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The Quark Lepton Mass Problem and the Anti-Grand Unification Model

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Abstract: The fermion mass problem and the ideas of mass protection are briefly reviewed. The Fritzsch ansatz for the quark mass matrices and a recent variant, based on a lightest flavour mixing mechanism in which all the CKM mixing angles disappear in the chiral symmetry limit of vanishing up and down quark masses, are discussed. The Anti-Grand Unification Model (AGUT) and the Multiple Point Principle (MPP) used to calculate the values of the Standard Model gauge coupling constants in the theory are described. The application of the MPP to the pure Standard Model predicts the top quark mass to be 173 ± 5 GeV and the Higgs particle mass to be 135 ± 9 GeV. Mass protection by the chiral quantum numbers of the maximal AGUT gauge group $SMG \times U(1)_f$ provides a successful fit to the charged fermion mass spectrum, with an appropriate choice of Higgs fields to break the AGUT gauge group down to the Standard Model gauge group (SMG) close to the Planck scale. The puzzle of the neutrino masses and mixing angles presents a challenge to the AGUT model and approaches to this problem are briefly discussed.

1. Introduction

As I discussed in my talk at the previous Corfu workshop [1] in 1995, the pattern of observed quark and lepton masses, their mixing and three generation structure form the major outstanding problem of particle physics. The hierarchical structure of the charged fermion masses, ranging over five orders of magnitude from 1/2 MeV for the electron to 175 GeV for the top quark, and of the quark weak coupling matrix elements strongly suggests the existence of physics beyond the Standard Model (SM). Furthermore the growing experimental support for the existence of neutrino oscillations and hence for a non-zero neutrino mass, from SuperKamiokande and other data, provides direct evidence for non-Standard Model physics. So the experimental values of the SM fermion masses and mixing angles presently provide our best clues to the fundamental physics of flavour.

A fermion mass term

$$\mathcal{L}_{mass} = -m\overline{\psi}_L\psi_R + h.c. \qquad (1.1)$$

couples together a left-handed Weyl field ψ_L and a right-handed Weyl field ψ_R, which then satisfy the Dirac equation

$$i\gamma^\mu\partial_\mu\psi_L = m\psi_R \qquad (1.2)$$

If the two Weyl fields are not charge conjugates $\psi_L \neq (\psi_R)^c$ we have a Dirac mass term and the two fields ψ_L and ψ_R together correspond to a Dirac spinor. However if the two Weyl fields are charge conjugates $\psi_L = (\psi_R)^c$ we have a Majorana mass term and the corresponding four component Majorana spinor has only two degrees of freedom. Particles carrying an exactly conserved charge Q, like the electron, must be distinct from their anti-particles and can only have Dirac masses with ψ_L and ψ_R having equal charges $Q_L = Q_R$. However a neutrino could be a massive Majorana particle.

The left-handed and right-handed top quark, t_L and t_R carry unequal SM $SU(2) \times U(1)$ gauge charges:

$$Q_L \neq Q_R \qquad \text{(Chiral charges)} \qquad (1.3)$$

Electroweak gauge invariance protects the quarks and leptons from gaining a fundamental mass term ($\overline{t_L} t_R$ is not gauge invariant). This *mass protection* mechanism is of course broken by the Higgs effect, which naturally generates a mass for the top quark of the same order of magnitude as the SM Higgs field vacuum expectation value (vev). Thus the Higgs mechanism explains why the top quark mass is suppressed, relative to the fundamental (Planck, GUT...) mass scale of the physics beyond the SM, down to the scale of electroweak gauge symmetry breaking. However the further suppression of the other quark-lepton masses remains a mystery, which it is natural to attribute to mass protection by another approximately conserved (gauge) charge (or charges) beyond the SM, as discussed in section 3. In this talk I will appeal to the gauge charges of the Anti-Grand Unification Theory (AGUT) for this mass protection. The AGUT model and its connection with the Multiple Point Principle (MPP) is discussed in section 4. The MPP predictions for the top quark and Higgs particle masses within the pure SM are then discussed in section 5. The Higgs field sector required to break the AGUT gauge group down to that of the SM is described in section 6. The structure of the quark and charged lepton mass matrices resulting from AGUT mass protection is presented in section 7. I will then consider the neutrino mass problem in section 8 and conclude in section 9.

However let me begin, in the following section 2, by considering the structure of the fermion mass matrices and some of the ansätze suggested by phenomenology.

2. Mass matrix texture

The hierarchical structure of the Standard Model fermion mass spectrum naturally suggests that the fermion mass matrix elements have a similar hierarchical structure, each typically having a different order of magnitude. The smaller elements may then contribute so weakly to the physical masses and mixing angles that they can effectively be neglected and replaced by zero—texture zeros. The best known ansatz incorporating such a texture zero is the two generation Fritzsch hermitean ansatz [2]:

$$M_U = \begin{pmatrix} 0 & B \\ B^* & A \end{pmatrix} \qquad M_D = \begin{pmatrix} 0 & B' \\ B'^* & A' \end{pmatrix} \tag{2.1}$$

The assumed hierarchical structure gives the following conditions:

$$|A| \gg |B|, \qquad |A'| \gg |B'| \tag{2.2}$$

among the parameters. It follows that the two generation Cabibbo mixing is given by the well-known Fritzsch formula

$$|V_{us}| \simeq \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi} \sqrt{\frac{m_u}{m_c}} \right| \tag{2.3}$$

where $\phi = \arg B' - \arg B$. This relationship fits the experimental value well, provided that the phase ϕ is close to $\frac{\pi}{2}$. The generalisation of the Fritzsch ansatz to three generations:

$$M_U = \begin{pmatrix} 0 & C & 0 \\ C^* & 0 & B \\ 0 & B^* & A \end{pmatrix} \tag{2.4}$$

$$M_D = \begin{pmatrix} 0 & C' & 0 \\ C'^* & 0 & B' \\ 0 & B'^* & A' \end{pmatrix} \tag{2.5}$$

with the assumed hierarchy of parameters:

$$|A| \gg |B| \gg |C|, \qquad |A'| \gg |B'| \gg |C'| \tag{2.6}$$

however leads to an additional relationship

$$|V_{cb}| \simeq \left| \sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2} \sqrt{\frac{m_c}{m_t}} \right| \tag{2.7}$$

which is excluded by the data for any value of the phase ϕ_2. Consistency with experiment can, for example, be restored by introducing a non-zero 2-2 mass matrix element [3].

