

Dynamical Symmetry Breaking with Mirror Fermions

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ABSTRACT: Mirror fermions with masses around the weak scale could break dynamically the electro-weak symmetry if they were coupled with a new strong interaction. The purpose of this talk is to show what sort of dynamics are needed in order to render such theories phenomenologically viable.

KEYWORDS: Dynamical symmetry breaking, mirror fermions.

1. Introduction

A first speculation for the existence of mirror fermions appeared in the classical paper on parity violation [1] that led to the V-A interaction models. Efforts to eliminate completely mirror fermions from nature are for some reminiscent of efforts several decades ago to identify the anti-electron with the proton, and amounts to not realising that particles consistent with natural symmetries could actually exist independently. Such a gauge group and fermion extension, apart from fitting nicely into unification schemata, restores the left-right symmetry missing in the standard-model or in the simplest left-right symmetric models. In this way it also provides a well-defined continuum limit of the theory, something which is usually problematic due to the Nielsen-Ninomiya theorem [2].

The left-right symmetric approach to standard-model extensions renders the baryon-lepton number symmetry $U(1)_{B-L}$ more natural by gauging it, and has also been proposed as a solution to the strong CP problem when accompanied with the introduction of mirror fermions [3]. Moreover, it proves to be economical by identifying the source of the strong dynamics which break the electro-weak symmetry dynamically with a “horizontal” generation gauge group in the mirror sector which, apart from preventing the pairing-up of the standard-model generations with the mirror ones, provides also the intra-generation

mass hierarchies.

Furthermore, even if the number of fermion generations is related to the topology of a manifold in the context of superstring theories, the equality of the number of generations and mirror generations eliminates the need to resort to complicated Calabi-Yau compactifications in order to explain the chiral asymmetry of the standard model with use of the Atiyah-Singer theorem [4]. In such a superstring-inspired unification, possibly connected to $N = 2$ supergravity, the standard-model fermions would have both mirror and supersymmetric partners. The present approach corresponds to breaking supersymmetry and leaving the supersymmetric partners close to the unification scale, and bringing the mirror partners down to the weak scale, altering thus radically the expected phenomenology.

In this talk, new dynamics needed to make such a mechanism phenomenologically viable are discussed. In particular, it proves necessary to review somewhat the dynamical assumptions made in Ref.[5]. In that work it was unclear why the characteristic scale of the strong group responsible for the fermion gauge-invariant masses happened to be so close to the scale where the strong interactions breaking electro-weak symmetry became critical. Furthermore, the previous model could not provide a see-saw mechanism for the standard-model neutrinos, coupling unification would be difficult, it had problems with the isospin quantum numbers of the lighter fermions, and it

needed some fine-tuning in order to prevent some fermions from acquiring large masses.

In the present approach, only the mirror particles are coupled strongly and dynamically involved in the breaking of $SU(2)_L$. By eventually breaking the mirror-generation symmetries, small gauge-invariant (by this we mean here and in the following gauge-invariant under the standard-model gauge group, unless otherwise stated) masses are allowed which communicate the electro-weak symmetry breaking to the standard-model fermions by mixing them with their mirror partners. This model has neither “sterile” nor $SU(2)_L$ -doublet light mirror neutrinos, as in [6] for example, which would pose problems with experiment. After the mass hierarchies are computed within this context, phenomenological consequences like electro-weak precision parameters and CKM matrix elements are then analyzed.

2. Matter content

We start by considering the gauge group structure $SU(4)_{PS} \times SU(2)_L \times SU(2)_{2R} \times SU(3)_{2G} \times U(1)_G \times U(1)_{1R}$. The group $SU(4)_{PS}$ is the usual Pati-Salam group unifying quarks and leptons, and $SU(2)_L$ is the group of weak interactions. The gauged symmetry $SU(2)_{2R}$ acts only on the mirror fermions. It does not play any significant role in the dynamics that follow, and it could have equally well been broken at much higher scales, but only *via* an $SU(2)_{2R}$ -doublet scalar vacuum expectation value (vev), in order to avoid a see-saw mechanism producing too light Majorana mirror neutrinos. The corresponding symmetry for the ordinary fermions $SU(2)_{1R}$ has already been broken down to $U(1)_{1R}$ by an $SU(2)_{1R}$ -triplet vev at higher scales, in order to allow the see-saw mechanism to produce light Majorana standard-model neutrinos.

The group $SU(3)_{2G}$ is a horizontal gauge symmetry also acting only on the mirror fermions, which becomes strong at around 2 TeV. All other groups are taken to have weak couplings at this energy. The corresponding symmetry for the standard-model fermions $SU(3)_{1G}$ has already been broken down to $U(1)_G$ at higher scales, at once or sequentially, in order to avoid large FCNC.

Under the above gauge structure, the following fermions are introduced, which are left-handed gauge (and not mass) eigenstates and transform like

Generations

$$\begin{aligned} \psi_L &: (\mathbf{4}, \mathbf{2}, \mathbf{1}, \mathbf{1}, q_g, 0) \\ \psi_R^c &: (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1}, \mathbf{1}, -q_g, \begin{smallmatrix} -1 \\ +1 \end{smallmatrix}) \end{aligned}$$

Mirror generations

$$\begin{aligned} \psi_L^M &: (\mathbf{4}, \mathbf{1}, \mathbf{2}, \mathbf{3}, 0, 0) \\ \psi_R^M{}^c &: (\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1}, \mathbf{3}, 0, 0) \end{aligned}$$

where $g = 1, 2, 3$ is a generation index, with $q_1 = \kappa$, $q_{2,3} = -\kappa_{\mp} \equiv -(\kappa \mp 1)/2$, and κ an arbitrary abelian charge corresponding to the group $U(1)_G$. The superscript M denotes the mirror partners of the ordinary fermions, and c denotes charge conjugation. For the sake of compactness here and in the following the two members of the doublets of the broken $SU(2)_{1R}$ symmetry are included in the same parenthesis.

