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Softly Broken $N = 1$ Supersymmetric QCD

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ABSTRACT: We study softly broken $N = 1$ supersymmetric QCD and its dual. Their vacuum structures have been investigated. Infrared behavior of soft parameters have also been calculated.

1. Introduction

Recently, nonperturbative aspects of $N = 1$ supersymmetric (SUSY) QCD have been understood [1, 2]. It is very important to extend such analyses to non-SUSY QCD and study its nonperturbative aspects, confinement and chiral symmetry breaking. To this end, it is an interesting trial to study softly broken $N = 1$ SUSY QCD. Actually in refs.[3]-[9] $N = 1$ SUSY QCD with soft scalar masses as well as gaugino masses has been discussed and interesting results have been obtained.

In particular, the vacuum structure of $N = 1$ SUSY QCD broken by adding soft masses has been clarified for the theory with $SU(N_c)$ gauge group and N_f flavors of quark pairs in ref.[3]. For $N_f < N_c$, there is a nontrivial stable vacuum, while there is no vacuum in the SUSY limit [1]. For $N_f = N_c$ we can have two nontrivial vacua and there is no trivial vacuum as in the SUSY limit. In one vacuum, only the meson fields T develop their expectation values (VEVs) and in the other only the baryon fields B and $\bar{B}$ develop their VEVs. Which vacuum is realized depends on the soft mass ratio between T and B ($\bar{B}$). In both vacua chiral symmetry is broken and this situation is the same as the SUSY limit, where we have chiral symmetry breaking as well as confinement. On the other hand, for $N_f = N_c + 1$ we have only the trivial vacuum and chiral symmetry is not broken, while in the SUSY limit we have confinement without chiral symmetry breaking, i.e. s-confinement. For $N_f > N_c + 1$, we have only the trivial vacuum and the presence of the Seiberg duality is suggested even in SUSY QCD broken by soft mass terms.

Here we study these vacuum structures adding all the allowed soft SUSY breaking terms. Also, we investigate infrared (IR) behavior of these soft SUSY breaking terms in the case with a IR fixed point and its dual.

2. Vacuum structure

2.1 $N_f > N_c + 1$

At first we study softly broken $N = 1$ supersymmetric QCD for $N_f > N_c + 1$ [6]. We concentrate to the case with $N_c \geq 3$. We consider the $N = 1$ supersymmetric QCD with the gauge symmetry $SU(N_c)$ and N_f flavors of quark supermultiplets, $\widehat{Q}^i$ and $\widehat{\overline{Q}}_i$. This theory has the flavor symmetry $SU(N_f)_Q \times SU(N_f)_{\overline{Q}}$ and no superpotential. In the case with $N_f > N_c + 1$, the dual theory is described by the $N = 1$ SUSY theory with the gauge group $SU(N_f - N_c)$, N_f flavors of dual quark pairs $\widehat{q}_i$ and $\widehat{\overline{q}}^i$, and $N_f \times N_f$ singlet meson supermultiplets $\widehat{T}^i_j$. The dual theory has the same flavor symmetry as the electric theory and the dual theory has the superpotential,

$$W = \widehat{q}_i \widehat{T}^i_j \widehat{\overline{q}}^j . \tag{2.1}$$

In the dual theory, all the symmetries except R-symmetry allow the following soft SUSY breaking terms,

$$\mathcal{L}_{SB} = -m_q^2 \mathrm{tr}|q|^2 - m_{\overline{q}}^2 \mathrm{tr}|\overline{q}|^2 - m_T^2 \mathrm{tr}|T|^2 + (hq_i T^i_j \overline{q}^j + h.c.), \quad (2.2)$$

where q_i, $\overline{q}^i$ and T^i_j denote scalar components of $\widehat{q}_i$, $\widehat{\overline{q}}^i$ and $\widehat{T}^i_j$, respectively. Also the gaugino mass terms are added. For the kinetic term, we assume the canonical form with normalization factors k_q and k_T for q, $\bar{q}$ and T. Then we write the following scalar potential:

$$V(q,\bar{q},T) = \frac{1}{k_T}(|q|^2|\bar{q}|^2) + \frac{1}{k_q}(|qT|^2 + |\bar{q}T|^2) + \frac{\tilde{g}^2}{2}(q^\dagger \tilde{t}^a q - \bar{q}\tilde{t}^a \bar{q}^\dagger)^2 + m_q^2|q|^2 + m_{\bar{q}}^2|\bar{q}|^2 + m_T^2|T|^2 - (hq_i T^i_j \bar{q}^j + h.c.), \quad (2.3)$$

where the third term is the D-term and $\tilde{g}$ denotes the gauge coupling constant of the dual theory. The trilinear coupling terms $hqT\overline{q}$ play a crucial role in determining the minimum of the potential. We assume h is real.