There are several ansätze, with texture zeros [4], which give testable relations between the masses and mixing angles [1]. Here I will discuss a recent suggestion [5], which predicts all the CKM mixing matrix elements in terms of quark masses. It is a common belief, due to the success of eq. (2.3), that the smallness of the Cabibbo mixing matrix element V_{us} is due to the lightness of the u and d quarks. However not only the 1-3 generation mixing V_{ub} but also the 2-3 generation mixing V_{cb} happen to be small compared to V_{us}.

This led us to the idea that all the other mixings, and primarily the 2-3 mixing, could also be controlled by the up and down quark masses m_u and m_d and vanishes in the chiral symmetry limit $m_u = m_d = 0$. Therefore we consider an ansatz in which the diagonal mass matrix elements for the second and third generations are practically the same in the gauge (unrotated) and physical bases.

We propose that the three mass matrices for the Dirac fermions—the up quarks ($U = u,\ c,\ t$), the down quarks ($D = d,\ s,\ b$) and charged leptons ($E = e,\ \mu,\ \tau$)—are each hermitian with three texture zeros of the following form:

$$M_i = \begin{pmatrix} 0 & a_i & 0 \\ a_i^* & A_i & b_i \\ 0 & b_i^* & B_i \end{pmatrix} \qquad i = U,\ D,\ E \quad (2.8)$$

with the hierarchy $B_i \gg A_i \sim |b_i| \gg |a_i|$ between the elements. Our ansatz requires the diagonal elements ($A_i,\ B_i$), of the mass matrices M_i, to be proportional to the modulus square of the off-diagonal elements ($a_i,\ b_i$):

$$\frac{A_i}{B_i} = \left|\frac{a_i}{b_i}\right|^2 \qquad i = U,\ D,\ E \qquad (2.9)$$

It follows that the Cabibbo mixing is given by the Fritzsch formula eq. (2.3) which fits the experimental value well, provided that the CP violating phase ϕ is required to be close to $\frac{\pi}{2}$. Our most interesting prediction (with the mass ratios calculated at the electroweak scale [6]) is:

$$\begin{aligned} |V_{cb}| &\simeq \left|\sqrt{\frac{m_d}{m_b}} - e^{i\gamma}\sqrt{\frac{m_u}{m_t}}\right| \\ &\simeq \sqrt{\frac{m_d}{m_b}} = 0.038 \pm 0.007 \end{aligned} \qquad (2.10)$$

in good agreement with the current data $|V_{cb}| = 0.039 \pm 0.003$ [7]. If we also take the phase $\gamma = \arg b_D - \arg b_U$ to be $\frac{\pi}{2}$, the uncertainty in our prediction of eq. (2.10) is reduced from 0.007 to 0.004. Another prediction for the ratio:

$$\left|\frac{V_{ub}}{V_{cb}}\right| = \sqrt{\frac{m_u}{m_c}} \qquad (2.11)$$

is quite general for models with nearest-neighbour mixing.

An alternative scenario, in which the hermitian mass matrix for the up quarks is changed to be of the form:

$$M_U = \begin{pmatrix} 0 & 0 & c_U \\ 0 & A_U & 0 \\ c_U^* & 0 & B_U \end{pmatrix} \qquad (2.12)$$

leads to mixing angles given by the simple and compact formulae:

$$|V_{us}| \simeq \sqrt{\frac{m_d}{m_s}} \quad |V_{cb}| \simeq \sqrt{\frac{m_d}{m_b}} \quad |V_{ub}| \simeq \sqrt{\frac{m_u}{m_t}} \qquad (2.13)$$

While the values of $|V_{us}|$ and $|V_{cb}|$ are practically the same as in our first scenario and in good agreement with experiment, a new prediction for $|V_{ub}|$ (not depending on the value of the CP violating phase) should allow experiment to differentiate between the two scenarios in the near future.

3. Mass matrix texture from chiral flavour charges

As we pointed out in section 1, a natural resolution to the charged fermion mass problem is to postulate the existence of some approximately conserved chiral charges beyond the SM. These charges, which we assume to be the gauge quantum numbers in the fundamental theory beyond the SM, provide selection rules forbidding the transitions between the various left-handed and right-handed quark-lepton states, except for the top quark. In order to generate mass terms for the other fermion states, we have to introduce new Higgs fields, which break the fundamental gauge symmetry group G down to the SM group. We also need suitable intermediate fermion states to mediate the forbidden transitions, which we take to be vector-like Dirac fermions with a mass of order the fundamental scale M_F of the theory. In this way effective SM Yukawa coupling constants are generated, which are suppressed by the appropriate product of Higgs field vacuum expectation values measured in units of M_F.

Consider, for example, the model obtained by extending the Standard Model gauge group $SMG = SU(3) \times SU(2) \times U(1)$ with a gauged abelian flavour group $U(1)_f$. This $SMG \times U(1)_f$

gauge group is broken to SMG by the vev of a scalar field ϕ_S where $\langle\phi_S\rangle < M_F$ and ϕ_S carries $U(1)_f$ charge $Q_f(\phi_S) = 1$. Suppose further that the $U(1)_f$ charges of the Weinberg Salam Higgs field and the left- and right-handed bottom quark fields are:

$$Q_f(\phi_{WS}) = 0 \qquad Q_f(b_L) = 0 \qquad Q_f(b_R) = 2 \tag{3.1}$$

Then it is natural to expect the generation of a mass for the b quark of order:

$$\left(\frac{\langle\phi_S\rangle}{M_F}\right)^2 \langle\phi_{WS}\rangle \tag{3.2}$$

via a tree level diagram involving the exchange of two $\langle\phi_S\rangle$ tadpoles, in addition to the usual $\langle\phi_{WS}\rangle$ tadpole, with two appropriately charged vector-like fermion intermediate states [8] of mass M_F. We identify $\epsilon_f = \langle\phi_S\rangle/M_F$ as the $U(1)_f$ flavour symmetry breaking parameter. In general we expect mass matrix elements of the form:

$$M(i,j) = \gamma_{ij}\epsilon_f^{n_{ij}}\langle\phi_{WS}\rangle \tag{3.3}$$

between the ith left-handed and jth right-handed fermion components, where

$$\gamma_{ij} = \mathcal{O}(1), \quad n_{ij} = \mid Q_f(\psi_{L_i}) - Q_f(\psi_{R_j}) \mid \tag{3.4}$$

So the *effective* SM Yukawa couplings of the quarks and leptons to the Weinberg-Salam Higgs field $y_{ij} = \gamma_{ij}\epsilon_f^{n_{ij}}$ can consequently be small even though all *fundamental* Yukawa couplings of the "true" underlying theory are of $\mathcal{O}(1)$. However it appears [9] not possible to explain the fermion mass spectrum with an anomaly free set of flavour charges in an $SMG \times U(1)_f$ model with a single Higgs field ϕ_S breaking the $U(1)_f$ gauge symmetry. In fact it is possible to produce a realistic quark-lepton spectrum, but at the expense of introducing three Higgs fields with relatively prime $U(1)_f$ charges and most of the SM fermions carrying exceptionally large $U(1)_f$ charges. Another possibility is to introduce SMG-singlet fermions with non-zero values of the $U(1)_f$ charge to cancel the $U(1)_f^3$ gauge anomaly (as in $MSSM \times U(1)_f$ models [10], which also use anomaly cancellation via the Green-Schwarz mechanism [11]). However we shall consider the alternative of extending the SM gauge group further—in fact to that of the anti-grand unification model introduced in the next section.