One observes that the generation symmetries play a very important role at this stage, and this is to prevent the formation of large gauge-invariant masses. Pairing-up of standard-model and mirror generations is thus prohibited, in agreement with what is usually called “survival hypothesis” [7]. Note also that anomalies are canceled between the fermion generations. One could have taken alternatively $q_g = k$ for all three generations and let anomalies be canceled with the $U(1)_G$ charge of a hypothetical fourth fermion generation that has already decoupled from the physics we study. A relevant example will be discussed later.

Even though this quantum number assignment is reminiscent of technicolor with a strong group $SU(N)_{TC} \approx SU(3)_{2G}$, there is no corresponding extended technicolor (ETC) group, the new anti-particles transform under the same (and not the complex conjugate) representation of the strong group as the new particles, and there is a left-right interchange of weak isospin charges. In addition, the strong group in the present case eventually breaks, as it will be seen in the following. One should furthermore not confuse the present model with other “mirror”

fermion approaches, like in [8] for example, where all components of the new fermions are singlets under $SU(2)_L$ and interact only gravitationally or marginally with the standard-model particles, and which obviously cannot break the electro-weak symmetry dynamically.

At high energy scales that do not enter directly in this talk, the Pati-Salam group is assumed to break spontaneously like $SU(4)_{PS} \times U(1)_{1R} \rightarrow SU(3)_C \times U(1)_Y$, where $SU(3)_C$ and $U(1)_Y$ are the usual QCD and hypercharge groups respectively. Much later, at scales on the order of $\Lambda_G \approx 2$ TeV, the mirror generation group breaks sequentially, just after it becomes strong, like $SU(3)_{2G} \times U(1)_G \rightarrow SU(2)_{2G} \times U(1)_{G'} \rightarrow U(1)_{G''}^*$, where the star superscript denotes here and in the following a broken gauge symmetry. We keep track of the $U(1)_{G''}$ charges because, even though the corresponding gauge group is eventually broken, they could prove useful to the qualitative understanding of the fermion mass hierarchies in the model, as will be seen later. It is not attempted here to investigate how exactly these breakings occur, and for simplicity it is enough to assume that a Higgs mechanism is responsible for them, effective or not. The issue of generation symmetry breaking will be discussed again in the discussion section. The first spontaneous generation symmetry breaking $SU(3)_{2G} \times U(1)_G \rightarrow SU(2)_{2G} \times U(1)_{G'}$ occurs at a scale Λ_G , with an $SU(2)_L$ -singlet scalar state denoted by ϕ_3 and transforming like $(\mathbf{3}, \kappa)$ under the generation symmetry acquiring a non-zero vev. Note that the group $SU(3)_{2G}$ could in principle also self-break dynamically *via* the fermion-condensation channel $\mathbf{3} \times \mathbf{3} \rightarrow \bar{\mathbf{3}}$ if it were given time to become strongly coupled at this energy scale. This breaking channel would however leave the mirror fermions without $U(1)_{G'}$ charge, and we would like to avoid that for reasons that will become clear shortly. The fermions have the following quantum numbers under the new gauge symmetry $SU(3)_C \times SU(2)_L \times SU(2)_{2R} \times SU(2)_{2G} \times U(1)_{G'} \times U(1)_Y$:

The 3rd & 2nd generations

$$q_L^{3,2} : (\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{1}, -\kappa_{\pm}, 1/3)$$

$$l_L^{3,2} : (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}, -\kappa_{\pm}, -1)$$

$$q_R^{3,2\ c} : (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \kappa_{\pm}, \frac{-4/3}{+2/3})$$

$$l_R^{3,2\ c} : (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \kappa_{\pm}, \frac{0}{2})$$

The 3rd & 2nd mirror generations

$$q_L^{3,2M} : (\mathbf{3}, \mathbf{1}, \mathbf{2}, \mathbf{2}, -\kappa/2, 1/3)$$

$$l_L^{3,2M} : (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2}, -\kappa/2, -1)$$

$$q_R^{3,2M\ c} : (\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1}, \mathbf{2}, \kappa/2, -1/3)$$

$$l_R^{3,2M\ c} : (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2}, \kappa/2, 1)$$

The 1st generation

$$q_L^1 : (\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{1}, \kappa, 1/3)$$

$$l_L^1 : (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}, \kappa, -1)$$

$$q_R^1\ c : (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1}, -\kappa, \frac{-4/3}{+2/3})$$

$$l_R^1\ c : (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}, -\kappa, \frac{0}{2})$$

The 1st mirror generation

$$q_L^{1M} : (\mathbf{3}, \mathbf{1}, \mathbf{2}, \mathbf{1}, \kappa, 1/3)$$

$$l_L^{1M} : (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1}, \kappa, -1)$$

$$q_R^{1M\ c} : (\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1}, \mathbf{1}, -\kappa, -1/3)$$

$$l_R^{1M\ c} : (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}, -\kappa, 1)$$

where the superscripts 1,...,3 indicate the fermion generations. Moreover, the letters q and l stand

for quarks and leptons respectively. Note that $\bar{\psi}_R \psi_L^M$ mass terms are prohibited by the $SU(2)_{2G}$ symmetry for the second and third generations, and by the $SU(2)_{2R}$ symmetry for all generations.

