plasma of charged particles, capture each other in Bohr orbits and eventually annihilate. The annihilation is effective provided the mean free path of monopoles in the plasma of charged particles does not exceed their capture distance. This happens at cosmic temperatures $T \gtrsim 10^{12}$ GeV. The overall result is that, if the initial relative magnetic monopole density $r_{M,\mathrm{in}} \gtrsim 10^{-9} (\lesssim 10^{-9})$, the final one $r_{M,\mathrm{fin}} \sim 10^{-9} (\sim r_{M,\mathrm{in}})$. This combined with the causality bound yields $r_{M,\mathrm{fin}} \gtrsim 10^{-10}$. However, the requirement that monopoles do not dominate the energy density of the universe at nucleosynthesis gives

$$r_M(T \approx 1 \text{ MeV}) \lesssim 10^{-19} \ , \qquad (2.5)$$

and we obtain a clear discrepancy of about ten orders of magnitude.

2.4 Density Fluctuations

For structure formation [15] in the universe, we need a primordial density perturbation, $\delta\rho/\rho$, at all length scales with a nearly flat spectrum [18]. We also need some explanation of the temperature fluctuations, $\delta T/T$, of CBR observed by COBE [2] at angles $\theta \gtrsim \theta_d \approx 2\ ^o$ which violate causality (see Sec.2.1).

Let us expand $\delta\rho/\rho$ in plane waves

$$\frac{\delta\rho}{\rho}(\bar{r},t) = \int d^3k\delta_{\bar{k}}(t)e^{i\bar{k}\bar{r}} \ , \qquad (2.6)$$

where $\bar{r}$ is a 'comoving' vector in 3-space and $\bar{k}$ is the 'comoving' wave vector with $k = |\bar{k}|$ being the 'comoving' wave number ($\lambda = 2\pi/k$ is the 'comoving' wave length whereas the physical wave length is $\lambda_{\rm phys} = a(t)\lambda$). For $\lambda_{\rm phys} \leq H^{-1}$, the time evolution of $\delta_{\bar{k}}$ is described by the Newtonian equation

$$\ddot{\delta}_{\bar{k}} + 2H\dot{\delta}_{\bar{k}} + \frac{v_s^2k^2}{a^2}\delta_{\bar{k}} = 4\pi G\rho\delta_{\bar{k}} \ , \qquad (2.7)$$

where the second term in the left hand side (lhs) comes from the cosmological expansion and the third is the 'pressure' term (v_s is the velocity of sound given by $v_s^2 = dp/d\rho$, where p is the mean pressure). The rhs of this equation corresponds to the gravitational attraction.

For the moment, let us put H=0 (static universe). In this case, there exists a characteristic wave number k_J, the Jeans wave number, given by $k_J^2 = 4\pi Ga^2\rho/v_s^2$ and having the following property. For $k \geq k_J$, pressure dominates over gravitational attraction and the density perturbations just oscillate, whereas, for $k \leq k_J$, gravitational attraction dominates and the density perturbations grow exponentially. In particular, for p=0 (matter domination), $v_s = 0$ and all scales are Jeans unstable with

$$\delta_{\bar{k}} \propto \exp(t/\tau) \ , \ \tau = (4\pi G\rho)^{-1/2} \ . \qquad (2.8)$$

Now let us take $H \neq 0$. Since the cosmological expansion pulls the particles apart, we get a smaller growth:

$$\delta_{\bar{k}} \propto a(t) \propto t^{2/3} \ , \qquad (2.9)$$

in the matter dominated case. For a radiation dominated universe ($p \neq 0$), we get essentially no growth of the density perturbations. This means that, in order to have structure formation in the universe, which requires $\delta\rho/\rho \sim 1$, we must have

$$(\frac{\delta\rho}{\rho})_{\rm eq} \sim 4 \times 10^{-5}(\Omega_0 h)^{-2} \ , \qquad (2.10)$$

at the 'equidensity' point (where the energy densities of matter and radiation coincide), since the available growth factor for perturbations is given by $a_0/a_{\rm eq} \sim 2.5 \times 10^4(\Omega_0 h)^2$. Here $\Omega_0 = \rho_0/\rho_c$, where ρ_0 is the present energy density of the universe. The question then is where these primordial density fluctuations originate from.

3. Inflation

Inflation [12, 13] is an idea which solves simultaneously all four cosmological puzzles and can be summarized as follows. Suppose there is a real scalar field ϕ (the inflaton) with (symmetric) potential energy density $V(\phi)$ which is quite 'flat' near $\phi = 0$ and has minima at $\phi = \pm\langle\phi\rangle$ with $V(\pm\langle\phi\rangle) = 0$. At high enough T's, $\phi = 0$ in the universe due to the temperature corrections in $V(\phi)$. As T drops, the effective potential density approaches the T=0 potential but a little potential barrier separating the local minimum at $\phi = 0$ and the vacua at $\phi = \pm\langle\phi\rangle$ still remains. At some point, ϕ tunnels out to $\phi_1 \ll \langle\phi\rangle$ and a bubble with $\phi = \phi_1$ is created in the universe. The field then rolls over to the minimum of $V(\phi)$ very slowly (due to the flatness of the potential). During this slow roll over, the energy density $\rho \approx V(\phi = 0) \equiv V_0$ remains essentially constant for quite some time. The Lagrangian density

$$L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \qquad (3.1)$$

gives the energy momentum tensor

$$T_\mu{}^\nu = -\partial_\mu\phi\partial^\nu\phi + \delta_\mu{}^\nu\left(\frac{1}{2}\partial_\lambda\phi\partial^\lambda\phi - V(\phi)\right) \ , \qquad (3.2)$$

which during the slow roll over takes the form $T_\mu{}^\nu \approx -V_0\ \delta_\mu{}^\nu$. This means that $\rho \approx -p \approx V_0$, i.e., the pressure p is negative and equal in magnitude with the energy density ρ, which is consistent with Eq.(1.5). Since, as we will see, $a(t)$ grows very fast, the 'curvature' term, k/a^2, in Eq.(1.8) becomes subdominant and we get

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} V_0 \ , \tag{3.3}$$

which gives $a(t) \propto e^{Ht}$, $H^2 = (8\pi G/3)V_0 =$ constant. So the bubble expands exponentially for some time and $a(t)$ grows by a factor

$$\frac{a(t_f)}{a(t_i)} = \exp H(t_f - t_i) \equiv \exp H\tau \ , \tag{3.4}$$

between an initial (t_i) and a final (t_f) time.