We shall take the point of view that, in the fundamental theory beyond the SM, the Yukawa couplings allowed by gauge invariance are all of order unity and, similarly, all the mass terms allowed by gauge invariance are of order the fundamental mass scale of the theory—say the Planck scale. Then, apart from the element responsible for the top quark mass, the quark-lepton mass matrix elements are only non-zero due to the presence of other Higgs fields having vevs smaller (typically by one order of magnitude) than the fundamental scale. These Higgs fields will, of course, be responsible for breaking the fundamental gauge group G—whatever it may be—down to the SM group. In order to generate a particular effective SM Yukawa coupling matrix element, it is necessary to break the symmetry group G by a combination of Higgs fields with the appropriate quantum number combination $\Delta\vec{Q}$. When this "$\Delta\vec{Q}$" is different for two matrix elements they will typically deviate by a large factor. If we want to explain the observed spectrum of quarks and leptons in this way, it is clear that we need charges which—possibly in a complicated way—separate the generations and, at least for $t-b$ and $c-s$, also quarks in the same generation. Just using the usual simple $SU(5)$ GUT charges does not help because both (μ_R and e_R) and (μ_L and e_L) have the same $SU(5)$ quantum numbers. So we prefer to keep each SM irreducible representation in a separate irreducible representation of G and introduce extra gauge quantum numbers distinguishing the generations, by adding extra Cartesian-product factors to the SM gauge group.

4. Anti-Grand unification model

In the AGUT model the SM gauge group is extended in much the same way as Grand Unified $SU(5)$ is often assumed; it is just that we assume another non-simple gauge group $G = SMG^3 \times U(1)_f$, where $SMG \equiv SU(3) \times SU(2) \times U(1)$, becomes active near the Planck scale $M_{Planck} \simeq 10^{19}$ GeV. So we have a pure SM desert, without any supersymmetry, up to an order of magnitude or so below M_{Planck}. The existence of

the $SMG^3 \times U(1)_f$ group means that, near the Planck scale, each of the three quark-lepton generations has got its own gauge group and associated gauge particles with the same structure as the SM gauge group. There is also an extra abelian $U(1)_f$ gauge boson, giving altogether $3\times 8 = 24$ gluons, $3\times 3 = 9$ W's and $3\times 1+1 = 4$ abelian gauge bosons.

The couplings of the i'th proto-generation to the $SMG_i = SU(3)_i \times SU(2)_i \times U(1)_i$ group are identical to those to the SM group. Consequently we have a charge quantization rule, analogous to the SM charge quantisation rule (see eq. (4.2) below), for each of the three proto-generation weak hypercharge quantum numbers y_i. For the colourless particles we have the Millikan charge quantization of all charges being integer when measured in units of the elementary charge unit, but for coloured particles the charges deviate from being integer by $-1/3$ of the elementary charge for quarks and by $+1/3$ for antiquarks. This rule can be expressed by introducing the concept of triality t, which characterizes the representation of the centre of the colour $SU(3)$ group, and is defined so that $t = 0$ for the trivial representation or for decuplets, octets and so on, while $t = 1$ for triplet ($\underline{3}$) or anti-sextet etc. and $t = -1$ for anti-triplet ($\overline{\underline{3}}$) or sextet etc. Then the rule can be written in the form

$$Q + t/3 = 0 \qquad (\text{mod } 1) \qquad (4.1)$$

where Q is the electric charge $Q = y/2 + t_3/2$ (t_3 is the third component of the weak isospin, SU(2), and y is the weak hypercharge). So we may write this SM charge quantization rule as

$$y/2 + d/2 + t/3 = 0 \qquad (\text{mod } 1) \qquad (4.2)$$

where we have introduced the duality d, which is defined to be 0 when the weak isospin is integer and $d = 1$ when it is half integer.

At first sight, this $SMG^3\times U(1)_f$ group with its 37 generators seems to be just one among many possible SM gauge group extensions. However, it is actually not such an arbitrary choice, as it can be uniquely specified by postulating 4 reasonable requirements on the gauge group $G \supset SMG$. As a zeroth postulate, of course, we require that the gauge group extension must contain the Standard Model group as a subgroup $G \supset SMG$. In addition it should obey the following 4 postulates:

The first two are also valid for $SU(5)$ GUT:

1. G should transform the presently known (left-handed, say) Weyl particles into each other. Here we take the point of view that we do not look for the whole gauge group G, say, but only for that factor group $G' = G/H$ which transforms the already known quark and lepton Weyl fields in a nontrivial way. That is to say, we ask for the group obtained by dividing out the subgroup $H \subset G$ which leaves the quark and lepton fields unchanged. This factor group G' can then be identified with its representation of the Standard Model fermions, i.e. as a subgroup of the $U(45)$ group of all possible unitary transformations of the 45 Weyl fields for the Standard Model. If one took G to be one of the extensions of SU(5), such as SO(10) or the E-groups as promising unification groups, the factor group G/H would be SU(5) only; the extension parts can be said to only transform particles that are not in the Standard Model (and thus could be pure fantasy *a priori*).

2. No anomalies, neither gauge nor mixed. We assume that only straightforward anomaly cancellation takes place and, as in the SM itself, do not allow for a Green-Schwarz type anomaly cancellation [11].

But the next two are rather just opposite to the properties of the $SU(5)$ GUT, thus justifying the name Anti-GUT:

3. The various irreducible representations of Weyl fields for the SM group remain irreducible under G. This is the most arbitrary of our assumptions about G. It is motivated by the observation that combining SM irreducible representations into larger unified representations introduces symmetry relations between Yukawa coupling constants, whereas the particle spectrum does

not exhibit any exact degeneracies (except possibly for the case $m_b = m_\tau$). In fact AGUT only gets the naive $SU(5)$ mass predictions as order of magnitude relations: $m_b \approx m_\tau$, $m_s \approx m_\mu$, $m_d \approx m_e$.