The minimum of potential can be obtained along the following diagonal direction ,

$$q = \mathrm{diag}(q_{(1)}, q_{(2)}, \cdots, q_{(\tilde{N}_c)}), \quad (2.4)$$

$$\bar{q} = \mathrm{diag}(\bar{q}_{(1)}, \bar{q}_{(2)}, \cdots, \bar{q}_{(\tilde{N}_c)}), \quad (2.5)$$

$$T = \mathrm{diag}(T_{(1)}, T_{(2)}, \cdots, T_{(\tilde{N}_c)}), \quad (2.6)$$

where all the entries, $q_{(i)}$, $\bar{q}_{(i)}$ and $T_{(i)}$, can be made real. Along the D-flat direction, $q_{(i)} = \bar{q}_{(i)} = X_i$, the potential is written as

$$V(X,T) = \sum_{i=1}^{\tilde{N}_c}[\frac{1}{k_T}X_i^4 + (m_q^2 + m_{\bar{q}}^2)X_i^2 + m_T^2 T_{(i)}^2 + \frac{2}{k_q}T_{(i)}^2 X_i^2 - 2hT_{(i)}X_i^2]. \quad (2.7)$$

This potential always has the nontrivial vacuum with $X_i \neq 0$ and $T_{(i)} \neq 0$ if

$$h \gg \frac{m_q^2 + m_{\overline{q}}^2}{2k_q}. \quad (2.8)$$

In addition, we have the nontrivial vacuum with $X_i \neq 0$ and $T_{(i)} \neq 0$ for a certain region of intermediate values of h, if m_T^2/k_T is sufficiently large compared with $(m_q^2 + m_{\overline{q}}^2)/2k_q$ [8]. Otherwise, in particular for a sufficiently small value of h we have the trivial vacuum with $T = 0$ and $q = 0$.

Now we compare between softly broken $N = 1$ supersymmetric QCD and its dual. In the original theory the soft scalar mass terms as well as gaugino mass terms are all we can add as soft SUSY breaking terms, i.e.

$$\mathcal{L}_{SB} = -m_Q^2|Q|^2 - m_{\bar{Q}}^2|\bar{Q}|^2. \quad (2.9)$$

Let us discuss the unbroken phase $q = \bar{q} = T = 0$. In this case the structure of massless fermions and global symmetries except gauginos and R-symmetry is not changed compared with the SUSY limit. Thus, this case leads to the same anomaly structure for the unbroken global symmetry $SU(N_f)_q \times SU(N_f)_{\bar{q}} \times U(1)_B$ as the SUSY limit. On the other hand, we have the unbroken phase $Q = \bar{Q} = 0$ for $m_Q^2 > 0$ and $m_{\bar{Q}}^2 > 0$. In this case no local or global symmetry is broken except the R-symmetry, which is broken by gaugino mass terms. Moreover, all the quarks remain massless. Thus the anomaly structure is the same as for the SUSY limit, e.g. $SU(N_f)^3$ and $SU(N_f)^2 U(1)_B$. Therefore, this dual pair has the same anomaly structure in the unbroken phase even in the presence of soft SUSY breaking terms. That seems to imply the presence of Seiberg's duality in this phase even after SUSY breaking with the A-terms. This observation has been already made in ref. [3], although the A-terms were not included in the discussions.

Let us extend the above consideration to the broken phase and notice that large symmetry breaking takes place in the broken phase. Here we simplify the issue and consider the following model. When adding the soft SUSY breaking terms, we break the flavor symmetry $SU(\tilde{N}_c)_q \times SU(\tilde{N}_c)_{\bar{q}}$ into $SU(\tilde{N}_c - 1)_q \times U(1)_q \times SU(\tilde{N}_c - 1)_{\bar{q}} \times U(1)_{\bar{q}}$. Then we assume the first flavor has soft scalar masses, m_{q1} and $m_{\bar{q}1}$, different from the others, m_q and $m_{\bar{q}}$ [4]. Recall that the i-th flavor is decoupled from the other flavors in all the conditions and equations to realize the broken phase. Here we assume that only the soft scalar masses of the first flavor, m_{q1} and $m_{\bar{q}1}$, satisfy the breaking conditions. In this case only the vacuum expectation values X_1 and $T_{(1)}$ are developed. That leads to the gauge symmetry breaking, $SU(\tilde{N}_c) \rightarrow SU(\tilde{N}_c - 1)$ $(= SU(N_f - 1 - N_c))$. Furthermore, $(N_f - 1)$ flavors of dual quarks and $(N_f - 1) \times (N_f - 1)$ singlet fermions χ_T remain massless. These massless fermions have the global symmetry $SU(N_f-1)_q\times SU(N_f-1)_{\bar{q}}\times U(1)_{B'}$. Massless dual quarks, ψ_q and $\psi_{\bar{q}}$, and singlet fermions χ_T transform as $(\bar{N}_f, 0, N_c/(\tilde{N}_c-1))$, $(0, N_f, -N_c/(\tilde{N}_c-1))$ and $(N_f, \bar{N}_f, 0)$ under this global symmetry, respectively. All the scalar fields become massive. This structure of massless fermions obviously corresponds to the SUSY model with $SU(N_f-1-N_c)$ gauge group and (N_f-1) flavors of quarks. This SUSY model is dual to SUSY QCD theory with $SU(N_c)$ gauge group and (N_f-1) flavors of quarks.

