Right-handed (RH) mixings are not relevant in the framework of the standard model (SM). Also, RH currents have not been observed experimentally (yet?). So, why are RH mixings interesting?

What are RH mixings?
To diagonalize a general complex (mass) matrix $M$ one needs a bi-unitary transformation, i.e. two unitary matrices $U_{L,R}$, such that

$$ U_L^\dagger M U_R = M_{\text{diagonal}} $$

or

$$ U_L^\dagger M M^\dagger U_L = (M_{\text{diag}})^2 = U_R^\dagger M^\dagger M U_R. $$

Only in the case of hermitian (symmetric) matrices is $U_R$ related to $U_L$

$$ M = M^\dagger (M^T) \Rightarrow U_R = U_L (U_L^*)^{-1}. $$

RH fermions are singlets in the SM and only LH charged currents are involved in the weak interactions

$$ \mathcal{L}_W = W^\dagger \mu \bar{\nu}_L \gamma^\mu V_{CKM} d_L + \text{h.c.} $$

where

$$ V_{CKM} = U_L^\dagger U_R^d. $$

The $U_R$'s do not play a role in the SM. However, the fermionic mass matrices are generated here by unknown Yukawa couplings and therefore are completely arbitrary. Hence, the SM must be extended to “explain” the fermionic masses and mixings, an extension which is already suggested by

- Grand Unification: $\alpha_1(M_W), \alpha_2(M_W), \alpha_3(M_W) \to \alpha(M_{\text{GUT}})$
- Yukawa Unification: $m_r(M_{\text{GUT}}) \simeq m_b(M_{\text{GUT}})$
- L-R restoration at $M_L \gg M_W$
- Mixed massive neutrinos (seesaw) $\nu_i^\nu$ with: $M_{\nu_R} \gg M_W$ e.t.c.

Many different “models” are known to give the right masses of the charged fermions and $V_{CKM}$ (within the experimental errors) $^{[1]}$ and this is an indication that the mass problem is far from being solved. Part of this freedom is due to the fact that these suggestions disregard the RH rotations.

Most models use hermitian mass matrices for no other reasons than simplicity$^{[2]}$. However, recently more and more asymmetric mass matrices are used (mainly to have additional freedom for the neutrino sector)$^{[3]}$. Asymmetric mass matri-
ces imply \( U_L \neq U_R \), so that here the \( U_R \)'s are a clue to distinguish between different models.

It is true that RH currents have not been observed till now\(^1\) but this means only that the relevant gauge bosons are heavy and/or mix very little with the observed LH ones and/or the RH neutrinos are very heavy. The limits on RH gauge bosons are clearly very model dependent \(^4\).

As an example, let us consider the leptons in the fermions in the \( L \)-sector therefore phenomenologically:

\[ \text{the L-R symmetries are restored. RH mixing seems effective at energies where currentswill not be directly observed at low energies they play an important role at energies where the L-R symmetries are restored.} \]

RH mixings effect therefore phenomena like:

- Proton decay
- Neutrino seesaw \(^2\)
- Leptogenesis via decays of RH neutrinos as the origin of baryon asymmetry \(^5\) e.t.c., which are indirectly observable.

Now, it is clear that the symmetries which dictate the mass matrices are effective at scales relevant for the theories beyond the SM. In those theories the RH mixings are not arbitrary any more, there are also no reason to assume that they are small. Actually even large RH mixings are not unnatural and are the standard in \( P_{LR} \) invariant theories \(^6\). We claim also that the large leptonic mixing (recently observed by SuperKamiokande \(^6\)) may be related to large RH rotations in the quark-sector.

What is \( P_{LR} \)?

In the framework of Current Algebra it is common to assign the baryons to a \( P \)-invariant \((3,3)\oplus(3,3)\) representation under the \textit{global} chiral group:

\[ SU_L(3) \times SU_R(3) \times P \ . \]

The baryons acquire their masses when the chiral group is broken into its diagonal subgroup \( SU_{L+R}(3) \), under which the baryons constitute \( 8 \oplus 1 \) Dirac spinors.