At a scale quite close to Λ_G , the $SU(2)_{2G} \times U(1)_{G'}$ group spontaneously breaks sequentially to $U(1)_{G''}$ and this down to $U(1)_{G''}^*$ by two $SU(2)_{L-}$ singlet scalar states, denoted by ϕ_2^\pm and transforming like $(\mathbf{2}, \pm 1/2)$ under the generation symmetry, which acquire non-zero vevs. The quantum numbers of the third and second generation mirror fermions after these breakings are given by

The 2nd mirror generation

$$q_L^{2M} : (\mathbf{3}, \mathbf{1}, \mathbf{2}, -\kappa_-, 1/3)$$

$$l_L^{2M} : (\mathbf{1}, \mathbf{1}, \mathbf{2}, -\kappa_-, -1)$$

$$q_R^{2M}{}^c : (\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1}, \kappa_-, -1/3)$$

$$l_R^{2M}{}^c : (\mathbf{1}, \mathbf{2}, \mathbf{1}, \kappa_-, 1)$$

The 3rd mirror generation

$$q_L^{3M} : (\mathbf{3}, \mathbf{1}, \mathbf{2}, -\kappa_+, 1/3)$$

$$l_L^{3M} : (\mathbf{1}, \mathbf{1}, \mathbf{2}, -\kappa_+, -1)$$

$$q_R^{3M}{}^c : (\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1}, \kappa_+, -1/3)$$

$$l_R^{3M}{}^c : (\mathbf{1}, \mathbf{2}, \mathbf{1}, \kappa_+, 1)$$

while the first mirror generation and all the standard-model generation quantum numbers are left unchanged.

The breakings of the mirror generation symmetries described above induce at lower energies, among others, effective four-fermion operators F of the form

$$F = \frac{\lambda}{\Lambda_G^2} (\bar{\psi}_R^M \psi_L^M) (\bar{\psi}_L^M \psi_R^M) \quad (2.1)$$

for the three mirror fermion generations, where λ are effective four-fermion couplings and the generation indices are omitted for simplicity. The

fermion bilinears in parentheses above transform like a $(\mathbf{2}, \mathbf{2})$ under $SU(2)_L \times SU(2)_{2R}$.

The next step is to assume that, in a manner analogous to top-color scenarios [9], the $SU(2)_{2G}$ group is strongly coupled just before it breaks, and it is therefore plausible to take the effective four-fermion couplings λ to be critical for the mirror fermions of the third and second generations, like in the Nambu-Jona-Lasinio model (NJL). Therefore, condensates of mirror fermions like $\langle \bar{\psi}_L^M \psi_R^M \rangle$ can form which break the symmetry $SU(2)_L \times SU(2)_{2R} \times U(1)_Y$ dynamically down to the usual $U(1)_{EM}$ group of electromagnetism. Consequently, apart from the electro-weak bosons one has now also $SU(2)_{2R}$ massive bosons, but these are only exchanged between mirror fermions.

The fermion condensates described above give to the mirror fermions symmetry-breaking masses of order $M \approx r\Lambda_F$ via the operators F , with r a constant not much smaller than unity if one wants to avoid excessive fine-tuning of the four-fermion interactions. Effective operators of the form $\bar{\psi}_R^{1M} \psi_L^{1M} \bar{\psi}_L^{2,3M} \psi_R^{2,3M} / \Lambda_G^2$ induced by the broken $SU(3)_{2G}$ interaction feed down gauge-symmetry-breaking masses to the first mirror generation. The fact that all mirror fermions get large masses of the same order of magnitude due to the critical interactions avoids fine-tuning problems that would appear if mass hierarchies were introduced by allowing only some of them to become massive, as is done in [10]. Moreover, to avoid breaking QCD and electromagnetism, it is assumed that most-attractive-channel arguments prevent quark-lepton condensates of the form $\langle \bar{q}_L^M l_R^M \rangle$ from appearing.

If generation symmetries were left intact, the mass matrix \mathcal{M} for all the fermions would have the form

$$\begin{array}{cc} & \psi_L \quad \psi_L^M \\ \begin{array}{c} \bar{\psi}_R \\ \bar{\psi}_R^M \end{array} & \begin{pmatrix} 0 & 0 \\ 0 & M \end{pmatrix}, \end{array}$$

where the 4 elements shown are blocks of 3×3 matrices in generation space and M the dynamical mirror-fermion mass due to the strong generation interactions. However, the broken generation symmetries allow the formation of gauge-invariant masses, and the mass matrix \mathcal{M} takes

the form:

$$\begin{pmatrix} \bar{\psi}_R \\ \bar{\psi}_R^M \end{pmatrix} \begin{pmatrix} \psi_L & \psi_L^M \\ 0 & m_1 \\ m_2 & M \end{pmatrix},$$

where the diagonal elements are gauge-symmetry breaking and the off-diagonal gauge-invariant.

The off-diagonal mass matrices can be generated by Yukawa couplings λ_{ij} associated with spinor bilinears of fermions with their mirror partners which are coupled to the scalar states $\phi_{2,3}$ responsible for the spontaneous generation symmetry breakings. The corresponding gauge-invariant term in the Lagrangian has the form

$$\sum_{i,j} \lambda_{ij} \bar{\psi}_{iR} \psi_{jL}^M \phi_{2,3}, \quad (2.2)$$

where the indices i, j count the corresponding fermions in the model. The elements of the matrices $m_{1,2}$ will be taken in general to be quite smaller than the ones in the matrix M , with the exception of the entries related to the top quark.

After diagonalization of the mass matrix shown above, in which the lighter mass eigenstates are identified with the standard-model fermions, a *see-saw* mechanism produces small masses for the ordinary fermions and larger ones for their mirror partners. A specific example for illustration purposes is produced in the next section. The situation is reminiscent of universal *see-saw* models, but it involves fermions having quantum-number assignments which should not in principle pose problems with the Weinberg angle $\sin^2 \theta_W$ [11], [12].

Some remarks relative to the (1,1) block entry of the mass matrix are in order. First, there are no $\langle \bar{\psi}_R \psi_L \rangle$ condensates at these high energy scales. Then, after careful inspection of the quantum numbers carried by the gauge bosons of the broken groups one observes that there are no four-fermion effective operators of the form $(\bar{\psi}_R \psi_L)(\bar{\psi}_L^M \psi_R^M)/\Lambda_G^2$ or any other gauge-invariant operators for any generation which would feed gauge-symmetry-breaking masses to the ordinary fermions.