4. G is the maximal group satisfying the other 3 postulates.

With these four postulates a somewhat complicated calculation shows that, modulo permutations of the various irreducible representations in the Standard Model fermion system, we are led to our gauge group $SMG^3 \times U(1)_f$. Furthermore it shows that the SM group is embedded as the diagonal subgroup of SMG^3, as required in our AGUT model. The AGUT group breaks down an order of magnitude or so below the Planck scale to the SM group. The anomaly cancellation constraints are so tight that, apart from various permutations of the particle names, the $U(1)_f$ charge assignments are uniquely determined up to an overall normalisation and sign convention. In fact the $U(1)_f$ group does not couple to the left-handed particles or any first generation particles, and the $U(1)_f$ quantum numbers can be chosen as follows:

$$Q_f(\tau_R) = Q_f(b_R) = Q_f(c_R) = 1 \qquad (4.3)$$

$$Q_f(\mu_R) = Q_f(s_R) = Q_f(t_R) = -1 \qquad (4.4)$$

The AGUT group breaks down an order of magnitude or so below the Planck scale to the diagonal subgroup of the SMG^3 subgroup (the diagonal subgroup is isomorphic to the usual SM group). For this breaking we shall use a relatively complicated system of Higgs fields with names W, T, ξ, and S. In order to fit neutrino masses as well, we need an even more complicated system. It should however be said that, although at the very high energies just under the Planck energy each generation has its own gluons, own W's etc., the breaking makes only one linear combination of a certain colour combination of gluons "survive" down to low energies. So below circa 1/10 of the Planck scale, it is only these linear combinations that are present and thus the couplings of the gauge particles—at low energy only corresponding to these combinations—are the same for all three generations. You can also say that the phenomenological gluon is a linear combination with amplitude $1/\sqrt{3}$ for each of the AGUT-gluons of the same colour combination. That then also explains why the coupling constant for the phenomenological gluon couples with a strength that is $\sqrt{3}$ times smaller than for the AGUT-gluons (see eq. (4.5) below) if, as we effectively assume, the three AGUT $SU(3)$ couplings were equal to each other.

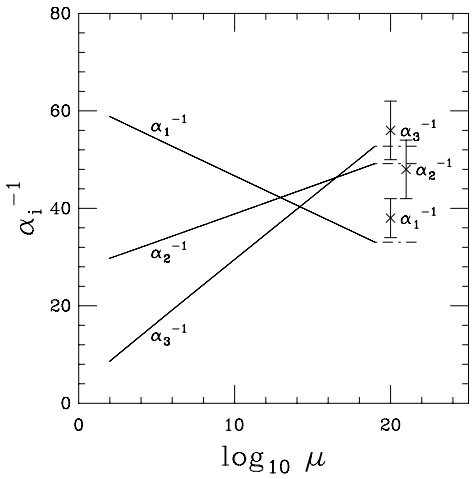

Figure 1: Evolution of the Standard Model fine structure constants α_i (α_1 in the SU(5) inspired normalisation) from the electroweak scale to the Planck scale. The anti-GUT model predictions for the values at the Planck scale, $\alpha_i^{-1}(M_{Planck})$, are shown with error bars.

The SM gauge coupling constants do not, of course, unify, because we have not combined the groups U(1), SU(2) and SU(3) together into a simple group, but their values have been successfully calculated using the Multiple Point Principle [12]. According to the MPP, the coupling constants should be fixed such as to ensure the existence of many vacuum states with the same energy density; in the Euclideanised version of the theory, there is a corresponding phase transition. So if several vacua are degenerate, there is a multiple point. The couplings at the multiple points have been calculated in lattice gauge theory for the groups $SU(3)$, $SU(2)$ and $U(1)$ separately. We imagine that the lattice has a truly physical significance in providing a cut-off for our model at the Planck scale. The SM fine structure constants correspond to those of the di-

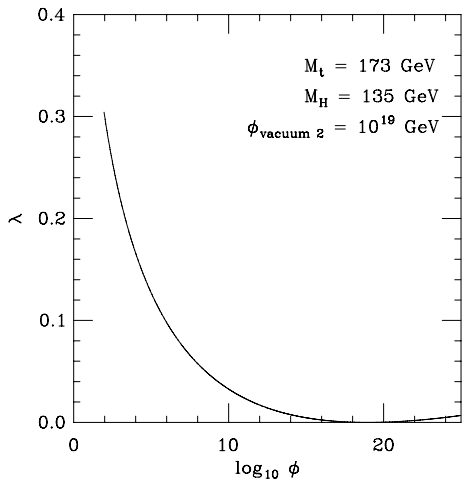

Figure 2: Plot of λ as a function of the scale of the Higgs field ϕ for degenerate vacua with the second Higgs VEV at the Planck scale $\phi_{vac\,2} = 10^{19}$ GeV.

agonal subgroup of the SMG^3 group and, for the non-abelian groups, this gives:

$$\alpha_i(M_{Planck}) = \frac{\alpha_i^{Multiple\ Point}}{3} \qquad i = 2,\ 3 \tag{4.5}$$

The situation is more complicated for the abelian groups, because it is possible to have gauge invariant cross-terms between the different $U(1)$ groups in the Lagrangian density such as:

$$\frac{1}{4g^2} F_{\mu\nu}^{gen\ 1}(x) F_{gen\ 2}^{\mu\nu}(x) \tag{4.6}$$

So, in first approximation, for the SM $U(1)$ fine structure constant we get:

$$\alpha_1(M_{Planck}) = \frac{\alpha_1^{Multiple\ Point}}{6} \tag{4.7}$$

The agreement of these AGUT predictions with the data is shown in figure 1.

5. The MPP Prediction for the Top Quark and Higgs masses in the Standard Model

The application of the MPP to the pure Standard Model [13], with a cut-off close to M_{Planck}, implies that the SM parameters should be adjusted, such that there exists another vacuum state degenerate in energy density with the vacuum in which we live. This means that the effective SM Higgs potential $V_{eff}(|\phi|)$ should, have a second minimum degenerate with the well-known first minimum at the electroweak scale $\langle|\phi_{vac\,1}|\rangle = 246$ GeV. Thus we predict that our vacuum is barely stable and we just lie on the vacuum stability curve in the top quark, Higgs particle (pole) mass (M_t, M_H) plane. Furthermore we expect the second minimum to be within an order of magnitude or so of the fundamental scale, i.e. $\langle|\phi_{vac\,2}|\rangle \simeq M_{Planck}$. In this way, we essentially select a particular point on the SM vacuum stability curve and hence the MPP condition predicts precise values for M_t and M_H.