The inflationary scenario just described here, known as new [19] inflation (with the inflaton field starting from the origin, ϕ=0), is certainly not the only realization of the idea of inflation. Another interesting possibility is to consider the universe as it emerges at the Planck time t_P, where the fluctuations of gravity cease to exist. We can imagine a region of size $\ell_P \sim M_P^{-1}$ where the inflaton field acquires a large and almost uniform value and carries negligible kinetic energy. Under certain circumstances this region can inflate (exponentially expand) as ϕ rolls down towards its vacuum value. This type of inflation with the inflaton starting from large values is known as the chaotic [20] inflationary scenario.

We will now show that, with an adequate number of e-foldings, $N = H\tau$, the first three cosmological puzzles are easily resolved (we leave the question of density perturbations for later).

3.1 Resolution of the Horizon Problem

The particle horizon during inflation (exponential expansion)

$$d(t) = e^{Ht} \int_{t_i}^{t} \frac{dt'}{e^{Ht'}} \approx H^{-1} \exp H(t - t_i) \ , \tag{3.5}$$

for $t - t_i \gg H^{-1}$, grows as fast as $a(t)$. At the end of inflation $(t = t_f)$, $d(t_f) \approx H^{-1} \exp H\tau$ and the field ϕ starts oscillating about the minimun of the potential at $\phi = \langle\phi\rangle$. It then decays and 'reheats' [21] the universe at a temperature $T_r \sim 10^9$ GeV [22]. The universe, after that, goes back to normal big bang cosmology. The horizon $d(t_f)$ is stretched during the period of ϕ-oscillations by some factor $\sim 10^9$ depending on details and between T_r and the present era by a factor T_r/T_0. So it finally becomes equal to $H^{-1}e^{H\tau}10^9(T_r/T_0)$, which should exceed $2H_0^{-1}$ in order to solve the horizon problem. Taking $V_0 \approx M_X^4$, $M_X \sim 10^{16}$ GeV, we see that, with $N = H\tau \gtrsim 55$, the horizon problem is evaded.

3.2 Resolution of the Flatness Problem

The 'curvature' term of the Friedmann equation, at present, is given by

$$\frac{k}{a^2} \approx \left(\frac{k}{a^2}\right)_{bi} e^{-2H\tau}\ 10^{-18} \left(\frac{10^{-13}\ \text{GeV}}{10^9\ \text{GeV}}\right)^2 , \tag{3.6}$$

where the terms in the rhs correspond to the 'curvature' term before inflation, and its growth factors during inflation, during ϕ-oscillations and after 'reheating' respectively. Assuming $(k/a^2)_{bi} \sim (8\pi G/3)\rho \sim H^2 \quad (\rho \approx V_0)$, we get $k/a_0^2H_0^2 \sim 10^{48}\ e^{-2H\tau}$ which gives $(\rho_0 - \rho_c)/\rho_c \equiv \Omega_0 - 1 = k/a_0^2H_0^2 \ll 1$, for $H\tau \gg 55$. In fact, strong inflation implies that the present universe is flat with a great accuracy.

3.3 Resolution of the Monopole Problem

It is obvious that, with a number of e-foldings $\gtrsim 55$, the primordial monopole density is diluted by at least 70 orders of magnitude and they become totally irrelevant. Also, since $T_r \ll m_M$ (=the monopole mass), there is no production of magnetic monopoles after 'reheating'.

4. Detailed Analysis of Inflation

The Hubble parameter is not exactly constant during inflation as we, naively, assumed so far. It actually depends on the value of ϕ:

$$H^2(\phi) = \frac{8\pi G}{3} V(\phi) \ . \tag{4.1}$$

To find the evolution equation for ϕ during inflation, we vary the action

$$\int \sqrt{-\det(g)}\ d^4x \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + M(\phi)\right) , \tag{4.2}$$

where g is the metric tensor and $M(\phi)$ represents the coupling of ϕ to 'light' matter causing its decay. We find

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma_\phi \dot{\phi} + V'(\phi) = 0 \ , \qquad (4.3)$$

where the prime denotes derivation with respect to ϕ and Γ_ϕ is the decay width [23] of the inflaton. Assume, for the moment, that the decay time of ϕ, $t_d = \Gamma_\phi^{-1}$, is much greater than H^{-1}, the expansion time for inflation. Then the term $\Gamma_\phi \dot{\phi}$ can be ignored and Eq.(4.3) reduces to

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \ . \qquad (4.4)$$

Inflation is by definition the situation where $\ddot{\phi}$ is subdominant to the 'friction' term $3H\dot{\phi}$ in this equation (and the kinetic energy density is subdominant to the potential one). Eq.(4.4) then further reduces to the inflationary equation [24]

$$3H\dot{\phi} = -V'(\phi) \ , \qquad (4.5)$$

which gives

$$\ddot{\phi} = -\frac{V''(\phi)\dot{\phi}}{3H(\phi)} + \frac{V'(\phi)}{3H^2(\phi)} H'(\phi)\dot{\phi} \ . \qquad (4.6)$$

Comparing the two terms in the rhs of this equation with the 'friction' term in Eq.(4.4), we get the conditions for inflation (slow roll conditions):

$$\eta \equiv \frac{M_P^2}{8\pi} \left| \frac{V''(\phi)}{V(\phi)} \right| \leq 1 \ , \ \epsilon \equiv \frac{M_P^2}{16\pi} \left(\frac{V'(\phi)}{V(\phi)} \right)^2 \leq 1 \ . \qquad (4.7)$$

The end of the slow roll over occurs when either of the these inequalities is saturated. If ϕ_f is the value of ϕ at the end of inflation, then $t_f \sim H^{-1}(\phi_f)$.