Let us consider now the corresponding original theory. If at the SUSY breaking scale, the flavor symmetry is broken in the same way as the one of the dual theory, $SU(N_f-1)_q\times SU(N_f-1)_{\bar{q}}$, nothing would prevent the appearance of the following superpotential:

$$W = M_1 \widehat{Q}^1 \widehat{\bar{Q}_1}. \qquad (2.10)$$

Note that in this case the B-term, $-M_B^2 Q^1\bar{Q}_1$, can also appear as the soft terms in the lagrangian $\mathcal{L}_{\mathcal{SB}}$. Thus, the (mass)^2 matrix of the first flavor of squarks, M_{11}^2 is written as

$$M_{11}^2 = \begin{pmatrix} m_{Q1}^2 + M_1^2 & -M_B^2 \\ -M_B^2 & m_{\bar{Q}1}^2 + M_1^2 \end{pmatrix}. \qquad (2.11)$$

If $\det(M_{11}^2) > 0$, the potential minimum corresponds to $Q^1=\bar{Q}_1=0$ and the gauge symmetry $SU(N_c)$ remains unbroken. In this case (N_f-1) flavors of quarks remain massless and these massless fermions have the global symmetry $SU(N_f-1)_q\times SU(N_f-1)_{\bar{q}}\times U(1)_B$. All scalar fields become massive. This model has the same anomaly structure as the softly broken dual theory in the broken phase, e.g. for $SU(N_f-1)^3$ and $SU(N_f-1)^2U(1)_B$, where $U(1)_B$ should be replaced by $U(1)_{B'}$ in the dual theory. That seems to suggest the presence of Seiberg's duality after SUSY breaking even in the broken phase.

Similarly the case with $\det(M_{11}^2) < 0$ and $m_{Q1}^2 + m_{\bar{Q}1}^2 + 2M_1^2 > 2|M_B^2|$ corresponds to the dual theory for the unbounded-from-below direction.

We have considered the case where only one flavor of squarks develop their vacuum expectation values. We can easily extend the above discussion to the case when more flavors of squarks develop their vacuum expectation values. Then we can obtain similar relations between softly broken original and dual theories in the broken phase.

2.2 $N_f = N_c + 1$

For $N_f = N_c+1$, the $N=1$ supersymmetric QCD is described in terms of $(N_c+1)\times(N_c+1)$ meson fields $\widehat{T}^i_j$ and (N_c+1) baryon fields $\widehat{B}_i$ and $\widehat{\overline{B}}^i$. They have the superpotential,

$$W = \frac{1}{\Lambda^{2N_c-1}}(\widehat{B}_i\widehat{T}^i_j\widehat{\overline{B}}^j - \det\widehat{T}). \qquad (2.12)$$

The flavor symmetry allows the SUSY breaking trilinear coupling

$$h' B_i T^i_j \overline{B}^j. \qquad (2.13)$$

Thus we consider here the following SUSY breaking terms,

$$\mathcal{L}_{SB} = -m_B^2 \mathrm{tr}|B|^2 - m_{\overline{B}}^2 \mathrm{tr}|\overline{B}|^2 - m_T^2 \mathrm{tr}|T|^2 + (h' B_i T^i_j \overline{B}^j + h.c.). \qquad (2.14)$$

The trilinear SUSY breaking term $h' B_i T^i_j \overline{B}^j$ corresponds to $hq_iT^i_j\overline{q}^j$ in the dual theory with $N_f > N_c+1$.

The minimum of the potential can be obtained along the diagonal direction. Here we consider the following direction,

$$B_i\overline{B}^j - \partial\det T/\partial T^j_i = 0. \qquad (2.15)$$

For simplicity, we consider the case where

$$B_i = \overline{B}^i = B, \quad T_{(i)} = T, \text{ for all } i\text{'s}, \qquad (2.16)$$

and B and T are real. In this case we have the potential,

$$\frac{V}{N_c+1} = 2\lambda_B^2 T^{N_c+2} - 2h' T^{N_c+1} + (m_B^2 + m_{\overline{B}}^2) T^{N_c} + m_T^2 T^2, \qquad (2.17)$$

where $\lambda_B = 1/(k_B\Lambda^{2N_c-1})$. Here the direction (2.15) means $T\geq 0$. The flavor symmetry allows the possibility of the further SUSY breaking term $h'_T\det T$, corresponding to the nonperturbative superpotential. If we add the SUSY breaking term $h'_T \det T$ corresponding to the nonperturbative superpotential, we have the extra term $h'_T T^{N_c+1}$. That corresponds to only the shift,

$$h' \to h' + h'_T, \tag{2.18}$$

in the scalar potential (2.17).

We have the nontrivial vacuum with nonvanishing B, $\overline{B}$ and T, i.e. the flavor symmetry breaking, if h' is sufficiently large compared with the soft scalar masses. Thus, the trilinear coupling term $h'BT\overline{B}$ plays an important role in the chiral symmetry breaking for the case with $N_f = N_c+1$ like the term $hqT\bar{q}$ for the case with $N_f > N_c+1$. In both cases with $N_f = N_c+1$ and $N_f > N_c+1$, the same trilinear term appears to be important.

2.3 $N_f \leq N_c$

In ref.[3] the $N_f = N_c$ and $N_f < N_c$ supersymmetric QCD are broken softly by adding soft scalar masses. It has been shown that in both cases with $N_f = N_c$ and $N_f < N_c$ we have only the nontrivial vacua leading to the chiral symmetry breaking.

The $N_f = N_c$ supersymmetric QCD can be described in terms of the baryon pair $\widehat{B}$ and $\widehat{\overline{B}}$ and $N_c \times N_c$ meson fields $\widehat{T}^j_i$. We have the quantum constraint [1, 2],

$$\widehat{B}\widehat{\overline{B}} - \det \widehat{T} = \Lambda^{2N_c}. \tag{2.19}$$

The flavor symmetry allows the SUSY breaking term $h_B B\overline{B}$. Thus we can add $h_B B\overline{B}$ as well as soft mass terms. However, the vacuum structure is same as the case where we do not add it. We always have two nontrivial vacua.