For the purposes of our discussion it is sufficient to consider the renormalisation group improved tree level effective potential $V_{eff}(\phi)$. We are interested in values of the Higgs field of the order $|\phi_{vac\,2}| \simeq M_{Planck}$, which is very large compared to the electroweak scale, and for which the quartic term strongly dominates the ϕ^2 term; so to a very good approximation we have:

$$V_{eff}(\phi) \simeq \frac{1}{8}\lambda(\mu = |\phi|)|\phi|^4 \tag{5.1}$$

The running Higgs self-coupling constant $\lambda(\mu)$ and the top quark running Yukawa coupling constant $g_t(\mu)$ are readily computed by means of the renormalisation group equations, which are in practice solved numerically, using the second order expressions for the beta functions.

The vacuum degeneracy condition is imposed by requiring:

$$V_{eff}(\phi_{vac\,1}) = V_{eff}(\phi_{vac\,2}) \tag{5.2}$$

Now the energy density in vacuum 1 is exceedingly small compared to $\phi_{vac\,2}^4 \simeq M_{Planck}^4$. So we basically get the degeneracy condition, eq. (5.2), to mean that the coefficient $\lambda(\phi_{vac\,2})$ of $\phi_{vac\,2}^4$ must be zero with high accuracy. At the same ϕ-value the derivative of the effective potential $V_{eff}(\phi)$ should be zero, because it has a minimum there. Thus at the second minimum of the effective potential the beta function β_λ also vanishes:

$$\beta_\lambda(\mu = \phi_{vac\,2}) = \lambda(\phi_{vac\,2}) = 0 \tag{5.3}$$

which gives to leading order the relationship:

$$\frac{9}{4}g_2^4 + \frac{3}{2}g_2^2 g_1^2 + \frac{3}{4}g_1^4 - 12g_t^4 = 0 \tag{5.4}$$

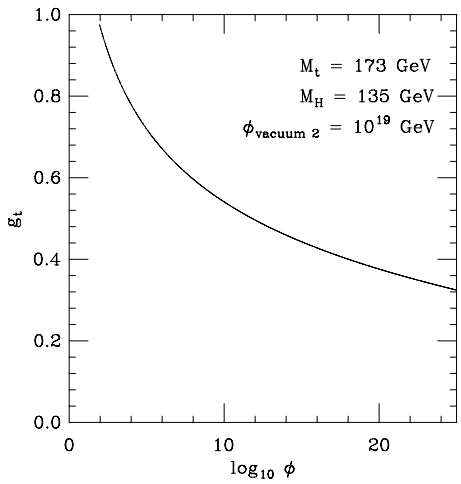

Figure 3: Plot of g_t as a function of the scale of the Higgs field ϕ for degenerate vacua with the second Higgs VEV at the Planck scale $\phi_{vac\,2} = 10^{19}$ GeV.

between the top quark Yukawa coupling and the electroweak gauge coupling constants $g_1(\mu)$ and $g_2(\mu)$ at the scale $\mu = \phi_{vac\,2} \simeq M_{Planck}$. We use the renormalisation group equations to relate the couplings at the Planck scale to their values at the electroweak scale. Figures 2 and 3 show the running coupling constants $\lambda(\phi)$ and $g_t(\phi)$ as functions of $\log(\phi)$. Their values at the electroweak scale give our predicted combination of pole masses [13]:

$$M_t = 173 \pm 5 \text{ GeV} \quad M_H = 135 \pm 9 \text{ GeV} \quad (5.5)$$

6. AGUT gauge symmetry breaking by Higgs fields

There are obviously many different ways to break down the large group $SMG \times U(1)_f$ to the much smaller SMG. However, we can greatly simplify the situation by assuming that, like the quark and lepton fields, the Higgs fields belong to singlet or fundamental representations of all non-abelian groups. The non-abelian representations are then determined from the $U(1)_i$ weak hypercharge quantum numbers, by imposing the charge quantization rule eq. (4.2) for each of the SMG_i groups. So now the four abelian charges, which we express in the form of a charge vector

$$\vec{Q} = \left(\frac{y_1}{2}, \frac{y_2}{2}, \frac{y_3}{2}, Q_f\right)$$

can be used to specify the complete representation of G. The constraint that we must eventually recover the SM group as the diagonal subgroup of the SMG_i groups is equivalent to the constraint that all the Higgs fields (except for the Weinberg-Salam Higgs field which of course finally breaks the SMG) should have charges y_i satisfying:

$$y = y_1 + y_2 + y_3 = 0 \quad (6.1)$$

in order that their SM weak hypercharge y be zero.

We wish to choose the quantum numbers of the Weinberg-Salam (WS) Higgs field ϕ_{WS} so that it matches the difference in charges between the left-handed and right-handed physical top quarks. This will ensure that the top quark mass in the SM is not suppressed relative to the WS Higgs field VEV. However we note that there is a finesse of our fit to the quark-lepton spectrum, according to which the right-handed component of the experimentally observed t-quark is actually the one having second generation $SU(3)$ quantum numbers and is thus really the proto-right-handed charm quark c_R. In a similar way the right-handed component of the experimentally observed charm quark has the third generation $SU(3)$ representation and is really the proto-right-handed top quark t_R. It is only the *right*-handed top and charm quarks that are permuted in this way, while for example the left-handed components are not. We have to make this identification of the proto-generation fields c_R and t_R; otherwise we cannot suppress the b quark and τ lepton masses. This is because, for the proto-fields, the charge differences between t_L and t_R are the same as between b_L and b_R and also between τ_L and τ_R. So now it is simple to calculate the quantum numbers of the WS Higgs field ϕ_{WS}:

$$\vec{Q}_{\phi_{WS}} = \vec{Q}_{c_R} - \vec{Q}_{t_L} = \left(0, \frac{2}{3}, 0, 1\right) - \left(0, 0, \frac{1}{6}, 0\right) = \left(0, \frac{2}{3}, -\frac{1}{6}, 1\right) \quad (6.2)$$

This means that the WS Higgs field will in fact be coloured under both $SU(3)_2$ and $SU(3)_3$. After breaking the symmetry down to the SM group,

we will be left with the usual WS Higgs field of the SM and another scalar which will be an octet of $SU(3)$ and a doublet of $SU(2)$. This should not present any phenomenological problems, provided this scalar doesn't cause symmetry breaking and doesn't have a mass less than about 1 TeV. In particular an octet of $SU(3)$ cannot lead to baryon decay. In our model we take it that what in the Standard Model are seen as many very small Yukawa-couplings to the Standard Model Higgs field really represent chain Feynman diagrams, composed of propagators with Planck scale heavy particles (fermions) interspaced with order of unity Yukawa couplings to Higgs fields with the names W, T, ξ, and S, which are postulated to break the AGUT to the Standard Model Group. The small effective Yukawa couplings in the Standard Model are then generated as products of small factors, given by the ratios of the vacuum expectation values of W, T, and ξ to the masses occurring in the propagators for the Planck scale fermions in the chain diagrams [8].