The number of e-foldings during inflation can be calculated as follows:

$$N(\phi_i \to \phi_f) \equiv \ell n \left(\frac{a(t_f)}{a(t_i)} \right) = \int_{t_i}^{t_f} H dt =$$

$$\int_{\phi_i}^{\phi_f} \frac{H(\phi)}{\dot{\phi}} d\phi = -\int_{\phi_i}^{\phi_f} \frac{3H^2(\phi) d\phi}{V'(\phi)} \ , \qquad (4.8)$$

where Eqs.(3.4), (4.5) and the definition of $H = \dot{a}/a$ were used. For simplicity, we can shift the field ϕ so that the global minimum of the potential is displaced at $\phi = 0$. Then, if $V(\phi) = \lambda \phi^\nu$ during inflation, we have

$$N(\phi_i \to \phi_f) = -\int_{\phi_i}^{\phi_f} \frac{3H^2(\phi) d\phi}{V'(\phi)} =$$

$$-8\pi G \int_{\phi_i}^{\phi_f} \frac{V(\phi) d\phi}{V'(\phi)} = \frac{4\pi G}{\nu} (\phi_i^2 - \phi_f^2) \ . \qquad (4.9)$$

Assuming that $\phi_i \gg \phi_f$, this reduces to $N(\phi) = (4\pi G/\nu)\phi^2$.

5. Coherent Field Oscillations

After the end of inflation at cosmic time t_f, the term $\ddot{\phi}$ takes over and Eq.(4.4) reduces to $\ddot{\phi} + V'(\phi) = 0$, which means that ϕ starts oscillating coherently about the global minimum of the potential. In reality, due to the 'friction' term, ϕ performs damped oscillations with a rate of energy density loss given by

$$\dot{\rho} = \frac{d}{dt} \left(\frac{1}{2}\dot{\phi}^2 + V(\phi) \right) = -3H\dot{\phi}^2 = -3H(\rho + p) \ , \qquad (5.1)$$

where $\rho = \dot{\phi}^2/2 + V(\phi)$ and the pressure $p = \dot{\phi}^2/2 - V(\phi)$. Averaging p over one oscillation of ϕ and writing [25] $\rho + p = \gamma\rho$, we get $\rho \propto a^{-3\gamma}$ and $a(t) \propto t^{2/3\gamma}$ (see Sec.1.2).

The number γ for an oscillating field can be written as (assuming a symmetric potential)

$$\gamma = \frac{\int_0^T \dot{\phi}^2 dt}{\int_0^T \rho dt} = \frac{\int_0^{\phi_{\max}} \dot{\phi} d\phi}{\int_0^{\phi_{\max}} (\rho/\dot{\phi}) d\phi} \ , \qquad (5.2)$$

where T and $\phi_{\max}$ are the period and the amplitude of the oscillation respectively. From the equation $\rho = \dot{\phi}^2/2 + V(\phi) = V_{\max}$, where $V_{\max}$ is the maximal potential energy density, we obtain $\dot{\phi} = \sqrt{2(V_{\max} - V(\phi))}$. Substituting this in Eq.(5.2) we get [25]

$$\gamma = \frac{2\int_0^{\phi_{\max}} (1 - V/V_{\max})^{1/2} d\phi}{\int_0^{\phi_{\max}} (1 - V/V_{\max})^{-1/2} d\phi} \ . \qquad (5.3)$$

For a potential of the simple form $V(\phi) = \lambda\phi^\nu$, γ is readily found to be given by $\gamma = 2\nu/(\nu + 2)$. Consequently, in this case, $\rho \propto a^{-6\nu/(\nu+2)}$ and $a(t) \propto t^{(\nu+2)/3\nu}$. For $\nu = 2$, in particular, one has γ=1, $\rho \propto a^{-3}$, $a(t) \propto t^{2/3}$ and the oscillating field behaves like pressureless 'matter'. This is not unexpected since a coherent oscillating massive free field corresponds to a distribution of static massive particles. For ν=4, however, we obtain $\gamma = 4/3$, $\rho \propto a^{-4}$, $a(t) \propto t^{1/2}$ and the system resembles 'radiation'. For $\nu = 6$, one has $\gamma = 3/2$, $\rho \propto a^{-4.5}$, $a(t) \propto t^{4/9}$ and the expansion is slower than in a radiation dominated universe (the pressure is higher than in 'radiation').

6. Decay of the Field ϕ

Reintroducing the 'decay' term $\Gamma_\phi \dot{\phi}$, Eq.(4.3) can be written as

$$\dot{\rho} = \frac{d}{dt}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) = -(3H + \Gamma_\phi)\dot{\phi}^2 \ , \quad (6.1)$$

which is solved [21, 25] by

$$\rho(t) = \rho_f \left(\frac{a(t)}{a(t_f)}\right)^{-3\gamma} \exp[-\gamma\Gamma_\phi(t - t_f)] \ , \quad (6.2)$$

where ρ_f is the energy density at the end of inflation at cosmic time t_f. The second and third factors in the rhs of this equation represent the dilution of the field energy due to the expansion of the universe and the decay of ϕ to light particles respectively.