For $N_f < N_c$ the $N = 1$ supersymmetric QCD has the nonperturbative superpotential,

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c-N_f}}{\det T}\right)^{1/(N_c-N_f)}. \tag{2.20}$$

This potential has no stable point for a finite value of T. However, if we add the soft SUSY breaking scalar mass term,

$$\mathcal{L}_{SB} = -m_T^2 \mathrm{tr}|T|^2, \tag{2.21}$$

we have a stable vacuum for a finite value of T as already shown in ref.[3]. The soft mass terms are all the SUSY breaking terms allowed by the symmetries.

Thus, in the case with $N_f = N_c$ and $N_f < N_c$ we always have the nontrivial vacua, i.e. $T \neq 0$ or $B \neq 0$ for $N_f = N_c$ and $T \neq 0$ for $N_f < N_c$. Because the vacuum with $N_f \leq N_c$ corresponds to nontrivial vacuum of the theory with $N_f = N_c + 1$.

3. Infrared behavior of soft parameters

Here we investigate IR behavior of soft parameters in the conformal window. There is a conjecture that supersymmetric QCD has nontrivial IR fixed point

$$\beta_g = 0 \text{ for } g \neq 0, \tag{3.1}$$

for $(3/2)N_c < N_f < 3N_c$.

Recently, all-order β-functions soft parameters have been obtained in terms of β-functions and anomalous dimensions γ^i_j of the rigid SUSY theory [10, 11],

$$\beta_M = 2\mathcal{O}\left(\frac{\beta_g}{g}\right), \tag{3.2}$$

$$\begin{aligned}\beta_h^{ijk} &= \gamma^i{}_l h^{ljk} + \gamma^j{}_l h^{ilk} + \gamma^k{}_l h^{ijl} \\ &- 2\gamma_1^i{}_l Y^{ljk} - 2\gamma_1^j{}_l Y^{ilk} \\ &- 2\gamma_1^k{}_l Y^{ijl}, \end{aligned} \tag{3.3}$$

$$(\beta_{m^2})^i{}_j = \left[\Delta + X\frac{\partial}{\partial g}\right]\gamma^i{}_j, \tag{3.4}$$

$$\mathcal{O} = \left(Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial Y^{lmn}}\right), \tag{3.5}$$

$$\begin{aligned}\Delta &= 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2 \frac{\partial}{\partial g^2} \\ &+ \tilde{Y}_{lmn}\frac{\partial}{\partial Y_{lmn}} + \tilde{Y}^{lmn}\frac{\partial}{\partial Y^{lmn}}, \end{aligned} \tag{3.6}$$

where Y^{ijk} is the Yukawa coupling, M is the gaugino mass, $(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j$, $Y_{lmn} = (Y^{lmn})^*$, and

$$\begin{aligned}\tilde{Y}^{ijk} &= (m^2)^i{}_l Y^{ljk} + (m^2)^j{}_l Y^{ilk} \\ &+ (m^2)^k{}_l Y^{ijl}. \end{aligned} \tag{3.7}$$

If the following equations,

$$\begin{aligned}\beta_g \frac{dY^{ijk}(g)}{dg} &= \beta^{ijk} \\ &= Y^{ijk}(g)[\gamma_i(g) + \gamma_j(g) + \gamma_k(g)]. \end{aligned} \tag{3.8}$$

$$h^{ijk} = -M(Y^{ijk})' \equiv -M\frac{dY^{ijk}(g)}{d\ln g} , \quad (3.9)$$

$$m_i^2 = |M|^2\{(1+\tilde{X}(g))(g/\beta_g)(\gamma_i(g)) + \frac{1}{2}[(g/\beta_g)\gamma_i(g)]' \}, \quad (3.10)$$

are satisfied, then the differential operators $\mathcal{O}$ and Δ can be written as

$$\mathcal{O} = \frac{M}{2}\frac{d}{d\ln g} , \quad (3.11)$$

$$\Delta + X\frac{\partial}{\partial g} = |M|^2[\ \frac{1}{2}\frac{d^2}{d(\ln g)^2} + (1+\tilde{X}(g))\frac{d}{d\ln g}\ ] , \quad (3.12)$$

where $g\tilde{X}(g) = X/|M|^2$ and X is a function of g, $Y(g)$, $Y^*(g)$, $h(M,g)$, $h^*(M,g)$ and $m^2(|M|^2,g)$.