The quantum numbers of our invented Higgs fields W, T, ξ and S are chosen—and it is remarkable that we succeeded so well—so as to make the order of magnitude for the suppressions of the mass matrix elements of the various mass matrices fit to the phenomenological requirements.

After the choice of the quantum numbers for the replacement of the Weinberg Salam Higgs field in our model, eq. (6.2), the further quantum numbers needed to be picked out of the vacuum in order to give, say, mass to the b-quark is denoted by $\vec{b}$ and analogously for the other particles. For example:

$$\vec{b} = \vec{Q}_{b_L} - \vec{Q}_{b_R} - \vec{Q}_{WS} \tag{6.3}$$

$$\vec{c} = \vec{Q}_{c_L} - \vec{Q}_{t_R} + \vec{Q}_{WS} \tag{6.4}$$

$$\vec{\mu} = \vec{Q}_{\mu_L} - \vec{Q}_{\mu_R} - \vec{Q}_{WS} \tag{6.5}$$

Here we denoted the quantum numbers of the quarks and leptons as e.g. $\vec{Q}_{c_L}$ for the left handed components of the proto-charmed quark. Note, as we remarked above, that $\vec{c}$ has been defined using the t_R proto-field, since we have essentially swapped the right-handed charm and top quarks. Also the charges of the WS Higgs field are added rather than subtracted for up-type quarks.

Next we attempted to find some Higgs field quantum numbers which, if postulated to have "small" vevs compared to the Planck scale masses of the intermediate particles, would give a reasonable fit to the order of magnitudes of the mass matrix elements. We were thereby led to the proposal:

$$\vec{Q}_W = \frac{1}{3}(2\vec{b} + \vec{\mu}) = \left(0, -\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}\right) \tag{6.6}$$

$$\vec{Q}_T = \vec{b} - \vec{Q}_W = \left(0, -\frac{1}{6}, \frac{1}{6}, -\frac{2}{3}\right) \tag{6.7}$$

$$\begin{aligned} \vec{Q}_\xi = \vec{Q}_{d_L} - \vec{Q}_{s_L} &= \left(\frac{1}{6}, 0, 0, 0\right) - \left(0, \frac{1}{6}, 0, 0\right) \\ &= \left(\frac{1}{6}, -\frac{1}{6}, 0, 0\right) \end{aligned} \tag{6.8}$$

From the Fritzsch relation [2] $V_{us} \simeq \sqrt{\frac{m_d}{m_s}}$ discussed in section 2, it is suggested that the two off-diagonal mass matrix elements connecting the d-quark and the s-quark be equally big. We achieve this approximately in our model by introducing a special Higgs field S, with quantum numbers equal to the difference between the quantum number differences for these 2 matrix elements in the down quark matrix. Then we postulate that this Higgs field has a vev of order unity in fundamental units, so that it does not cause any suppression but rather ensures that the two matrix elements get equally suppressed. Henceforth we will consider the vevs of the new Higgs fields as measured in Planck scale units and so we have:

$$< S >= 1 \tag{6.9}$$

and

$$\begin{aligned} \vec{Q}_S &= [\vec{Q}_{s_L} - \vec{Q}_{d_R}] - [\vec{Q}_{d_L} - \vec{Q}_{s_R}] \\ &= \left(\frac{1}{6}, -\frac{1}{6}, 0, -1\right) \end{aligned} \tag{6.10}$$

The existence of a non-suppressing field S means that we cannot control phenomenologically when

this S-field is used. Thus the quantum numbers of the other Higgs fields W, T, ξ and ϕ_{WS} given above have only been determined modulo those of the field S.

7. Quark and lepton mass matrices in AGUT

We define the mass matrices by considering the mass terms in the SM to be given by:

$$\mathcal{L} = \overline{Q}_L M_U U_R + \overline{Q}_L M_D D_R + \overline{L}_L M_E E_R + \text{h.c.} \tag{7.1}$$

The mass matrices can be expressed in terms of the effective SM Yukawa matrices and the WS Higgs VEV by:

$$M_f = Y_f \frac{< \phi_{WS} >}{\sqrt{2}} \tag{7.2}$$

We can now calculate the suppression factors for all elements in the Yukawa matrices, by expressing the charge differences between the left-handed and right-handed fermions in terms of the charges of the Higgs fields. They are given by products of the small numbers denoting the vevs of the fields W, T, ξ in fundamental units and the order unity vev of S. In the following matrices we simply write W instead of $< W >$ etc. for the vevs in Planck units. With the quantum number choice given above, the resulting matrix elements are—but remember that "random" complex order unity factors are supposed to multiply all the matrix elements—for the uct-quarks:

$$Y_U \simeq \begin{pmatrix} SWT^2\xi^2 & WT^2\xi & W^2T\xi \\ SWT^2\xi^3 & WT^2 & W^2T \\ S\xi^3 & 1 & WT \end{pmatrix} \tag{7.3}$$

the dsb-quarks:

$$Y_D \simeq \begin{pmatrix} SWT^2\xi^2 & WT^2\xi & T^3\xi \\ SWT^2\xi & WT^2 & T^3 \\ SW^2T^4\xi & W^2T^4 & WT \end{pmatrix} \tag{7.4}$$

and the charged leptons:

$$Y_E \simeq \begin{pmatrix} SWT^2\xi^2 & WT^2\xi^3 & S^2WT^4\xi \\ SWT^2\xi^5 & WT^2 & S^2WT^4\xi^2 \\ S^3WT^5\xi^3 & W^2T^4 & WT \end{pmatrix} \tag{7.5}$$

We can now set $S = 1$ and fit the nine quark and lepton masses and three mixing angles, using 3 parameters: W, T and ξ. That really means we have effectively omitted the Higgs field S and replaced the maximal AGUT gauge group $SMG^3 \times U(1)_f$ by the reduced AGUT group $SMG_{12} \times SMG_3 \times U(1)$, which survives the spontaneous breakdown due to S. In order to find the best possible fit we must use some function which measures how good a fit is. Since we are expecting an order of magnitude fit, this function should depend only on the ratios of the fitted masses to the experimentally determined masses. The obvious choice for such a function is:

$$\chi^2 = \sum \left[\ln \left(\frac{m}{m_{\text{exp}}} \right) \right]^2 \tag{7.6}$$

where m are the fitted masses and mixing angles and m_{exp} are the corresponding experimental values. The Yukawa matrices are calculated at the fundamental scale which we take to be the Planck scale. We use the first order renormalisation group equations (RGEs) for the SM to calculate the matrices at lower scales.