An analogous symmetry can be applied to fermions in the \( L-R \) symmetric gauge theories. As an example, let us consider the leptons in the \( E_6 \) GUT \(^7\). Those are LH Weyl spinors that transform like \((1,3,3)\) under the maximal subgroup of \( E_6 \),

\[ E_6 \supset SU_C(3) \times SU_L(3) \times SU_R(3) \ . \]

Whereas \( P \)-reflection for the global symmetry leads per definition to \( SU_L(3) \leftrightarrow SU_R(3) \) exchange, in the gauge theories \( L, R \) are only an historical notation. The chirality of the local currents is fixed by the representation content of the fermions under \( SU_L(3) \times SU_R(3) \). Hence, for gauge theories we have to require, in addition to Parity exchange, also, \( SU_L(3) \leftrightarrow SU_R(3) \). The irreducible representation of the leptons under \( SU_C(3) \times SU_L(3) \times SU_R(3) \times P_{LR} \) is

\[ (1,3,3)_LH \oplus (1,3,3)_RH \ , \]

which requires two families.

Under the diagonal \( SU_C(3) \times SU_{L+R}(3) \) one obtains then \( 8 \oplus 1 \) of Dirac spinors. Applying this to the \( e \) and \( \mu \) families this is realized in analogy with the hadrons as follows.

\[
\begin{align*}
\text{Such a model was actually constructed in 1977} & \text{ when the third heavy family was not yet observed. It is quite a general belief now that this top-family is the only one acquiring masses through direct coupling to the Higgs representation, while the light families get their masses through second order “corrections”. It is then natural that these two light families obey symmetries like } P_{LR} \ . \text{ When those symmetries are broken, the particles gain their physical masses and mixings.}\quad (5) \\
\text{The } P_{LR} \text{ operation can be formally defined in terms of two families} & \text{ as} \\
& P_{LR} f^i(x) P_{LR}^{-1} = \epsilon^{ij} \sigma_2 \tilde{f}^j(x) \ .
\end{align*}
\]

The \( P_{LR} \) invariant Lagrangian looks then as follows

\[ L_Y = y_{12} \bar{\psi}_1 \Phi_{12} \psi^2 - y_{21} \bar{\psi}_2 \Phi_{21} \psi^1 + h.c. \quad (6) \]

\(^2\)We know that in SUSY theories as well, sfermions of the two light families must be quite degenerate to avoid FCNCs.
The corresponding mass matrices are hence pure off-diagonal in this limit

\[
M_2^u = \begin{pmatrix} 0 & -m_u \\ m_c & 0 \end{pmatrix}, \quad M_2^d = \begin{pmatrix} 0 & -m_d \\ m_s & 0 \end{pmatrix}, \\
M_2^c = \begin{pmatrix} 0 & -m_c \\ m_\mu & 0 \end{pmatrix}, \quad M_2^e = \begin{pmatrix} 0 & -m_e \\ m_\tau & 0 \end{pmatrix}.
\]

These matrices can be diagonalized by the transformations

\[
\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -m_1 \\ m_2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix},
\]

and those are equivalent to the exchanges

\[
u_{LH} \leftrightarrow e_{LH}^c, \quad d_{LH}^c \leftrightarrow s_{LH}^c, \quad e_{LH} \leftrightarrow \mu_{LH}^c,
\]

which mean full RH rotations. Applying this to the effective dim.6 B-violating Lagrangian of $SO(10)$ and noting that only the two light families are relevant for the proton decay, two decay modes result:

\[
P \rightarrow \bar{\nu}_\mu K^+ \quad \text{and} \quad P \rightarrow \mu^+ K^0.
\]

Now, to make such a model realistic one must break $P_{LR}$ by a small amount, to allow for Cabibbo mixing and add the heavy t-family. Also, to induce gauge unification (without SUSY) an intermediate breaking scale, $M_I \approx 10^{12}$ GeV is required. This is however also the right RH neutrino mass scale for the seesaw mechanism and leptogenesis as well as the scale of the invisible Axion window.