All pre-existing 'radiation' (known as 'old radiation') was diluted by inflation, so the only 'radiation' present is the one produced by the decay of ϕ and is known as 'new radiation'. Its energy density satisfies [21, 25] the equation

$$\dot{\rho}_r = -4H\rho_r + \gamma\Gamma_\phi\rho \ , \quad (6.3)$$

where the first term in the rhs represents the dilution of radiation due to the cosmological expansion while the second one is the energy density transfer from ϕ to 'radiation'. Taking $\rho_r(t_f)$=0, this equation gives [21, 25]

$$\rho_r(t) = \rho_f \left(\frac{a(t)}{a(t_f)}\right)^{-4}$$

$$\int_{t_f}^{t} \left(\frac{a(t')}{a(t_f)}\right)^{4-3\gamma} e^{-\gamma\Gamma_\phi(t'-t_f)} \ \gamma\Gamma_\phi dt' \ . \quad (6.4)$$

For $t_f \ll t_d$ and $\nu = 2$, this expression is approximated by

$$\rho_r(t) = \rho_f \left(\frac{t}{t_f}\right)^{-8/3} \int_0^t \left(\frac{t'}{t_f}\right)^{2/3} e^{-\Gamma_\phi t'} dt' \ , \quad (6.5)$$

which, using the formula

$$\int_0^u x^{p-1} e^{-x} dx = e^{-u} \sum_{k=0}^{\infty} \frac{u^{p+k}}{p(p+1)\cdots(p+k)} \ , \quad (6.6)$$

can be written as

$$\rho_r = \frac{3}{5} \ \rho \ \Gamma_\phi t \left[1 + \frac{3}{8} \ \Gamma_\phi t + \frac{9}{88} \ (\Gamma_\phi t)^2 + \cdots\right] \ , \quad (6.7)$$

with $\rho = \rho_f (t/t_f)^{-2} \exp(-\Gamma_\phi t)$ being the energy density of the field ϕ which performs damped oscillations and decays into 'light' particles.

The energy density of the 'new radiation' grows relative to the energy density of the oscillating field and becomes essentially equal to it at a cosmic time $t_d = \Gamma_\phi^{-1}$ as one can deduce from Eq.(6.7). After this time, the universe enters into the radiation dominated era and the normal big bang cosmology is recovered. The temperature at t_d, $T_r(t_d)$, is historically called the 'reheat' temperature although no supercooling and subsequent reheating of the universe actually takes place. Using the time to temperature relation in Eq.(1.12) for a radiation dominated universe we find that

$$T_r = \left(\frac{45}{16\pi^3 g_*}\right)^{1/4} (\Gamma_\phi M_P)^{1/2} \ , \quad (6.8)$$

where g_* is the effective number of degrees of freedom. For a potential of the type $V(\phi) = \lambda\phi^\nu$, the total expansion of the universe during the period of damped field oscillations is

$$\frac{a(t_d)}{a(t_f)} = \left(\frac{t_d}{t_f}\right)^{\frac{\nu+2}{3\nu}} \ . \quad (6.9)$$

7. Density Perturbations

We are ready to sketch how inflation solves the density fluctuation problem described in Sec.2.4. As a matter of fact, inflation not only homogenizes the universe but also provides us with the primordial density fluctuations necessary for the structure formation in the universe. To understand the origin of these fluctuations, we must first introduce the notion of 'event horizon'. Our 'event horizon', at a cosmic time t, includes all points with which we will eventually communicate sending signals at t. The 'instantaneous' (at cosmic time t) radius of the 'event horizon' is

$$d_e(t) = a(t) \int_t^\infty \frac{dt'}{a(t')} \cdot \quad (7.1)$$

It is obvious, from this formula, that the 'event horizon' is infinite for matter or radiation dominated universe. For inflation, however, we obtain a slowly varying 'event horizon' with radius $d_e(t) = H^{-1} < \infty$. Points, in our 'event horizon' at t, with which we can communicate sending signals at t, are eventually pulled away by the 'exponential' expansion and we cease to be able to communicate with them again emitting signals at later times. We say that these points (and the corresponding scales) crossed outside the 'event horizon'. The situation is very similar to that of a black hole. Indeed, the exponentially expanding (de Sitter) space is like a black hole turned inside out. This means that we are inside and the black hole surrounds us from all sides. Then, exactly as in a black hole, there are quantum fluctuations of the 'thermal type' governed by the 'Hawking temperature' [26, 27] $T_H = H/2\pi$. It turns out [28, 29] that the quantum fluctuations of all massless fields (the inflaton is nearly massless due to the 'flatness' of the potential) are $\delta\phi = H/2\pi = T_H$. These fluctuations of ϕ lead to energy density fluctuations $\delta\rho = V'(\phi)\delta\phi$. As the scale of this perturbations crosses outside the 'event horizon', they become [30] classical metric perturbations.

The evolution of these fluctuations outside the 'inflationary horizon' is quite subtle and involved due to the gauge freedom in general relativity. However, there is a simple gauge invariant quantity [31] $\zeta \approx \delta\rho/(\rho + p)$, which remains constant outside the horizon. Thus, the density fluctuation at any present physical ('comoving') scale ℓ, $(\delta\rho/\rho)_\ell$, when this scale crosses inside the post-inflationary particle horizon (p=0 at this instance) can be related to the value of ζ when the same scale crossed outside the inflationary 'event horizon' (symbolically at $\ell \sim H^{-1}$). This latter value of ζ can be found using Eq.(4.5) and turns out to be

$$\zeta \mid_{\ell\sim H^{-1}} = \left(\frac{\delta\rho}{\dot{\phi}^2}\right)_{\ell\sim H^{-1}} = \left(\frac{V'(\phi)H(\phi)}{2\pi\dot{\phi}^2}\right)_{\ell\sim H^{-1}} = -\left(\frac{9H^3(\phi)}{2\pi V'(\phi)}\right)_{\ell\sim H-1} . \tag{7.2}$$