3. Hierarchies, mixings and precision tests

The mass hierarchies produced by the model are computed next, since they provide the basis of any phenomenological analysis. The gauge-symmetry breaking mass submatrices M are hermitian because of parity symmetry. The gauge-invariant ones, denoted by $m_{1,2}$ should be symmetric due to the quantum numbers assigned to the fermions, but not necessarily real. Complex matrix elements allow therefore in general for weak CP violation. Assuming that $SU(2)_L \times SU(2)_{2R}$ effects can be neglected in the gauge-invariant mass generation process or that their effect is just homogeneously multiplicative, one also has the relation $m_2 = c m_1^\dagger$ between the gauge-invariant submatrices, with c a real constant. This means that the determinant of the mass matrix \mathcal{M} is real, eliminating thus the strong CP problem in this approximation, at least at tree level.

For simplicity, the mass matrices in the following are taken real and having the form

$$\mathcal{M}_i = \begin{pmatrix} 0 & m_i \\ m_i & M_i \end{pmatrix}, \quad i = U, D, l \quad (3.1)$$

for the up-type quarks (U), down-type quarks (D) and charged leptons (l). We give as a numerical example forms for the off-diagonal gauge-invariant mass submatrices of the up-type and down-type quark sectors for illustration purposes (with obvious correspondence between column and row numbers with generation indices):

$$\begin{aligned} m_U(\text{GeV}) &= \begin{pmatrix} 2.3 & 5.7 & 1.1 \\ 5.7 & 20 & 1.3 \\ 1.1 & 1.3 & 360 \end{pmatrix} \\ m_D(\text{GeV}) &= \begin{pmatrix} 1.6 & 1.6 & 0.51 \\ 1.6 & 4 & 1.3 \\ 0.51 & 1.3 & 35 \end{pmatrix}. \end{aligned} \quad (3.2)$$

Without loss of generality, the dynamical assumption is made here that the $SU(2)_L$ -breaking mass submatrices are diagonal and have the form

$$\begin{aligned} M_U(\text{GeV}) &= \begin{pmatrix} 360 & 0 & 0 \\ 0 & 650 & 0 \\ 0 & 0 & 650 \end{pmatrix} \\ M_D(\text{GeV}) &= \begin{pmatrix} 200 & 0 & 0 \\ 0 & 360 & 0 \\ 0 & 0 & 360 \end{pmatrix}. \end{aligned} \quad (3.3)$$

The gauge-symmetry breaking masses of the first mirror generation are taken to be smaller than the ones of the two heavier generations because they are fed down by effective operators that are not critical like the ones for the other mirror generations.

It is also expected that the dynamics provide some custodial symmetry breaking which is responsible for the mass difference in the up- and down-quark sectors. The $U(1)_Y$ could be in principle the source of this difference, but we do not speculate on how this is precisely realised here. One has to further stress that the splitting of M_U and M_D is not *a priori* needed to produce the top-bottom quark mass hierarchy, but it is introduced only to better fit the experimental constraints on the electro-weak parameters, as will be seen later.

These mass matrices give, after diagonalization and without the need for any fine-tuning, the following quark and mirror-quark masses (given in units of GeV):

Standard-model quarks	Mirror quarks
$m_t = 160, m_c = 0.77, m_u = 0.001$	$m_{t^M} = 810,$
$m_{c^M} = 651, m_{u^M} = 360$	
$m_b = 3.4, m_s = 0.07, m_d = 0.003$	$m_{b^M} = 363,$
$m_{s^M} = 360, m_{d^M} = 200.$	

The ordinary quark masses given are slightly smaller than the ones usually quoted because the values reported here are relevant to the characteristic scale of the new strong dynamics which is around 2 TeV, and one has therefore to account for their running with energy. The formalism presents no inherent difficulty whatsoever producing larger masses for these fermions.

The generalization of the standard-model CKM quark-mixing matrix in this scenario is a unitary 6×6 matrix of the form $V_G = K_U^T K_D$, where the matrices $K_{U,D}$ diagonalize the fermion mass matrices like $\mathcal{M}_i = K_i^T J_i K_i$, $i = U, D$, with $J_{U,D}$ being the two 6×6 diagonal mass matrices of the up- and down-quark sector. The generalized CKM matrix has the form

$$V_G = \begin{pmatrix} V_{CKM} & V_1 \\ V_2 & V_{CKM}^M \end{pmatrix}, \quad (3.4)$$

and the usual standard-model CKM matrix V_{CKM} is one of its submatrices given (in absolute val-

ues) by

$$|V_{CKM}| = \begin{pmatrix} 0.98 & 0.22 & 0.003 \\ 0.22 & 0.97 & 0.042 \\ 0.006 & 0.038 & 0.95 \end{pmatrix}, \quad (3.5)$$

which is consistent with present experimental constraints.

The mixing between the first and second generations is larger than the one between the second and third generations, and this can be easily traced back to the relative elements of $m_{U,D}$. Furthermore, one has to be particularly cautious when using the flavor symbol ‘t’ and the flavor name ‘top quark’ for the heaviest standard-model-quark mass eigenstate, since $t_L(t_R^c)$ has a non-negligible $SU(2)_L$ singlet (doublet) component as expected due to the large $\bar{t}_R t_L^M = t_R^M t_L$ mass terms, and this is reflected on the reported value of $|V_{tb}| = 0.95$. This is particularly apparent in the third-generation fermions to which correspond larger gauge-invariant masses, since the fermion-mirror fermion mixings are given roughly by the ratio m_{ii}/M_{ii} . Present experimental data give $|V_{tb}| = 0.99 \pm 0.15$ [13]. More precise future measurements of this quantity should show deviations from its standard-model value which is very close to 1 assuming unitarity of the mixing matrix V_{CKM} . Larger mirror-fermion masses can diminish this effect by reducing the corresponding mixing of the mirrors with the ordinary fermions.