It has been further shown in ref. [11] that the unknown term $\tilde{X}$ has to have the form

$$\tilde{X} = \frac{1}{2}(\ln(\beta_g/g))' - 1 , \quad (3.13)$$

in order that the expression (3.10) is RG invariant. Therefore, eq.(3.10) becomes [11],

$$m_i^2 = \frac{1}{2}|M|^2(g/\beta_g)(\gamma_i(g))' . \quad (3.14)$$

In ref. [12], with the use of the Novikov-Shifman-Vainstein-Zakharov (NSVZ) β-function [13] for the gauge coupling

$$\beta_g^{\rm NSVZ} = \frac{g^3}{16\pi^2}\left[\frac{\sum_\ell T(R_\ell)(1-2\gamma_\ell) - 3C(G)}{1-g^2C(G)/8\pi^2}\right] , \quad (3.15)$$

it has been shown that the sum rule

$$\sum_{\ell=i,j,k} m_\ell^2 = |M|^2\{\ \frac{1}{1-g^2C(G)/(8\pi^2)}\frac{d\ln Y^{ijk}}{d\ln g} + \frac{1}{2}\frac{d^2\ln Y^{ijk}}{d(\ln g)^2}\ \} + \sum_\ell \frac{m_\ell^2 T(R_\ell)}{C(G)-8\pi^2/g^2}\frac{d\ln Y^{ijk}}{d\ln g} \quad (3.16)$$

is RG invariant, if the $\tilde{X}$ on the subspace defined by $Y = Y(g)$ and eq. (3.9) takes the form [12]

$$\tilde{X} = \frac{-|M|^2C(G) + \sum_\ell m_\ell^2 T(R_\ell)}{C(G)-8\pi^2/g^2} . \quad (3.17)$$

Eq. (3.17) is consistent with (3.13). For SQCD (without Yukawa couplings), the β_{m^2} in the NSVZ scheme becomes [12]

$$\beta_{m_i^2}^{\rm NSVZ} = \frac{|M|^2}{1-g^2C(G)/(8\pi^2)}\frac{d\gamma_i^{\rm NSVZ}}{d\ln g} + \frac{|M|^2}{2}\frac{d^2\gamma_i^{\rm NSVZ}}{d(\ln g)^2} + \sum_\ell \frac{m_\ell^2 T(R_\ell)}{C(G)-8\pi^2/g^2}\frac{d\gamma_i^{\rm NSVZ}}{d\ln g} . \quad (3.18)$$

Seiberg [1] has conjectured on the existence of an IR fixed point in the β-function of SQCD. To recall his proposal, consider the NSVZ β-function (3.15) for SQCD:

$$\beta(g) = -\frac{g^3}{16\pi^2}\frac{3N_c - N_f + 2N_f\gamma(g^2)}{1-N_cg^2/8\pi^2} ,$$
$$\gamma(g^2) = -\frac{g^2}{16\pi^2}\frac{N_c^2-1}{N_c} + O(g^4) . \quad (3.19)$$

There is a non-trivial zero of the β-function for $N_f = (3-\epsilon)N_c$, $N_f, N_c >> 1$. In this regime, the β-function becomes

$$\beta(g) = -\frac{g^3}{16\pi^2}\frac{N_c\epsilon - g^2N_cN_f/8\pi^2}{1-N_cg^2/8\pi^2}. \quad (3.20)$$

Therefore, at order ϵ the fixed point g_*^2 is given by

$$N_cg_*^2 = \frac{8\pi^2}{3}\epsilon. \quad (3.21)$$

In fact it was argued in [1] that such a fixed point exists in the range $\frac{3}{2}N_c \le N_f \le 3N_c$ in SQCD. Also such a behavior was already conjectured to hold in ordinary QCD [14].

We will show that the RG invariant relations (3.9) and (3.14) are IR attractive, and they play a crucial role in investigating the behavior of the soft SUSY breaking parameters near an IR fixed point. The sum rule (3.16) will be used in the dual theory to derive from the behavior of the soft scalar masses near the fixed point (which is obtained by linearizing the problem) a condition which should be satisfied away from the fixed point in order to restore supersymmetry at the fixed point. We will discuss the IR stability of the RG invariant relations (3.9) and (3.14) in the conformal window for which an IR stable fixed point in the space of the gauge coupling for

SQCD, and in the space of the gauge and Yukawa couplings for the dual theory is supposed to exist. Our analysis does not relay on the explicit form of the β-functions for the soft SUSY breaking parameters. However, we assume that the perturbative relations (3.2)–(3.4) among the RG functions of the soft SUSY breaking parameters and those of the corresponding supersymmetric theory can be used to discuss IR physics.

To begin with, using the formulae (3.2)–(3.4) and the RG invariant solutions (3.9) and (3.14), we show that there always exists at least a trajectory in the space of the soft SUSY breaking parameters that approach the origin if the gauge and Yukawa couplings approach a non-trivial fixed point. To this end, we note that if eq. (3.9) is satisfied, then the differential operator $\mathcal{O}$ becomes a total derivative operator as we see from eq. (3.11). Then eq. (3.2) becomes nothing but the Hisano-Shifman relation [15]

$$M = M_0(\beta_g/g) , \tag{3.22}$$

where M_0 is a RG invariant quantity. Since $\beta_g \to 0$ as $g \to g_* \neq 0$ by assumption, we find that $M \to 0$ as $g \to g_*$. Similarly, eqs. (3.9) and (3.14) imply that

$$h^{ijk} = -M_0(\beta_g/g)(Y^{ijk})' \to 0, \tag{3.23}$$

$$m_i^2 = (1/2)|M_0|^2(\beta_g/g)(\gamma_i)' \to 0, \tag{3.24}$$

as $g \to g_*$. In what follows, we will investigate carefully whether the origin of the soft SUSY breaking parameters is a stable IR fixed point.