Taking into account an extra $2/5$ factor from the fact that the universe is matter dominated when the scale ℓ re-enters the horizon, we obtain

$$\left(\frac{\delta\rho}{\rho}\right)_\ell = \frac{16\sqrt{6\pi}}{5}\,\frac{V^{3/2}(\phi_\ell)}{M_P^3 V'(\phi_\ell)} . \tag{7.3}$$

The calculation of ϕ_ℓ, the value of the inflaton field when the 'comoving' scale ℓ crossed outside the 'event horizon', goes as follows. A 'comoving' (present physical) scale ℓ, at T_r, was equal to $\ell(a(t_d)/a(t_0)) = \ell(T_0/T_r)$. Its magnitude at the end of inflation ($t = t_f$) was equal to $\ell(T_0/T_r)(a(t_f)/a(t_d)) = \ell(T_0/T_r)(t_f/t_d)^{(\nu+2)/3\nu} \equiv \ell_{\rm phys}(t_f)$, where the potential $V(\phi) = \lambda\phi^\nu$ was assumed. The scale ℓ, when it crossed outside the inflationary horizon, was equal to $H^{-1}(\phi_\ell)$. We, thus, obtain

$$H^{-1}(\phi_\ell)e^{N(\phi_\ell)} = \ell_{\rm phys}(t_f) . \tag{7.4}$$

Solving this equation, one can calculate ϕ_ℓ and, thus, $N(\phi_\ell) \equiv N_\ell$, the number of e-foldings the scale ℓ suffered during inflation. In particular, for our present horizon scale $\ell \approx 2H_0^{-1} \sim 10^4$ Mpc, it turns out that $N_{H_0} \approx 50 - 60$.

Taking the potential $V(\phi) = \lambda\phi^4$, Eqs.(4.9), (7.3) and (7.4) give

$$\left(\frac{\delta\rho}{\rho}\right)_\ell = \frac{4\sqrt{6\pi}}{5}\lambda^{1/2}\left(\frac{\phi_\ell}{M_P}\right)^3 = \frac{4\sqrt{6\pi}}{5}\lambda^{1/2}\left(\frac{N_\ell}{\pi}\right)^{3/2} . \tag{7.5}$$

The measurements of COBE [2], $(\delta\rho/\rho)_{H_0} \approx 6 \times 10^{-5}$, then imply that $\lambda \approx 6 \times 10^{-14}$ for $N_{H_0} \approx 55$. Thus, we see that the inflaton must be a very weakly coupled field. In nonsupersymmetric GUTs, the inflaton is necessarily gauge singlet since otherwise radiative corrections will certainly make it strongly coupled. This is, undoubtedly, not a very satisfactory situation since we are forced to introduce an otherwise unmotivated extra *ad hoc* very weakly coupled gauge singlet. In supersymmetric GUTs, however, the inflaton could be identified [32] with a conjugate pair of gauge nonsinglet fields ϕ, $\bar{\phi}$, already existing in the theory and causing the gauge symmetry breaking. Absence of strong radiative corrections from gauge interactions is guaranteed, in this case, by the mutual cancellation of the D terms of these fields.

The spectrum of density fluctuations which emerge from inflation can also be analyzed. We will again take the potential $V(\phi) = \lambda\phi^\nu$. One then finds that $(\delta\rho/\rho)_\ell$ is proportional to $\phi_\ell^{(\nu+2)/2}$ which, combined with the fact that $N(\phi_\ell)$ is proportional to ϕ_ℓ^2 (see Eq.(4.9)), gives

$$\left(\frac{\delta\rho}{\rho}\right)_\ell = \left(\frac{\delta\rho}{\rho}\right)_{H_0} \left(\frac{N_\ell}{N_{H_0}}\right)^{\frac{\nu+2}{4}} . \tag{7.6}$$

The scale ℓ divided by the size of our present horizon ($\approx 10^4$ Mpc) should equal $\exp(N_\ell - N_{H_0})$. This gives $N_\ell/N_{H_0} = 1 + \ell n(\ell/10^4)^{1/N_{H_0}}$ which expanded around $\ell \approx 10^4$ Mpc and substituted in Eq.(7.6) yields

$$\left(\frac{\delta\rho}{\rho}\right)_\ell = \left(\frac{\delta\rho}{\rho}\right)_{H_0} \left(\frac{\ell}{10^4 \text{ Mpc}}\right)^{\alpha_s} , \tag{7.7}$$

with $\alpha_s = (\nu+2)/4N_{H_0}$. For $\nu = 4$, $\alpha_s \approx 0.03$ and, thus, the density fluctuations are essentially scale independent.

8. Density Fluctuations in 'Matter'

We will now discuss the evolution of the primordial density fluctuations after their scale enters the post-inflationary horizon. To this end, we introduce [33] the 'conformal' time, η, so that the Robertson-Walker metric takes the form of a conformally expanding Minkowski space:

$$ds^2 = -dt^2 + a^2(t)\ d\bar{r}^2 = a^2(\eta)\ (-d\eta^2 + d\bar{r}^2) , \tag{8.1}$$

where $\bar{r}$ is a 'comoving' 3-vector. The Hubble parameter now takes the form $H \equiv \dot{a}(t)/a(t) = a'(\eta)/a^2(\eta)$ and the Friedmann Eq.(1.8) can be rewritten as

$$\frac{1}{a^2}\left(\frac{a'}{a}\right)^2 = \frac{8\pi G}{3}\rho , \tag{8.2}$$

where primes denote derivation with respect to the 'conformal' time η. The continuity Eq.(1.5) takes the form $\rho' = -3\tilde{H}(\rho+p)$ with $\tilde{H} = a'/a$. For a matter dominated universe, $\rho \propto a^{-3}$ which gives $a = (\eta/\eta_0)^2$ and $a'/a = 2/\eta$ (η_0 is the present value of η).