In fact, indirect experimental indications for the existence of $SU(2)_L$ -singlet new fermions which can mix with the third standard-model-generation charged fermions t_L, b_L, τ_L , and $SU(2)_L$ -doublet new anti-fermions which can mix with t_R^c, b_R^c and τ_R^c could already exist in LEP/SLC precision data. One would be coming from the S and T parameters, which are consistent with anomalous top-quark couplings, as will be seen later, and the other coming from anomalous b -quark and τ -lepton couplings to the Z^0 boson corresponding to even 3σ effects [14]. The actual sign of the deviations depends on the relevant interaction strength of the two isospin partners of the mirror doublets with the standard-model fermions, but more details on this are given later. Deviations from the weak couplings of the lighter standard-model particles are heavily suppressed,

but they can be potentially large when the mirror partners are light. Bringing all the mirror partners to lower scales should be avoided nevertheless, since reproducing the weak scale would then require fine-tuning, as will be shown shortly.

The corresponding CKM matrix for the mirror sector V_{CKM}^M is equal (in absolute values) to

$$|V_{CKM}^M| = \begin{pmatrix} 1 & 0.001 & 0.001 \\ 0.001 & 1 & 0.039 \\ 0.001 & 0.036 & 0.95 \end{pmatrix}. \quad (3.6)$$

The third generation is here the main reason why this matrix is not diagonal (The entries (1,1) and (2,2) are close to unity because of the assumed diagonal form of $M_{U,D}$, but not exactly unity, so that the unitarity character of the mixing matrix V_G is preserved.) Furthermore, the matrices V_1 and V_2 mix the up-quark sector of the standard model with the down mirror-quark sector and *vice-versa*, but most of their entries are quite small and we do not list them here.

For the charged leptons, a diagonal gauge-symmetry breaking mass matrix is used again and a gauge-invariant mass matrix having the forms

$$M_l(\text{GeV}) = \begin{pmatrix} 180 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 200 \end{pmatrix}$$

$$m_l(\text{GeV}) = \begin{pmatrix} 0.25 & 0.25 & 0.1 \\ 0.25 & 3.8 & 1 \\ 0.1 & 1 & 17 \end{pmatrix}. \quad (3.7)$$

These give the following lepton and mirror-lepton mass hierarchy (at 2 TeV and in GeV units):
Standard-model charged leptons Mirror charged leptons

$$m_\tau = 1.45, \quad m_\mu = 0.07, \quad m_e = 3 \times 10^{-4} \quad m_{\tau^M} = 201, \quad m_{\mu^M} = 200, \quad m_{e^M} = 180.$$

The difference of the charged-lepton mass matrix with the down-quark mass matrix is attributed to QCD effects. The same mass hierarchies could have been produced with a diagonal submatrix m_l which would require less parameters, but for the sake of consistency a submatrix form similar to m_D is chosen. The neutrino mass and mixing matrix could be quite interesting and is left for future work, since the fact that neutrinos can have both Dirac and Majorana masses makes

theoretical considerations and calculations more involved.

We next proceed by giving an estimate for the dynamically generated weak scale v . A rough calculation using the Pagels-Stokar formula gives

$$v^2 \approx \frac{1}{4\pi^2} \sum_i^N M_i^2 \ln(\Lambda_G/M_i), \quad (3.8)$$

where N is the number of new weak doublets introduced and M_i their mass, where it has been assumed for simplicity that $m_{\nu_i^M} = m_{u_i^M}$ for all mirror neutrinos and where departures from pure weak eigenstates have been neglected. Consequently, for the masses found before and $\Lambda_G \approx 1.8$ TeV one gets $v \approx 250$ GeV, as is required. The mirror fermions can therefore be heavy enough to eliminate any need for excessive fine-tuning of the four-fermion interactions responsible for their masses.

The S parameter [15] could be problematic in this scenario however, since 12 new $SU(2)_L$ doublets are introduced. The main negative effect able to cancel the corresponding large positive contributions to S coming from ‘‘oblique’’ corrections is the existence of vertex corrections stemming from 4-fermion effective interactions, which can give rise to similar effects as the ones induced by light $SU(2)_L$ -invariant scalars known as ‘‘techniscalars’’ [16].

More precisely, it is argued that the effective Lagrangian of the theory, after the spontaneous breaking of the $U(1)_{G'}$ generation symmetry, contains terms which can lead to a shift to the couplings of the top and bottom quarks to the W^\pm and Z^0 bosons. In particular, there are four-fermion terms involving 3rd generation-quark flavor eigenstates and their mirror partners given by

$$\mathcal{L}_{\text{eff}} = - \left(\frac{\lambda_{n1}}{\Lambda_{n1}^2} t_L^{\bar{M}} \gamma^\mu t_L^M + \frac{\lambda_{c1}}{\Lambda_{c1}^2} b_L^{\bar{M}} \gamma^\mu b_L^M \right) t_R^{\bar{}} \gamma_\mu t_R -$$

$$- \left(\frac{\lambda_{c2}}{\Lambda_{c2}^2} t_L^{\bar{M}} \gamma^\mu t_L^M + \frac{\lambda_{n2}}{\Lambda_{n2}^2} b_L^{\bar{M}} \gamma^\mu b_L^M \right) b_R^{\bar{}} \gamma_\mu b_R \quad (3.9)$$

where the λ 's and Λ 's are the effective positive couplings and scales of the corresponding operators renormalized at the Z^0 boson mass, and the subscripts n, c indicate whether the participating fermions have the same hypercharge or not. Note

that terms like $\frac{\lambda_{n3}}{\Lambda_{n3}^2}(q_R^{\bar{M}}\tau^a\gamma^\mu q_R^M)(\bar{q}_L\tau^a\gamma_\mu q_L)$, where $\tau^a, a = 1, 2, 3$ are the three $SU(2)_L$ generators, cannot be generated here in perturbation theory, unlike analogous terms in extended technicolor models. Anyway, such terms would produce shifts only to the left-handed fermion couplings, and these are already too much constrained from LEP/SLC data to be of any interest here.