First we consider softly broken SQCD and examine the IR behavior of the gaugino mass M and soft scalar squared masses m_Q^2, $m_{\overline{Q}}^2$. Since there is no Yukawa coupling in SQCD, the differential operator (3.5) becomes a total derivative operator, and we have

$$\beta_M = Mg\frac{d}{dg}(\beta_g/g) = M(\beta_g/g)' . \tag{3.25}$$

The conjecture that g_* is a stable IR fixed point for SQCD implies that

$$\Gamma_M \equiv \frac{d\beta_g}{dg}|_* > 0 , \tag{3.26}$$

where $|_*$ means an evaluation at the fixed point. We may assume that

$$|\beta_g| \ \text{ and } \ |\frac{d\beta_g}{dg}| < \infty , \tag{3.27}$$

for the range of g we are considering. Eq. (3.25) implies the Hisano-Shifman relation [15], and so if M_0 in the r.h. side of (3.22) is a non-vanishing constant, then the gaugino mass M has to vanish at the fixed point, as we have seen above. Moreover, one sees from eqs. (3.25) and (3.26) that the fixed point $M_* = 0$ is a stable one, because

$$M \sim e^{\Gamma_M t} \to 0 \ \text{ as } \ t \to -\infty , \tag{3.28}$$

where in the lowest order approximation in the ϵ expansion we have $\Gamma_M = \epsilon^2/3$.

To discuss the IR behavior of $m^2_{Q,\overline{Q}}$, on the same level as M, we have to go the NSVZ scheme, and use the β-function (3.18). First we would like to show that the RG invariant relation (3.14) is IR attractive. To this end, we note that for the $\beta_g^{\rm NSVZ}$ given in eq. (3.20), the conditions (3.26) and (3.27) are satisfied if

$$\begin{aligned}\Gamma_\gamma &\equiv \frac{1}{2}\frac{d}{dg}(\gamma_Q + \gamma_{\overline{Q}})|_* \\ &= \frac{d}{dg}\gamma_Q|_* < 0,\end{aligned} \tag{3.29}$$

$$N_c - 8\pi^2/g^2 < 0 \tag{3.30}$$

are satisfied, where we have used $\gamma_Q = \gamma_{\overline{Q}}$. Then we consider the behavior of m_Q^2 and $m_{\overline{Q}}^2$ near the RG invariant relation (3.14),

$$m^2_{Q,\overline{Q}} = m^2_{(0)Q,\overline{Q}} + \delta m^2_{Q,\overline{Q}} , \tag{3.31}$$

$$m^2_{(0)Q,\overline{Q}} \equiv \frac{|M|^2}{2}(g/\beta_g^{\rm NSVZ})(\gamma_Q^{\rm NSVZ})' . \tag{3.32}$$

Linearizing the evolution equation near the RG invariant relation (3.14), we find that

$$\begin{aligned}\frac{d}{dt}\delta m_Q^2 &\simeq \frac{d}{dt}\delta m_{\overline{Q}}^2 \\ &\simeq \frac{1}{2}\Gamma_{m_Q^2}\ (\delta m_Q^2 + \delta m_{\overline{Q}}^2) ,\end{aligned} \tag{3.33}$$

$$\Gamma_{m_Q^2} \equiv \frac{g_* N_f \Gamma_\gamma}{N_c - 8\pi^2/g_*^2} . \tag{3.34}$$

Since $\Gamma_{m_Q^2}$ is positive (see eq. (3.29)), we find

$$\begin{aligned}&\delta m_Q^2 - \delta m_{\overline{Q}}^2 = \text{const.} , \\ &\delta m_Q^2 + \delta m_{\overline{Q}}^2 \sim e^{\Gamma_{m_Q^2} t} \to 0,\end{aligned} \tag{3.35}$$

as $t \to -\infty$. In the lowest order approximation in the ϵ expansion we have $\Gamma_{m_Q^2} = \epsilon^2/3$. Therefore, if the difference $\delta m^2_Q - \delta m^2_{\overline{Q}}$ is non-zero at some point, then we obtain

$$\delta m^2_Q = -\delta m^2_{\overline{Q}}, \tag{3.36}$$

in the IR limit. Since, however, $m^2_{(0),Q,\overline{Q}}$ vanish at the fixed point, which can be concluded from eqs. (3.22) (3.28), we see that

$$m^2_Q = m^2_{\overline{Q}}, \tag{3.37}$$

should be satisfied in order not to break the color symmetry in the IR limit. Then from eq. (3.35) we may conclude that

$$m^2_Q = m^2_{\overline{Q}} \sim e^{\Gamma_{m^2_Q} t} \to 0, \tag{3.38}$$

as $t \to -\infty$. To conclude, we have shown that superconformal symmetry revives at the IR fixed point if $m^2_Q = m^2_{\overline{Q}}$. Otherwise, the $SU(N_c)$ gauge symmetry and supersymmetry is broken.

Note that to show the stability of the IR fixed point ($M_* = m_{Q*} = m_{\overline{Q}*} = 0$), we have used only the sign of Γ_M and Γ_γ, which changes depending on whether the fixed point is UV or IR stable. Therefore, in both type of fixed points, $M_* = m_{Q*} = m_{\overline{Q}*} = 0$ is a stable fixed point if $m^2_Q = m^2_{\overline{Q}}$ is satisfied.