The Newtonian Eq.(2.7) can now be written in the form

$$\delta''_{\bar{k}}(\eta) + \frac{a'}{a}\delta'_{\bar{k}}(\eta) - 4\pi G\rho a^2 \delta_{\bar{k}}(\eta) = 0 , \tag{8.3}$$

and the growing (Jeans unstable) mode $\delta_{\bar{k}}(\eta)$ is proportional to η^2 and can be expressed [34] as

$$\delta_{\bar{k}}(\eta) = \epsilon_H \left(\frac{k\eta}{2}\right)^2 \hat{s}(\bar{k}) , \tag{8.4}$$

where $\hat{s}(\bar{k})$ is a Gaussian random variable satisfying

$$< \hat{s}(\bar{k}) >= 0 \ , \ \ < \hat{s}(\bar{k})\hat{s}(\bar{k}') >= \frac{1}{k^3}\delta(\bar{k}-\bar{k}') , \tag{8.5}$$

and ϵ_H is the amplitude of the perturbation when its scale crosses inside the post-inflationary horizon. The latter can be seen as follows. A 'comoving' (present physical) length ℓ crosses inside the post-inflationary horizon when $a\ell/2\pi = H^{-1} = a^2/a'$ which gives $\ell/2\pi \equiv k^{-1} = a/a' = \eta_H/2$ or $k\eta_H/2 = 1$, where η_H is the 'conformal' time at horizon crossing. This means that, at horizon crossing, $\delta_{\bar{k}}(\eta_H) = \epsilon_H \hat{s}(\bar{k})$. For scale invariant perturbations, the amplitude ϵ_H is constant. The gauge invariant perturbations of the scalar gravitational potential are given [33] by the Poisson's equation,

$$\Phi = -4\pi G \frac{a^2}{k^2}\rho\delta_{\bar{k}}(\eta) . \tag{8.6}$$

From the Friedmann Eq.(8.2), we then obtain

$$\Phi = -\frac{3}{2}\epsilon_H \hat{s}(\bar{k}) . \tag{8.7}$$

The spectrum of the density perturbations can be characterized by the correlation function ($\bar{x}$ is a 'comoving' 3-vector)

$$\xi(\bar{r}) \equiv < \tilde{\delta}^*(\bar{x},\eta)\tilde{\delta}(\bar{x}+\bar{r},\eta) > , \tag{8.8}$$

where

$$\tilde{\delta}(\bar{x},\eta) = \int d^3k \delta_{\bar{k}}(\eta) e^{i\bar{k}\bar{x}} . \tag{8.9}$$

Substituting Eq.(8.4) in Eq.(8.8) and then using Eq.(8.5), we obtain

$$\xi(\bar{r}) = \int d^3k e^{-i\bar{k}\bar{r}} \epsilon_H^2 \left(\frac{k\eta}{2}\right)^4 \frac{1}{k^3} , \tag{8.10}$$

and the spectral function $P(k,\eta) = \epsilon_H^2(\eta^4/16)k$ is proportional to k for ϵ_H constant. We say that, in this case, the 'spectral index' $n = 1$ and we have a Harrison-Zeldovich [18] flat spectrum. In the general case, $P \propto k^n$ with $n = 1 - 2\alpha_s$ (see Eq.(7.7)). For $V(\phi) = \lambda\phi^4$, we get $n \approx 0.94$.

9. Temperature Fluctuations

The density inhomogeneities produce temperature fluctuations in the CBR. For angles $\theta \gtrsim 2\ ^o$, the dominant effect is the scalar Sachs-Wolfe [35] effect. Density perturbations on the 'last scattering surface' cause scalar gravitational potential fluctuations, Φ, which, in turn, produce temperature fluctuations in the CBR. The physical reason is that regions with a deep gravitational potential will cause the photons to lose energy as they climb up the well and, thus, appear cooler. For $\theta \lesssim 2\,^o$, the dominant effects are: i) Motion of the last scattering surface causing Doppler shifts, and ii) Intrinsic fluctuations of the photon temperature, T_γ, which are more difficult to calculate since they depend on microphysics, the ionization history, photon streaming and other effects.

The temperature fluctuations at an angle θ due to the scalar Sachs-Wolfe effect turn out [35] to be $(\delta T/T)_\theta = -\Phi_\ell/3$, ℓ being the 'comoving' scale on the 'last scattering surface' which subtends the angle θ [$\ell \approx 100\, h^{-1}(\theta/\text{degrees})$ Mpc] and Φ_ℓ the corresponding scalar gravitational potential fluctuations. From Eq.(8.7), we then obtain $(\delta T/T)_\theta = (\epsilon_H/2)\hat{s}(\bar{k})$, which using Eq.(8.4) gives the relation

$$\left(\frac{\delta T}{T}\right)_\theta = \frac{1}{2}\delta_{\bar{k}}(\eta_H) = \frac{1}{2}\left(\frac{\delta\rho}{\rho}\right)_{\ell\sim 2\pi k^{-1}} \cdot \qquad (9.1)$$

The COBE scale (present horizon) corresponds to $\theta \approx 60\,^o$. Eqs.(4.9), (7.3) and (9.1) give

$$\left(\frac{\delta T}{T}\right)_\ell \propto \left(\frac{\delta\rho}{\rho}\right)_\ell \propto \frac{V^{3/2}(\phi_\ell)}{M_P^3 V'(\phi_\ell)} \propto N_\ell^{\frac{\nu+2}{4}}\ . \qquad (9.2)$$

Analyzing the temperature fluctuations in spherical harmonics, the quadrupole anisotropy due to the scalar Sachs-Wolfe effect can be obtained:

$$\left(\frac{\delta T}{T}\right)_{Q-S} = \left(\frac{32\pi}{45}\right)^{1/2} \frac{V^{3/2}(\phi_\ell)}{M_P^3 V'(\phi_\ell)} \cdot \qquad (9.3)$$