We cannot simply use the 3 matrices given by eqs. (7.3)–(7.5) to calculate the masses and mixing angles, since only the order of magnitude of the elements is defined. Therefore we calculate statistically, by giving each element a random complex phase and then finding the masses and mixing angles. We repeat this several times and calculate the geometrical mean for each mass and mixing angle. In fact we also vary the magni-

Table 1: Best fit to conventional experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

	Fitted	Experimental
m_u	3.6 MeV	4 MeV
m_d	7.0 MeV	9 MeV
m_e	0.87 MeV	0.5 MeV
m_c	1.02 GeV	1.4 GeV
m_s	400 MeV	200 MeV
m_μ	88 MeV	105 MeV
M_t	192 GeV	180 GeV
m_b	8.3 GeV	6.3 GeV
m_τ	1.27 GeV	1.78 GeV
V_{us}	0.18	0.22
V_{cb}	0.018	0.041
V_{ub}	0.0039	0.0035

tude of each element randomly, by multiplying by a factor chosen to be the exponential of a number picked from a Gaussian distribution with mean value 0 and standard deviation 1.

We then vary the 3 free parameters to find the best fit given by the χ^2 function. We get the lowest value of χ^2 for the VEVs:

$$\langle W \rangle = 0.179 \tag{7.7}$$

$$\langle T \rangle = 0.071 \tag{7.8}$$

$$\langle \xi \rangle = 0.099 \tag{7.9}$$

The result [14] of the fit is shown in table 1. This fit has a value of:

$$\chi^2 = 1.87 \tag{7.10}$$

This is equivalent to fitting 9 degrees of freedom (9 masses + 3 mixing angles - 3 Higgs vevs) to within a factor of $\exp(\sqrt{1.87/9}) \simeq 1.58$ of the experimental value. This is better than might have been expected from an order of magnitude fit.

We can also fit to different experimental values of the 3 light quark masses by using recent results from lattice QCD, which seem to be consistently lower than the conventional phenomenological values. The best fit in this case [14] is shown in table 2. The corresponding values of the Higgs vevs are:

$$\langle W \rangle = 0.123 \tag{7.11}$$

$$\langle T \rangle = 0.079 \tag{7.12}$$

$$\langle \xi \rangle = 0.077 \tag{7.13}$$

and this fit has a larger value of:

$$\chi^2 = 3.81 \tag{7.14}$$

But even this is good for an order of magnitude fit.

Table 2: Best fit using alternative light quark masses extracted from lattice QCD. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

	Fitted	Experimental
m_u	1.9 MeV	1.3 MeV
m_d	3.7 MeV	4.2 MeV
m_e	0.45 MeV	0.5 MeV
m_c	0.53 GeV	1.4 GeV
m_s	327 MeV	85 MeV
m_μ	75 MeV	105 MeV
M_t	192 GeV	180 GeV
m_b	6.4 GeV	6.3 GeV
m_τ	0.98 GeV	1.78 GeV
V_{us}	0.15	0.22
V_{cb}	0.033	0.041
V_{ub}	0.0054	0.0035

8. Neutrino mass and mixing

Physics beyond the SM can generate an effective light neutrino mass term

$$\mathcal{L}_{\nu-mass} = \sum_{i,j} \psi_{i\alpha}\psi_{j\beta}\epsilon^{\alpha\beta}(M_\nu)_{ij} \tag{8.1}$$

in the Lagrangian, where $\psi_{i,j}$ are the Weyl spinors of flavour i and j, and $\alpha, \beta = 1, 2$. Fermi-Dirac statistics means that the mass matrix M_ν must be symmetric. In models with chiral flavour symmetry we typically expect the elements of the mass matrices to have different orders of magnitude. The charged lepton matrix is then expected to give only a small contribution to the lepton mixing. As a result of the symmetry of the neutrino mass matrix and the hierarchy of the mass matrix elements it is natural to have an almost degenerate pair of neutrinos, with nearly maximal mixing [15]. This occurs when an off-diagonal element dominates the mass matrix.

A neutrino mass matrix of this texture is generated in the AGUT model, by tree level diagrams involving the exchange of two Weinberg Salam Higgs tadpoles and the appropriate combination of Planck scale Higgs field tadpoles. The combination which leads to the mass term $(M_\nu)_{ij}$ between ν_{Li} and ν_{Lj} is determined by the equation

$$\left(\sum \vec{Q}_\theta\right)_{ij} = \vec{Q}_{\nu Li} + \vec{Q}_{\nu Lj} + 2\vec{Q}_{\phi_{WS}} \tag{8.2}$$

Here the sum is over the charge vectors for the combination of Planck scale Higgs fields (W, T, ξ and S) exchanged. In this way we obtain the

neutrino mass matrix

$$M_\nu \simeq \frac{\langle\phi_{WS}\rangle^2}{M_{Pl}} \begin{pmatrix} W^2\xi^4T^4 & W^2\xi T^4 & W^2\xi^3 T \\ W^2\xi T^4 & WT^5 & W^2T \\ W^2\xi^3 T & W^2T & W^2T^2\xi^2 \end{pmatrix} \tag{8.3}$$

where we have set $< S >= 1$. The off-diagonal element $(M_\nu)_{23} = (M_\nu)_{32}$ clearly dominates this matrix, so that we have large mu-tau mixing (between the nearly degenerate mass eigenstates ν_2 and ν_3). The mixing matrix U_ν is given by

$$U_\nu \sim \begin{pmatrix} 1 & \frac{\xi^3}{\sqrt{2}} & \frac{\xi^3}{\sqrt{2}} \\ -\xi^3 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\xi T^3 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{8.4}$$

We also have the ratio of neutrino mass squared differences

$$\frac{\Delta m^2_{23}}{\Delta m^2_{12}} \sim 2T\xi^2 \sim 1.4 \times 10^{-3} \tag{8.5}$$

giving a hierarchy that is not suitable for the simultaneous solution of the solar and atmospheric neutrino problems.