Adopting the effective Lagrangian approach for the heavy, strongly interacting sector of the theory [17], the two mirror-fermion currents are expressed in terms of effective chiral fields Σ like

$$\begin{aligned} t_L^{\bar{M}}\gamma^\mu t_L^M &= i\frac{v^2}{2}\text{Tr}\left(\Sigma^\dagger\frac{1+\tau^3}{2}D^\mu\Sigma\right) \\ b_L^{\bar{M}}\gamma^\mu b_L^M &= i\frac{v^2}{2}\text{Tr}\left(\Sigma^\dagger\frac{1-\tau^3}{2}D^\mu\Sigma\right) \end{aligned} \quad (3.10)$$

where the covariant derivative D^μ is defined by

$$D^\mu\Sigma = \partial^\mu\Sigma + ig\frac{\tau^a}{2}W_a^\mu\Sigma - ig'\Sigma\frac{\tau^3}{2}B^\mu. \quad (3.11)$$

The g and g' above are the couplings corresponding to the gauge fields W_a^μ and B^μ of the groups $SU(2)_L$ and $U(1)_Y$ respectively. The chiral field $\Sigma = e^{2i\tilde{\pi}/v}$ transforms like $L\Sigma R^\dagger$ with $L \in SU(2)_L$ and $R \in U(1)_Y$ as usual, with hypercharge $Y = \tau^3/2$ and $\tilde{\pi} = \tau^a\pi^a/2$ containing the would-be Nambu-Goldstone modes π^a "eaten" by the electro-weak bosons.

In the unitary gauge $\Sigma = 1$, and the currents given above induce shifts in the standard-model Lagrangian of the form

$$\begin{aligned} \delta\mathcal{L} &= (gW_3^\mu - g'B^\mu)(\delta g_R^t \bar{t}_R \gamma_\mu t_R + \\ &\quad \delta g_R^b \bar{b}_R \gamma_\mu b_R) \end{aligned} \quad (3.12)$$

with the non-standard fermion-gauge boson couplings expressed by

$$\begin{aligned} \delta g_R^t &= \frac{v^2}{4}\left(\frac{\lambda_{n1}}{\Lambda_{n1}^2} - \frac{\lambda_{c1}}{\Lambda_{c1}^2}\right) \\ \delta g_R^b &= -\frac{v^2}{4}\left(\frac{\lambda_{n2}}{\Lambda_{n2}^2} - \frac{\lambda_{c2}}{\Lambda_{c2}^2}\right). \end{aligned} \quad (3.13)$$

After Fierz rearrangement of the terms in the effective Lagrangian \mathcal{L}_{eff} , the scales $\Lambda_{n1, n2, c1, c2}$ can be seen as masses of effective scalar $SU(2)_L$ -singlet spinor bilinears consisting of a mirror and an ordinary fermion. These are reminiscent of "techniscalars" as to their quantum numbers. The

effective four-fermion couplings are not only determined by the corresponding $U(1)_G$ charges, since the present situation is closer to gauged NJL models where unbroken strong gauge interactions can influence considerably four-fermion terms. One may observe that, unlike the present situation, the corresponding scalar effective operators induced by ETC interactions in ordinary technicolor theories would be $SU(2)_L$ -doublets and would not produce shifts to the right-handed fermion couplings. The scalar operators appearing here could in principle correspond to mesons bound with the QCD force, but their constituents are very heavy and are expected in principle to decay weakly before they have time to hadronize.

It is as a matter of fact difficult to predict the values of the effective couplings of the operators that determine the fermion anomalous couplings, since they are influenced by non-perturbative dynamics. The values of the various terms are here chosen for illustration purposes to be $\frac{\lambda_{n1}v^2}{\Lambda_{n1}^2} = 1$, $\frac{\lambda_{c1}v^2}{\Lambda_{c1}^2} = 3.2$, $\frac{\lambda_{n2}v^2}{\Lambda_{n2}^2} = 0.22$, $\frac{\lambda_{c2}v^2}{\Lambda_{c2}^2} = 0.1$. The terms corresponding to operators involving the standard-model top quark (see subscripts $n1, c1$) are assumed larger than the ones involving the standard-model bottom quark (subscripts $n2, c2$). This might be related to the fact that, as was already seen in the mass matrices, $\bar{t}_R t_L^M$ gauge-invariant mass terms are much larger than $\bar{b}_R b_L^M$ terms, which is needed in order to reproduce the correct top-bottom mass hierarchy. This would also explain why four-fermion terms involving first- and second-generation quarks are neglected in this analysis.

Using the values above one finds the anomalous couplings $\delta g_R^b = -0.03$ and $\delta g_R^t = -0.55$. The coupling δg_R^b is within its best-fit value $\delta g_R^b = 0.036 \pm 0.068$ (this is a combined fit including information on δg_L and the S and T parameters [18]). It works here against δg_R^t since it contributes positively, by a comparatively small amount, to the S parameter. It is already so tightly constrained that, even if it finally turns out to be positive, as suggested by [14] and which is easily achievable here by an appropriate choice of the relevant four-fermion couplings, it will not change our conclusions substantially. The coupling δg_R^t is of course not yet constrained, and

it is therefore a good candidate for a possible source of the large vertex corrections needed in this model.

One should expect therefore that, apart from the model-independent “oblique” contributions to the electro-weak precision parameters S and $T = \Delta\rho/\alpha$ (where α is the fine structure constant), denoted by S^0 and T^0 , these parameters receive also important vertex corrections $S^{t,b}$ and $T^{t,b}$ due to the top and bottom quarks, which should be given in terms of the anomalous couplings calculated above. The “oblique” positive corrections to S are given by $S^0 = 0.1N$ for N new $SU(2)_L$ doublets, assuming QCD-like strong dynamics. On the other hand, the mass difference between the up- and down-type mirror fermions produces a positive contribution to T^0 . Considerations in the past literature with mirror fermions or vector-like models which can give very small or negative S^0 and T^0 do not concern us here because they are, unlike the present case, based on the decoupling theorem due to the existence of large gauge-invariant masses [5], [19].