The basic idea for treating the IR behavior of the soft SUSY breaking parameters of the dual theory is the same as the case of SQCD, where we assume that the kinetic term in the dual theory takes the canonical form. A slight difference is that in this case there is a Yukawa coupling Y in the theory, and hence a trilinear coupling h in the softly broken case.

Following Seiberg, we assume that there exists an IR fixed point in the space of $\tilde{g}$ and Y, the gauge coupling for the dual theory is denoted by $\tilde{g}$, and the gaugino mass by $\tilde{M}$. The non-triviality of the fixed point of β_Y implies

$$(\gamma_q + \gamma_{\overline{q}} + \gamma_T)|_* = 0 \ . \tag{3.39}$$

Further, the stability of the IR fixed point requires, among other things, that

$$\Gamma_{\tilde{M}} \equiv \frac{d\beta_{\tilde{g}}}{d\tilde{g}}|_* > 0 \ . \tag{3.40}$$

We in addition assume that

$$\begin{aligned} 2\Gamma_h \equiv \frac{\partial \beta_Y}{\partial Y}|_* &= \frac{\partial}{\partial Y}[Y(\gamma_q + \gamma_{\overline{q}} + \gamma_T)]|_* \\ &= Y_* \frac{\partial}{\partial Y}(\gamma_q + \gamma_{\overline{q}} + \gamma_T)|_* > 0 \ . \end{aligned} \tag{3.41}$$

As in the case of SQCD, we assume that $\beta_{\tilde{g}}$, β_Y together with their derivatives with respect to $\tilde{g}$ and Y in the space of $\tilde{g}$ and Y we are interested in exist. For the NSVZ scheme, eq. (3.40) means that

$$\frac{d}{d\tilde{g}}(\gamma_q + \gamma_{\overline{q}})|_* < 0 \ , \tag{3.42}$$

where $\tilde{N}_c - 8\pi^2/\tilde{g}^2 < 0$ is assumed as in the case of SQCD (3.30).

Now we consider the RG invariant relation (3.9) and show that it is IR attractive. Defining

$$\begin{aligned} h &= h_0 + \delta h \ , \\ h_0 &= -\tilde{M} Y' = -\tilde{M}\tilde{g}\frac{dY}{d\tilde{g}}, \end{aligned} \tag{3.43}$$

and linearizing the evolution equations, we find

$$\frac{d\tilde{M}}{dt} \simeq \tilde{M}(\beta_{\tilde{g}}/\tilde{g})' - 2\delta h \frac{\partial}{\partial Y}(\beta_{\tilde{g}}/\tilde{g}) \ , \tag{3.44}$$

$$\frac{d\delta h}{dt} \simeq (1 + 2Y\frac{\partial}{\partial Y})[\gamma_q + \gamma_{\overline{q}} + \gamma_T] \ \delta h \ . \tag{3.45}$$

Near the fixed point, δh behaves like

$$\delta h \sim e^{\Gamma_h t} \to 0 \ \text{ as } \ t \to -\infty \ . \tag{3.46}$$

Consequently, the gaugino mass behaves like

$$\tilde{M} \sim C_1 e^{\Gamma_{\tilde{M}} t} + C_2 e^{\Gamma_h t} \to 0, \tag{3.47}$$

as $t \to -\infty$, where C_1 and C_2 are integration constants. Therefore, we find that

$$\tilde{M}_* = h_* = 0, \tag{3.48}$$

is a stable fixed point. In the lowest order approximation in the $\tilde{\epsilon}$ expansion we have $\Gamma_{\tilde{M}} = \tilde{\epsilon}^2/3$ and $\Gamma_h = \tilde{\epsilon}/3$, where $\tilde{\epsilon} = 3 - N_f/\tilde{N}_c$.

Next we consider $m^2_{q,\overline{q},T}$. Since near the IR fixed point the RG invariant relation (3.9) (or h_0 given in eq. (3.43)) is attractive, we may use $h = h_0$ in the linearization procedure. We then go to the NSVZ scheme, consider a deviation from the RG invariant relation (3.14), and define

$$\begin{aligned} m^2_i &= m^2_{(0)i} + \delta m^2_i \ , \\ m^2_{(0)i} &\equiv \frac{1}{2}|\tilde{M}|^2(\tilde{g}/\beta^{\rm NSVZ}_{\tilde{g}})(\gamma^{\rm NSVZ}_i)' \ , \end{aligned} \tag{3.49}$$

where $i = q, \overline{q}, T$. We find

$$\begin{aligned} \frac{d}{dt}\delta m^2_q \simeq \frac{d}{dt}\delta m^2_{\overline{q}} &\simeq \frac{1}{2}\Gamma_{m^2_q} \ (\delta m^2_q + \delta m^2_{\overline{q}}) \ , \\ \frac{d}{dt}\delta m^2_T &\simeq \frac{1}{2}\Gamma_{m^2_T} \ (\delta m^2_q + \delta m^2_{\overline{q}}) \ , \end{aligned} \tag{3.50}$$

where

$$\Gamma_{m_q^2} \equiv \frac{\tilde{g}_* N_f \Gamma_{\gamma_q}}{\tilde{N}_c - 8\pi^2/\tilde{g}_*^2} ,$$
$$\Gamma_{m_T^2} \equiv \frac{\tilde{g}_* \Gamma_{\gamma_T}}{\tilde{N}_c - 8\pi^2/\tilde{g}_*^2} ,$$
$$\Gamma_{\gamma_{q,T}} \equiv (d\gamma_{q,T}^{\rm NSVZ}/d\tilde{g})|_* , \qquad (3.51)$$