For $V(\phi) = \lambda\phi^\nu$, this becomes

$$\left(\frac{\delta T}{T}\right)_{Q-S} = \left(\frac{32\pi}{45}\right)^{1/2} \frac{\lambda^{1/2}\phi_\ell^{\frac{\nu+2}{2}}}{\nu M_P^3} =$$

$$\left(\frac{32\pi}{45}\right)^{1/2} \frac{\lambda^{1/2}}{\nu M_P^3}\left(\frac{\nu M_P^2}{4\pi}\right)^{\frac{\nu+2}{4}} N_\ell^{\frac{\nu+2}{4}}\ . \qquad (9.4)$$

Comparing this with COBE [2] measurements, $(\delta T/T)_Q \approx 6.6\times 10^{-6}$, we obtain $\lambda \approx 6\times 10^{-14}$, for $\nu = 4$, and number of e-foldings suffered by our present horizon scale during the inflationary phase $N_{\ell\sim H_0^{-1}} \equiv N_Q \approx 55$.

There are also 'tensor' [36] fluctuations in the temperature of CBR. The quadrupole tensor anisotropy is

$$\left(\frac{\delta T}{T}\right)_{Q-T} \approx 0.77\ \frac{V^{1/2}(\phi_\ell)}{M_P^2} \cdot \qquad (9.5)$$

The total quadrupole anisotropy is given by

$$\left(\frac{\delta T}{T}\right)_Q = \left[\left(\frac{\delta T}{T}\right)^2_{Q-S} + \left(\frac{\delta T}{T}\right)^2_{Q-T}\right]^{1/2}, \qquad (9.6)$$

and the ratio

$$r = \frac{(\delta T/T)^2_{Q-T}}{(\delta T/T)^2_{Q-S}} \approx 0.27\ \left(\frac{M_P V'(\phi_\ell)}{V(\phi_\ell)}\right)^2 \cdot \qquad (9.7)$$

For $V(\phi) = \lambda\phi^\nu$, we obtain $r \approx 3.4\ \nu/N_H \ll 1$, and the 'tensor' contribution to the temperature fluctuations of the CBR is negligible.

10. Hybrid Inflation

10.1 The non Supersymmetric Version

The most important disadvantage of the inflationary scenarios described so far is that they need extremely small coupling constants in order to reproduce the results of COBE [2]. This difficulty was overcome some years ago by Linde [37] who proposed, in the context of nonsupersymmetric GUTs, an inflationary scenario known as hybrid inflation. The idea was to use two real scalar fields χ and σ instead of one that was normally used. The field χ provides the vacuum energy which drives inflation while σ is the slowly varying field during inflation. The main advantage of this scenario is that it can reproduce the observed temperature fluctuations of the CBR with 'natural' values of the parameters in contrast to previous realizations of inflation (like the new [19] or chaotic [20] inflationary scenarios).

The potential utilized by Linde is

$$V(\chi,\sigma) = \kappa^2\left(M^2 - \frac{\chi^2}{4}\right)^2 + \frac{\lambda^2\chi^2\sigma^2}{4} + \frac{m^2\sigma^2}{2}\ , \qquad (10.1)$$

where κ, λ are dimensionless positive coupling constants and M, m are mass parameters. The vacua lie at $\langle\chi\rangle = \pm 2M$, $\langle\sigma\rangle = 0$. Putting m=0, for the moment, we observe that the potential possesses an exactly flat direction at $\chi = 0$ with $V(\chi = 0, \sigma) = \kappa^2 M^4$. The mass squared of the field χ along this flat direction is given by $m_\chi^2 = -\kappa^2 M^2 + \lambda^2\sigma^2/2$ and remains nonnegative for $\sigma \geq \sigma_c = \sqrt{2}\kappa M/\lambda$. This means that, at $\chi = 0$ and $\sigma \geq \sigma_c$, we obtain a valley of minima with flat bottom. Reintroducing the mass parameter m in Eq.(10.1), we observe that this valley acquires a nonzero slope. A region of the universe, where χ and σ happen to be almost uniform with negligible kinetic energies and with values close to the bottom of the valley of minima, follows this valley in its subsequent evolution and undergoes inflation.

The quadrupole anisotropy of CBR produced during this hybrid inflation can be estimated, using Eq.(9.3), to be

$$\left(\frac{\delta T}{T}\right)_Q \approx \left(\frac{16\pi}{45}\right)^{1/2} \frac{\lambda\kappa^2 M^5}{M_P^3 m^2} \; . \qquad (10.2)$$

The COBE [2] result, $(\delta T/T)_Q \approx 6.6 \times 10^{-6}$, can then be reproduced with $M \approx 2.86 \times 10^{16}$ GeV (the supersymmetric GUT vev) and $m \approx 1.3\,\kappa\sqrt{\lambda} \times 10^{15}$ GeV $\sim 10^{12}$ GeV for $\kappa, \lambda \sim 10^{-2}$.

Inflation terminates abruptly at $\sigma = \sigma_c$ and is followed by a 'waterfall', i.e., a sudden entrance into an oscillatory phase about a global minimum. Since the system can fall into either of the two available global minima with equal probability, topological defects are copiously produced if they are predicted by the particular particle physics model one is considering.

10.2 The Supersymmetric Version

The hybrid inflationary scenario is [38] 'tailor made' for application to supersymmetric GUTs except that the mass of σ, m, is unacceptably large for supersymmetry, where all scalar fields acquire masses of order $m_{3/2} \sim 1$ TeV (the gravitino mass) from soft supersymmetry breaking.