In any case, the mass scale is much too small to give suitable masses for the atmospheric neutrino problem. This is because, even if the $(M_\nu)_{23}$ element was unsuppressed by Planck scale Higgs vevs, the see-saw mass

$$\frac{< \phi_{WS} >^2}{M_{Planck}} \sim 3 \times 10^{-6} \text{ eV} \tag{8.6}$$

would still be too small. So, it is necessary to introduce a new mass scale into the AGUT model in order to obtain observable neutrino masses and mixings. This may be done by extending the AGUT Higgs spectrum to include a weak isotriplet Higgs field Δ with SM weak hypercharge $\frac{y}{2} = -1$. However there is some unnaturalness in obtaining a value for $< \Delta^0 >$ from the scalar potential some orders of magnitude greater than the see-saw mass of eq. (8.6)

Furthermore we need extra structure for the lepton mass matrices and must relax the assumption that all the independent matrix elements are of different orders of magnitude. For example M_ν may have two order of magnitude degenerate elements $A \sim B$ with a texture of the form:

$$M_\nu = \begin{pmatrix} \times & A & B \\ A & \times & \times \\ B & \times & \times \end{pmatrix} \tag{8.7}$$

where $\times$ indicates texture zeros. The mass eigenvalues are given by:

$$m_{\nu i} = \pm\sqrt{A^2 + B^2},\ 0, \qquad (i = 1, 2, 3) \tag{8.8}$$

although these will be slightly altered when the effects of the small elements represented by texture zeros are included. With these eigenvalues we clearly have a hierarchy in Δm^2's with the more degenerate pair being heavier:

$$\Delta m^2_{12} \ll \Delta m^2_{13} \sim \Delta m^2_{23}. \tag{8.9}$$

So we take $\Delta m^2_{12} = \Delta m^2_{solar}$, $\Delta m^2_{23} = A^2 + B^2 \sim 10^{-3}$ eV^2, where Δm^2_{solar} will depend on the type of solution we adopt for the solar neutrinos.

The corresponding neutrino mixing matrix (assuming that the charged lepton mass matrix M_E is quasi-diagonal) is:

$$\begin{aligned} U_\nu \sim & \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ = & \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}}\cos\theta & \frac{1}{\sqrt{2}}\cos\theta & -\sin\theta \\ \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{2}}\sin\theta & \cos\theta \end{pmatrix} \end{aligned} \tag{8.10}$$

where

$$\tan\theta = \frac{B}{A}. \tag{8.11}$$

From the first row we can see that ν_e is maximally mixed between ν_1 and ν_2, so that its mixing does not contribute to the atmospheric neutrino anomaly, and there will be no effect observable at Chooz. The atmospheric neutrino anomaly will be entirely due to large $\nu_\mu - \nu_\tau$ mixing and, in order that the mixing be large enough, we need $\sin^2 2\theta \geq 0.8$ $(90\% C.L)$ which requires

$$0.56 \leq \frac{B}{A} \leq 1.8 \tag{8.12}$$

so that, although A and B must be order of magnitude degenerate, it is not necessary to do any fine tuning. The solar neutrino problem is explained by vacuum oscillations, although whether it is an 'energy-independent' or a 'just-so' solution will depend on the small elements which we have neglected. It cannot be explained by an MSW type solution since the mixing between ν_e and ν_μ is too large for this type of solution, and

will remain too large even after the texture zeroes are removed. The particular case of $B = A$ for this texture corresponds to the popular bimaximal mixing solution [16] to the solar and atmospheric neutrino problems. This type of structure cannot explain the LSND result and does not give a significant contribution to hot dark matter, since the sum of the neutrino masses is given by

$$\sum m_\nu \sim 2\sqrt{A^2 + B^2} \sim 2\sqrt{\Delta m^2_{atm}} < 0.2 \text{ eV} \quad (8.13)$$

We have not been able to extend the Higgs sector of the AGUT model in such a way as to obtain a neutrino mass matrix M_ν with the above texture of eq. (8.7). However we have constructed [17] an anomaly free Abelian extension of the Standard Model, which naturally yields a mass matrix M_ν of this type. This $SMG \times U(1)^2$ model was inspired by the AGUT model and has exactly the same charged fermion spectrum as in the AGUT fit of Table 1. In order to rescue the AGUT neutrino mass and mixing predictions, it seems necessary to introduce yet another Higgs field and obtain the large mixing required for the atmospheric neutrino problem from the charged lepton mass matrix M_E. The solution to the solar neutrino problem can then be obtained from M_ν or from the mixing due to small elements in M_E. This, of course, has to be achieved without signicantly disturbing the quality of the AGUT fit to the charged fermion spectrum.

9. Conclusions

We emphasized the hierarchical structure of the quark-lepton mass spectrum and how it points to a mass protection mechanism, controlled by approximately conserved chiral (gauge) charges beyond the Standard Model. The structure of ansätze for the fermion mass matrices, suggested by the hierarchy of masses and mixing angles, was briefly discussed. A recent ansatz based on a lightest flavour mixing mechanism was discussed, which gives simple and compact formulae for all the CKM mixing angles in terms of the quark masses.

The anti-grand unification theory (AGUT), and how the associated multiple point principle (MPP) is used to predict the values of the three Standard Model gauge coupling constants, was described. Applied to the case of the pure Standard Model, the MPP leads to our predictions for the top quark and Higgs pole masses: $M_t = 173 \pm 5$ GeV and $M_H = 135 \pm 9$ GeV.

The AGUT group $SMG^3 \times U(1)_f$ is characterised by being the largest anomaly-free gauge group acting on just the 45 SM Weyl fermions, without any unification of the SM irreducible representations. This group assigns a unique set of anomaly free chiral gauge charges to the quarks and leptons. With an appropriate choice of Higgs field quantum numbers, the AGUT chiral charges naturally give a realistic charged fermion mass hierarchy. An order of magnitude fit in terms of 3 Higgs vevs is given in Table 1, which reproduces all the masses and mixing angles within a factor of two. The most characteristic feature of the fit is that, apart from the t and c quarks, the masses of the particles in the same generation are predicted to be degenerate (but only in order of magnitude) at the Planck scale. The worst feature is the deviation, by a factor of about 2, between the fitted and experimental values for m_s and V_{cb}.

On the other hand, the puzzle of the neutrino masses and mixing angles presents a challenge to the model. It is necessary to introduce a new mass scale into the AGUT model, using say a weak isotriplet Higgs field Δ, in order to generate a neutrino mass appropriate to atmospheric neutrino oscillations. Using a reduced model, based on the gauge group $SMG \times U(1)^2$, it is possible to obtain a reasonably natural solution to the solar and atmospheric neutrino problems and, at the same time, reproduce the successful AGUT fit to the charged fermion spectrum. However it is not possible to embed this Abelian extension of the SM into the AGUT, since one cannot choose a consistent set of non-Abelian representations for the Higgs fields. It appears that we shall have to relax the assumption that the charged lepton mass matrix is quasi-diagonal, in order to rescue the AGUT model.

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