and we have used $\gamma_q^{\rm NSVZ} = \gamma_{\overline{q}}^{\rm NSVZ}$. As one can easily find, there are two zero eigenvalues and one positive one ($= \Gamma_{m_q^2}$) in the linearized problem (3.50). One of the two zero eigenvalues corresponds to the solution that the difference $\delta m_q^2 - \delta m_{\overline{q}}^2$ is constant independent of t. Thus, as in the case of SQCD, we see that the dual color symmetry is broken unless

$$m_q^2 = m_{\overline{q}}^2, \qquad (3.52)$$

is satisfied. The other zero eigenvalue expresses the fact that m_T^2 may contain a piece which is constant independent of t in the IR limit. The presence of the constant part breaks supersymmetry at the IR fixed point. If the relation (3.37) in the softly broken SQCD is satisfied (so that supersymmetry is recovered at the IR fixed point), we have to demand the dual theory, too, to be supersymmetric at the IR fixed point. Therefore, we have the unique solution to (3.50) which preserve supersymmetry at the fixed point:

$$\delta m_q^2 = \delta m_{\overline{q}}^2 , \ \delta m_T^2 \sim e^{\Gamma_{m_q^2} t} \to 0, \qquad (3.53)$$

as $t \to -\infty$, with

$$\frac{\delta m_q^2}{\delta m_T^2} \simeq \frac{\Gamma_{m_q^2}}{\Gamma_{m_T^2}} = \frac{\Gamma_{\gamma_q}}{\Gamma_{\gamma_T}} , \qquad (3.54)$$

where Γ's are defined in eq. (3.51), and $\Gamma_{m_q^2} > 0$ because $\Gamma_{\gamma_q} < 0$ if $\gamma_q = \gamma_{\overline{q}}$ (see eq. (3.42)). In the lowest order approximation in the $\tilde{\epsilon}$ expansion we have $\Gamma_{m_q^2} = \tilde{\epsilon}/3$.

Eq. (3.54) being constant independent of t suggests the existence of a RG invariant relation. In fact eq. (3.54) is the consequence of the RG invariant sum rule (3.16). To see this, we insert $m_i^2 \ \ (i = q, \overline{q}, T)$ (3.49) with $m_q^2 = m_{\overline{q}}^2$ into the sum rule (3.16), and find that the sum rule reduces to

$$2\delta m_q^2 + \delta m_T^2 = \frac{\tilde{g}(2N_f \gamma_q^{\rm NSVZ} + \gamma_T^{\rm NSVZ})\delta m_q^2}{\beta_{\tilde{g}}^{\rm NSVZ}(C(\tilde{G}) - 8\pi^2/\tilde{g}^2)} . \qquad (3.55)$$

In the IR limit, the quantity on the r.h. side contains an expression 0/0. To obtain the correct limit, we compute

$$\frac{(d/d\tilde{g})(2N_f \gamma_q^{\rm NSVZ} + \gamma_T^{\rm NSVZ})}{(d/d\tilde{g})(\beta_{\tilde{g}}^{\rm NSVZ}/\tilde{g})} , \qquad (3.56)$$

at the fixed point. We find that the expression (3.56) at the fixed point can be written as

$$(C(\tilde{G}) - 8\pi^2/\tilde{g}^2) \ (\ 2 + \frac{\Gamma_{\gamma_T}}{\Gamma_{\gamma_q}} \ ) , \qquad (3.57)$$

implying that the sum rule (3.55) exactly becomes (3.54). Therefore, the soft scalar masses away from the IR fixed point have to satisfy the sum rule

$$m_q^2 + m_{\overline{q}}^2 + m_T^2 = \frac{|\tilde{M}|^2}{1 - \tilde{g}^2 C(\tilde{G})/(8\pi^2)} \frac{d\ln Y}{d\ln\tilde{g}} + \frac{|\tilde{M}|^2}{2}\frac{d^2\ln Y}{d(\ln\tilde{g})^2} + \frac{(N_f/2)(m_q^2 + m_{\overline{q}}^2)}{C(\tilde{G}) - 8\pi^2/\tilde{g}^2}(\frac{d\ln Y}{d\ln\tilde{g}}) , \qquad (3.58)$$

and also (3.52) so that all the soft scalar masses asymptotically vanish in the IR limit. As a result, superconformal symmetry in the dual theory, too, revives at the IR fixed point. If eq. (3.37) for SQCD, and eqs. (3.52) and (3.58) for the dual theory are not satisfied, there will be marginal operators that break supersymmetry as well as the local gauge symmetries in the IR limit.

4. Conclusions

We have studied vacuum structure of softly broken supersymmetric QCD. For $N_f > N_c$, we have trial and nontrivial vacua. The trilinear SUSY breaking terms are important to determine the potential minima. For $N_f \leq N_c$, we always have nontrivial vacua. We have a suggestion of duality in several phases even after SUSY breaking.

We have also investigated infrared behavior of soft SUSY breaking terms in the case with a nontrivial infrared fixed point. In this case the gaugino masses and the A-terms vanish in the infrared region of both sides of duals. Soft scalar masses also vanish if they are symmetric. Thus, in this case supersymmetry revives in the infrared region.

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