By summing up these effects therefore, one finds for S and T the expressions [18]

$$\begin{aligned} S &= S^0 + S^{t,b} = 0.1N + \\ &\quad \frac{4}{3\pi}(2\delta g_R^t - \delta g_R^b) \ln(\Lambda/M_Z) \\ T &= T^0 + T^{t,b} = \frac{3}{16\pi^2\alpha v^2} \sum_i^N (m_{U_i^M} - m_{D_i^M})^2 + \\ &\quad \delta g_R^t \frac{3m_t^2}{\pi^2\alpha v^2} \ln(\Lambda/m_t), \end{aligned} \quad (3.14)$$

where $m_{U_i^M}, D_i^M$ denote the masses of the up- and down-type mirror quarks, $N = 12$ in the present case, and Λ is the cut-off, which is expected to be close to Λ_G . Note that these expressions are valid for small anomalous couplings, but they are used in the following to illustrate the main effect of new sector even though the top-quark anomalous coupling is taken to be quite large. Moreover, there should be corrections to these formulas, mainly related to the mirror top quark, due to the fact that the mirror mass eigenstates are not pure gauge eigenstates. They are in the following neglected since they should be -at least partially- compensated by the fact that the top quark is also not a weak eigenstate but has a

weak-singlet admixture. It is also noted that contributions to S^0 and T^0 from the lepton sector are calculated assuming Dirac mirror neutrinos.

Nevertheless, one has to stress here that no isospin splitting whatsoever is required *a priori* in the mirror sector in order to get the top-bottom quark mass hierarchy, since this can be produced by differences in the gauge-invariant mass submatrices. The dynamical generation of this hierarchy does not lead to problems with the T parameter, and this can be traced to the fact that the fermion condensates which break dynamically the electro-weak symmetry are distinct from the electro-weak-singlet condensates responsible for the feeding-down of masses to the standard-model fermions. This is contrary to the usual ETC philosophy and closer to the conceptual basis of [20]. The reason this isospin asymmetry is introduced here is only to cancel the large negative contributions to the T parameter coming from the vertex corrections, as will be seen in the following.

By using the fermion masses and anomalous couplings calculated above, one finds that the parameters S and T are given by

$$\begin{aligned} S &\approx 1.2 - 0.45 \ln(\Lambda/M_Z) \\ T &\approx 14.7 \times (0.76 - 0.32 \ln(\Lambda/m_t)). \end{aligned} \quad (3.15)$$

The present best-fit values for the electroweak parameters are (note that this is again a combined fit including b-quark anomalous-coupling information [18])

$$\begin{aligned} S &= -0.40 \pm 0.55 \\ T &= -0.25 \pm 0.46. \end{aligned} \quad (3.16)$$

One observes therefore that for cut-off scales Λ equal or larger than about $\Lambda_G \approx 1.8$ TeV negative values for the S and T parameters consistent with experiment are feasible, i.e. $S \lesssim -0.15$, $T \lesssim -0.2$, and this is mainly due to the large negative anomalous coupling δg_R^t . Similar values for the electro-weak precision parameters could be achieved with smaller anomalous couplings accompanied with a larger cut-off Λ . This would lead to lighter mirror fermions in order to reproduce the weak scale correctly, something that would also automatically imply a larger fermion-mirror fermion mixing, but it would have the un-

desirable effect of increasing the fine tuning in the model.

One way to achieve a smaller S parameter is to note that the generation group is broken, leading to non-QCD-like strong dynamics. If this makes the mirror-fermion masses run much slower with momentum, it can reduce the positive contributions to the S parameter even by a factor of two [21]. In any case, the purpose of the numerical example presented is merely to illustrate that theories of this type may potentially produce negative S and T parameters.

4. Discussion

A possible origin of the gauge group structure introduced in this study and a breaking mechanism of the mirror generation groups are discussed next. The following unification gauge group is considered

$$SO(10)_1 \times SU(4)_{1G} \times SO(10)_2 \times SU(4)_{2G} \subset E_{8_1} \times E_{8_2},$$

under which the matter fields, contained initially in the adjoint representation of the E_8 groups, transform like $(\mathbf{16}, \bar{\mathbf{4}}, \mathbf{1}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{1}, \bar{\mathbf{16}}, \mathbf{4})$ for the ordinary and mirror fermions respectively. This corresponds to four ordinary and four mirror-fermion generations including $SU(2)_L$ -singlet neutrinos. Such a fermion content is Witten-anomaly free [22]. One has then the subsequent spontaneous breaking of $SO(10)_i \times SU(4)_{iG} \rightarrow G_i \times SU(2)_{iR} \times SU(3)_{iG}$ at unification scales, at once or sequentially, where $G_i \sim SU(4)_{iPS} \times SU(2)_{iL} \times U(1)_{iG}$, with $i = 1, 2$. In this context the three lighter mirror-fermion generations could have $U(1)_{2G}$ couplings from the start, with a natural value for the $U(1)_{iG}$ charges being $\kappa = 1/3$. Still at unification scales, the group $G_1 \times G_2$ breaks spontaneously down to its diagonal subgroup $G_D \sim SU(4)_{PS} \times SU(2)_L \times U(1)_G$. Subsequently, the fourth generations can pair-up and disappear from the low energy spectrum.