In this talk I would like to report on a systematic study of models with large RH rotations and their possible effects. I will give an example in terms of a “realistic” SO(10) Model with such mixings. By this I mean a conventional SO(10) theory that reproduces all the observed fermionic masses and LH mixings but at the same time generates large RH angles.

This can be obtained by requiring small deviations from the $P_{LR}$ invariant case. E.g. consider at the high unification scale the following mass matrices (those can be obtained using a global $U_F(1)$ or a discrete symmetry):

\[
m_d = \begin{pmatrix} 0 & -m_d & 0 \\ m_s & 0 & 0 \\ 0 & 0 & m_b \end{pmatrix},
\]

\[
m_u = \begin{pmatrix} a & m_1 & b \\ m_2 & 0 & 0 \\ c & 0 & m_3 \end{pmatrix},
\]

\[
m_e = \begin{pmatrix} 0 & -m_e & 0 \\ m_\mu & 0 & 0 \\ 0 & 0 & m_c \end{pmatrix}.
\]

These matrices give the following RH angles at the high scale

\[
\Theta_{12}^R = 1.57 \text{ rad}, \quad \Theta_{23}^R = 0.0 \text{ rad}, \quad \Theta_{13}^R = -1.5 \text{ rad}.
\]

We studied in detail the embedding of those matrices in the framework of an SO(10) model broken at $M_U$ to the Pati-Salam group and this in the second step to the SM at $M_I$:

\[
SO(10) \rightarrow \begin{array}{c} SU_C(4) \times SU_L(2) \times SU_R(2) \\ \rightarrow SM \end{array}
\]

The Higgs representations needed for the local breaking and the generation of the fermionic mass matrices, fix the two loop renormalization group equations (RGEs). Those are used for the two cases, one with D-Parity ($g_L = g_R$) and the other without it ($g_L \neq g_R$). We found:

with D-Parity:

\[
M_U = 1.04 \times 10^{15} \text{GeV},
\]

\[
M_I = 5.66 \times 10^{13} \text{GeV},
\]

\[
\alpha_U = 0.02841
\]

\[
M_U = 5.68 \times 10^{15} \text{GeV},
\]

\[
M_I = 2.09 \times 10^{13} \text{GeV},
\]

\[
\alpha_U = 0.04207
\]

Using then the fermionic mass matrices and $V_{CKM}$ at $M_W$ we evaluated the values of the matrix elements at $M_I$ and also give the RH mixing angles at this scale. Those values were used to calculate the proton and neutron B-violating branching ratios (see tab. 1 and tab. 2).
We obtained very similar results in those two cases and only the absolute rates depend on the details of the local breaking. Without D-Parity we obtain:

\[ t^{\text{Proton}}_{\text{total}} = 1.1 \times 10^{34} \pm 7 \pm 10^{17} \text{yr} \]  

(12)

For the uncertainties and threshold corrections we used the estimates of Langacker [11] and Lee et al [12].

Our main prediction are the branching ratios which are independent on those uncertainties and the details of the local breaking. The absolute rates indicate, however, that the results of the model are well in the range of observability of the new proton decay experiments [14]. The branching ratios are very similar to the “smoking gun” predictions of the SUSY GUTs [18] and in contradiction with the conventional GUTs where \( P \rightarrow e^+ \pi^0 \) dominates. Using a \( U_F(1) \) one can obtain naturally large LH leptonic mixings induced by large RH rotations in the d-quark sector [14]. We will study also effects of large RH mixings on the proton decay in SUSY SO(10). Those could play an important role in view of the fact that it was shown recently that RRRR and RRLL effective dim.5 operators can dominate proton decay in such models [13]. Also, effects of SUSY and non SUSY leptogenesis as the origin of the baryon asymmetry \( \frac{\Delta_n}{\Delta} \) will be considered.

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References


