Reduction of Couplings in Supersymmetric GUTs

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ABSTRACT: Applications of the principle of reduction of couplings to the standard model and supersymmetric grand unified theories are reviewed. Phenomenological applications of renormalization group invariant sum rules for soft supersymmetry-breaking parameters are also discussed.

1. Application to the Standard Model

High energy physicists have been using renormalizability as the predictive tool, and also to decide whether or not a quantity is calculable. It is possible, using the method of reduction of couplings [1]–[4],[5], to renormalize a theory with fewer number of counter terms then usually counted, implying that the traditional notion of renormalizability should be generalized in a certain sense. Consequently, the notion of the predictability and the calculability [6] may also be generalized with the help of reduction of couplings. Of course, whether the generalizations of these notions have anything to do with nature is another question. The question can be answered if one applies the idea of reduction of couplings to realistic models, make predictions that are specfic for reduction of couplings, and then wait till experimentalists find positive results.

In ref. [8] the idea of reduction of couplings has been applied to the $SU(3)_{\rm C}\times SU(2)_{\rm L}\times U(1)_{\rm Y}$ gauge model for the strong and electroweak interactions. As it is known, this theory has a lot of free parameters, and at first sight it seemed there exists no guiding principle how to reduce the couplings in this theory. There were two main problems associated with this program. The one was that it is not possible to assume a common asymptotic behavior for all couplings, and the other one is how to increase the predictive power of the model without running into the contradiction with the experimental knowledge (of that time) such as the masses of the known fermions. It was decided to use asymptotic freedom as a guiding principle, and assumed that QCD is most fundamental among the interactions of the standard model (SM). Since pure QCD is asymptotically free, we tried to switch on as many SM interactions as possible while keeping asymptotic freedom and added them to QCD. The result was almost unique: There exist two possibilities (or two asymptotically free (AF) surfaces in the space of couplings). It turned out that on the first surface, the $SU(2)_{\rm L}$ gauge coupling α_2 is bigger than the QCD coupling α_3, and on the second surface, α_2 has to identically vanish. The second possibility was chosen, because we found out that it is possible to include the $SU(2)_{\rm L}$ gauge coupling α_2 as a certain kind of "perturbation" into the AF system. Since the perturbating couplings should be regarded as free parameters, the reduction of couplings in this case can be achieved only partially ("partial reduction"). For the first case, it was not possible.

Thus, the largest AF system which is phenomenologically acceptable (at that time) contains α_3, the quark Yukawa couplings α_i $(i = d, s, b, u, c, t)$ and the Higgs self coupling α_λ. However, because of the hierarchy of the Yukawa couplings, we could not expect that all couplings can be expressed in terms of a power series of α_3 with-

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out running into the contradiction with that hierarchy. So we decided to apply to the reduction of couplings only to the system with α_3 , α_t and α_λ, and to regard the other couplings as perturbations like α_2.

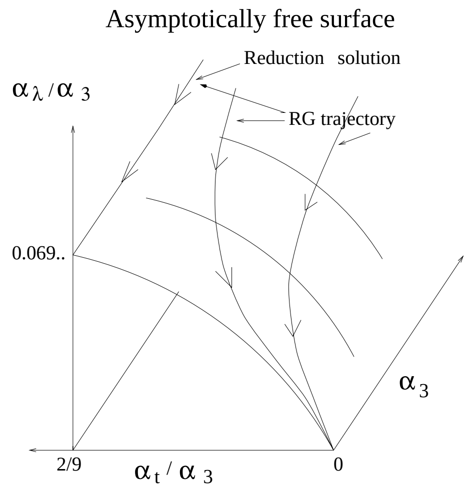

Figure 1: Reduction of α_t and α_λ in favor of α_3.

Fig. 1 shows the AF surface in the space of α_3, α_{t/α_3} and α_λ/α_3. The reduction of the top Yukawa and Higgs couplings in favor of the QCD coupling corresponds to the border line on the surface, i.e., the line defined by

$$\alpha_t/\alpha_3 = \frac{2}{9} ,$$
$$\alpha_\lambda/\alpha_3 = \frac{\sqrt{689}-25}{18} \simeq 0.0694 \qquad (1.1)$$

in the one-loop approximation. This border line was already known as the Pendleton-Ross infrared (IR) fixed point (line) [10]. Note that the existence of the AF surface (shown in Fig. 1) at least for α_3 closed to the origin is mathematically ensured (see also [11]), while the line for large α_3, Pendleton-Ross infrared IR line, can be an one-loop artifact which was pointed out by Zimmermann [12]. He showed explicitly in the two-loop approximation that this is indeed the case.

An asymptotically free renormalization group (RG) trajectory lies exactly on the surface.