To see this, consider a supersymmetric GUT with a (semi-simple) gauge group G of rank ≥ 5 with $G \to G_S$ (the standard model gauge group) at a scale $M \sim 10^{16}$ GeV. The spectrum of the theory below M is assumed to coincide with the spectrum of the minimal supersymmetric standard model (MSSM) plus standard model singlets so that the successful predictions for α_s, $\sin^2\theta_W$ are retained. The theory may also possess global symmetries. The breaking of G is achieved through the superpotential

$$W = \kappa S(-M^2 + \bar{\phi}\phi), \qquad (10.3)$$

where $\bar{\phi}, \phi$ is a conjugate pair of G_S singlet left handed superfields belonging to nontrivial representations of G and reduce its rank by their vevs and S is a gauge singlet left handed superfield. The coupling constant κ and the mass parameter M can be made positive by phase redefinitions. This superpotential has the most general form consistent with a $U(1)$ R-symmetry under which $W \to e^{i\theta}W$, $S \to e^{i\theta}S$, $\bar{\phi}\phi \to \bar{\phi}\phi$.

The potential derived from the superpotential W in Eq.(10.3) is

$$V = \kappa^2 \mid M^2 - \bar{\phi}\phi \mid^2 + \kappa^2 \mid S \mid^2 (\mid \phi \mid^2 + \mid \bar{\phi} \mid^2)$$
$$+\mathrm{D} - \mathrm{terms}. \qquad (10.4)$$

Restricting ourselves to the D flat direction $\bar{\phi}^* = \phi$ which contains the supersymmetric vacua and performing appropriate gauge and R- transformations, we can bring S, $\bar{\phi}$, ϕ on the real axis, i.e., $S \equiv \sigma/\sqrt{2}$, $\bar{\phi} = \phi \equiv \chi/2$, where σ, χ are normalized real scalar fields. The potential then takes the form in Eq.(10.1) with $\kappa = \lambda$ and $m = 0$ and, thus, Linde's potential for hybrid inflation is almost obtainable from supersymmetric GUTs but without the mass term of σ which is, however, of crucial importance since it provides the slope of the valley of minima necessary for driving the inflaton towards the vacua.

One way to obtain a valley of minima useful for inflation is [39] to replace the renormalizable trilinear superpotential term in Eq.(10.3) by the next order nonrenormalizable coupling. Another way, which we will adopt here, is [40] to keep the renormalizable superpotential in Eq.(10.3) and use the radiative corrections along the inflationary valley ($\bar{\phi} = \phi = 0$, $S > S_c \equiv M$). In fact, the breaking of supersymmetry by the 'vacuum' energy density $\kappa^2 M^4$ along this valley causes a mass splitting in the supermultiplets $\bar{\phi}$, ϕ. This results to the existence of important radiative corrections on the inflationary valley. At one-loop, and for S sufficiently larger than S_c, the inflationary potential is given [40, 41] by

$$V_{\rm eff}(S) = \kappa^2 M^4$$
$$\left[1 + \frac{\kappa^2}{16\pi^2}\left(\ln\left(\frac{\kappa^2 S^2}{\Lambda^2}\right) + \frac{3}{2} - \frac{S_c^4}{12 S^4} + \cdots\right)\right] , \tag{10.5}$$

where Λ is a suitable mass renormalization scale.

From Eqs.(10.5) and (9.3), we find the cosmic microwave quadrupole anisotropy:

$$\left(\frac{\delta T}{T}\right)_Q \approx 8\pi \left(\frac{N_Q}{45}\right)^{1/2} \frac{x_Q}{y_Q} \left(\frac{M}{M_P}\right)^2 \cdot \tag{10.6}$$

Here N_Q is the number of e-foldings suffered by our present horizon scale during inflation and $y_Q = x_Q(1 - 7/(12x_Q^2) + \cdots)$ with $x_Q = S_Q/M$, S_Q being the value of the scalar field S when the scale which evolved to the present horizon size crossed outside the de Sitter (inflationary) horizon. Also, from Eq.(10.5), one finds

$$\kappa \approx \frac{8\pi^{3/2}}{\sqrt{N_Q}}\, y_Q\, \frac{M}{M_P} \cdot \tag{10.7}$$

Inflation ends as S approaches S_c. Writing $S = xS_c$, $x = 1$ corresponds to the phase transition from G to G_S which, as it turns out, more or less coincides with the end of the inflationary phase (this is checked by noting the amplitude of the quantities ϵ and η in Eq.(4.7)). Indeed, the $50 - 60$ e-foldings needed for the inflationary scenario can be realized even with small values of x_Q. For definiteness, we take $x_Q \approx 2$. From COBE [2] one then obtains $M \approx 5.5 \times 10^{15}$ GeV and $\kappa \approx 4.5 \times 10^{-3}$ for $N_Q \approx 56$. Moreover, the primordial density fluctuation 'spectral index' $n \simeq 0.98$. We see that the relevant part of inflation takes place at $S \sim 10^{16}$ GeV. An important consequence of this is [38, 41, 42] that the supergravity corrections can be brought under control so as to leave inflation intact.

After the end of inflation the system falls towards the supersymmetric minima, oscillates about them and eventually decays 'reheating' the universe. The oscillating system (inflaton) consists of the two complex scalar fields S and $\theta = (\delta\bar{\phi} + \delta\phi)/\sqrt{2}$, where $\delta\bar{\phi} = \bar{\phi} - M$, $\delta\phi = \phi - M$, with mass $m_{infl} = \sqrt{2}\kappa M$.

In conclusion, it is important to note that the superpotential W in Eq.(10.3) leads to the hybrid inflationary scenario in a 'natural' way. This means that a) there is no need of extremely small coupling constants, b) W is the most general renormalizable superpotential which is allowed by the gauge and R- symmetries, c) supersymmetry guarantees that the radiative corrections do not invalidate inflation, but rather provide a slope along the inflationary trajectory which drives the inflaton towards the supersymmetric vacua, and d) supergravity corrections can be negligible leaving inflation intact.

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