In addition, the $SU(2)_{1R}$ group is assumed to break spontaneously down to $U(1)_{1R}$ via an $SU(2)_{1R}$ triplet, in order to allow for a see-saw mechanism for the standard-model neutrino masses. The gauged generation symmetry $SU(3)_{1G}$ acting on the standard-model fermions has also to

be broken spontaneously at high energy scales, at once or sequentially, in order to avoid large direct FCNC in the standard-model sector and to prevent the pairing-up of the fermion generations with their mirror partners. These two breakings signal parity violation, and are at the source of the fundamental asymmetry between ordinary and mirror fermions in nature. The unbroken $SU(3)_{2G}$ will allow the condensation of mirror fermions at much lower energy scales, and is remotely reminiscent of the ‘‘heavy color’’ group that was considered in [23] in order to conceal these new fermions. The difference here is that, far from concealing the mirror partners, we mix them with the ordinary fermions by eventually breaking the generation group, in order to generate masses for the standard-model particles. One can further note here that the gauge sector responsible for the eventual mirror-fermion condensation at lower energy scales corresponds to what is usually called ‘‘hidden sector’’ used for gaugino condensation in dynamical supersymmetry breaking models.

Since the breaking of the generation groups takes place just after the corresponding gauge couplings become strong, we speculate next on how a dynamical mechanism could be responsible for this effect. It is namely observed that operators of the form $\psi_L^M \psi_R$ transform like a $(\mathbf{3}, \kappa)$ under $SU(3)_{2G} \times U(1)_G$ and are singlets under the standard-model gauge symmetry. They have therefore the same quantum numbers as the scalar state ϕ_3 introduced in this talk. If these composite operators of fermions could gain non-zero vevs they would break the mirror generation symmetry dynamically to $SU(2)_{2G} \times U(1)_{G'}$.

Operators of the same form, transforming like $(\mathbf{2}, \pm 1/2)$ under the new generation symmetry and having the same quantum numbers as ϕ_2^\pm would break dynamically this gauge symmetry completely if they could also acquire subsequently non-zero vevs. (Note that if $U(1)_{G''}$ breaking occurs dynamically in the same fashion one would have to introduce for consistency slightly non-diagonal gauge-symmetry breaking mass submatrices $M_{U,D}$, which would however not alter qualitatively the results reported. Large non-diagonal elements both in the invariant and symmetry-breaking submatrices should be avoided

nevertheless, since they would create problems with the quasi-diagonal V_{CKM} .) It would be therefore possible to identify these fermionic condensates with the scalars introduced like $\langle \phi_{2,3} \rangle \approx \frac{\langle \psi_L^M \psi_R \rangle}{\Lambda_G^2}$.

The only strong interaction able to generate such condensates in the present framework is unfortunately QCD. It could be of course assisted by non-negligible $U(1)_G$ interactions. The problem is that a strongly coupled abelian group at the TeV scale could in principle pose problems with a Landau pole below the unification scale, unless it is soon embedded in a non-abelian group or other unknown dynamics are involved in the process. On the other hand, non-abelian instead of abelian generation groups common to ordinary and mirror fermions are also problematic. A common $SU(2)_G$ would not have a negative β -function, and a common $SU(3)_G$ would pose problems with too large FCNC in the first generation. The question is therefore whether QCD could be responsible for the generation symmetry breaking. One would then have a situation where the generation symmetry breaks due to strong dynamics at scales much smaller than the scale Λ_G where it becomes strong, since the QCD characteristic scale Λ_{QCD} is on the order of 1 GeV, which is similar to the situation discussed in [5]. Examples for scenarios of this kind, even though *a priori* not excluded, have not been seen in nature yet.

However, in order to make such a scheme consistent with what has been already discussed, non-perturbative effects should shift the masses of the bosons corresponding to the broken symmetries of the generation group from Λ_{QCD} roughly three orders of magnitude up to its characteristic scale Λ_G . It is the fact that the generation group has already become strongly coupled at higher scales that allows us to speculate that the condensate $\langle \psi_L^M \psi_R \rangle$, if it ever forms due to QCD and to $U(1)_G$, and possibly having a large top-quark component, could acquire values much larger than the QCD scale by such effects. An argument supporting this view could be that, after the generation group breaking, there is no symmetry protecting gauge-invariant composite operators from acquiring large vevs, and radia-

tive corrections could in principle shift these vevs anywhere from Λ_{QCD} to Λ_G .

This hierarchy of scales between Λ_{QCD} and Λ_G would also potentially explain without fine-tuning the smallness of most of the gauge-invariant masses in comparison with the gauge-symmetry breaking ones, with the exception of the top and bottom quarks. In a scenario of this type leptons would get their masses *via* four-fermion operators induced by the broken Pati-Salam group. In any case, if QCD, possibly assisted by $U(1)_G$, is unrelated to the formation of such condensates because of the large energy-scales discrepancy, the only way to avoid fundamental Higgs fields in this approach would be a new strong interaction not present in this scenario or other unknown dynamics.

To summarize, we were motivated by several theoretical arguments and possibly by some experimental indications that there are new physics around the TeV scale, and we showed how one can extend the gauge sector of the standard model and its fermionic content in a left-right symmetric context. We argue that doubling the matter degrees of freedom should be considered positively if, instead of just burdening the theory with more parameters, it renders it more symmetric while simultaneously solving several problems like fine-tuning, mass generation, and possibly absence of strong CP violation and eventual unification.

It was shown that the model sets up a precise theoretical framework for the calculation of fermion mass hierarchies and mixings. It gives furthermore rise to dynamics which could potentially reconcile the S - and T - parameter theoretical estimates with their experimental values without excessive fine tuning. Moreover, the doubling of the fermionic spectrum it predicts provides decay modes which should in principle be detectable in colliders like *LHC* and *NLC*. This fact, together with more precise future measurements of possible FCNC and anomalous couplings in the third fermion generation render the model experimentally testable.

Within the present approach, a deeper understanding of the generation of the gauge-invariant mass matrices m and the effective couplings leading to anomalous third-generation standard-model

fermion couplings to the Z^0 boson is still needed. This would settle the question on whether the large positive loop corrections to the S parameter in this model can be adequately canceled by vertex corrections without unnatural fine-tuning. Furthermore, the investigation on how the mirror generation groups break just after they become strong and the unification of couplings at high energies in a way consistent with present bounds on the proton life-time are important questions left for future studies.

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