Fig. 2 shows trajectories projected on the $\alpha_3 - \alpha_t/\alpha_3$ plane. It may be worthwhile to mention that the branches above the Pendleton-Ross IR line (the lines left to it in Fig. 2) are used by Bardeen *et. al.* [13] to interpret the Higgs

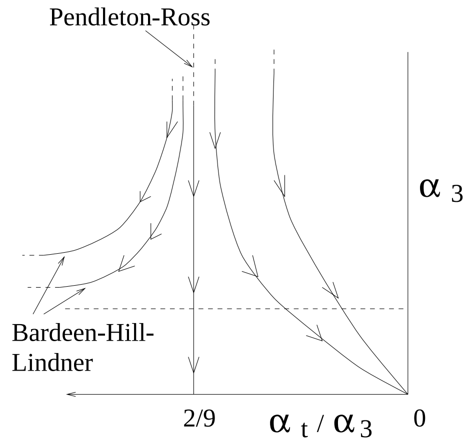

Figure 2: Asymptotically free surface in the $\alpha_3 - \alpha_t/\alpha_3$ space.

particle as a bound state of the top and anti-top quarks. From Fig. 2 one can see that the higher the energy scale where the top Yukawa coupling diverges (the horizontal dotted line in Fig. 2 will be lowered), the similar is the prediction of the top mass in two methods. However, I should be emphasized that how to include the corrections to this lowest order system (especially those due to the non-vanishing $SU(2)_{\rm L}$ and $U(1)_{\rm Y}$ gauge couplings) depends on the ideas behind, so that the actual predictions are different. Including these corrections within the one-loop approximation we calculated α_t/α_3 and α_h/α_3 in terms of α_3 and the perturbating free couplings. Then using the formulae

$$M_t^2/M_Z^2 = 2\cos^2\theta_{\rm W}\alpha_t/\alpha_2 ,$$
$$M_h^2/M_Z^2 = 2\cos^2\theta_{\rm W}\alpha_h/\alpha_2 , \qquad (1.2)$$

the top quark and Higgs masses, M_t and M_h have been calculated, from the known values of the parameters such as the Z boson mass M_Z and the Weinberg mixing angle $\theta_{\rm W}$. It was obtained [8]

$$M_t \simeq 81 \ \ \text{GeV} , M_h \simeq 61 \ \ \text{GeV} . \qquad (1.3)$$

Later including higher order corrections such as two-loop corrections it was found that the earlier predictions (1.3) become $M_t = 98.6 \pm 9.2$ GeV and $M_h = 64.5 \pm 1.5$ GeV, which should be compared with the present knowledge [14]

$$M_t = 173.8 \pm 5.2 \ \ \text{GeV} ,$$
$$M_h \gtrsim s \ 77.5 \ \ \text{GeV} . \qquad (1.4)$$

2. Why Supersymmetry is an Ideal Place for Application?

2.1 Naturalness and supersymmetry

Let us now come to the application of reduction of parameters to supersymmetric theories, and let us accept for a while the usual argument for low energy supersymmetry, which is based on the naturalness notion of 't Hooft [15]. 't Hooft [15] said that there exist a natural scale in a given theory, and that the natural energy scale of spontaneously broken gauge theories which contain the SM is usually less than few TeV. The argument is the following. Suppose the scale at which the SM goes over to a more fundamental theory is Λ. That is, there are in the fundamental theory particles with masses of this order. Now consider the propagator $\Delta(p^2)$ of a scalar boson field with the physical mass $m_{\rm B}$ much smaller than Λ, and suppose that it is normalized at Λ so that the propagator assumes a simple form at $p^2=-\Lambda^2$:

$$\lim_{p^2\to-\Lambda^2}\Delta(p^2)\to\frac{iZ(\Lambda^2)}{p^2-m_{\rm B}^2(\Lambda^2)}~, \qquad (2.1)$$

where Z is the normalization constant for the wave function. The physical mass squared $m_{\rm B}^2$ can be expressed as

$$m_{\rm B}^2 = m_{\rm B}^2(\Lambda^2)+\delta m_{\rm B}^2~. \qquad (2.2)$$

Then we ask ourselves how accurately we have to tune the value of $m_{\rm B}^2(\Lambda^2)$ to obtain a desired accuracy in the physical mass squared $m_{\rm B}^2$. This depends on $\delta m_{\rm B}^2$, of course. 't Hooft said that for a theory to be natural the ratio $m_{\rm B}^2(\Lambda^2)/m_{\rm B}^2$ should be of $O(1)$, which implies that $|\delta m_{\rm B}^2|<m_{\rm B}^2$. If quadratic divergences are involved in the theory, the correction $\delta m_{\rm B}^2$ will be proportional not only to the masses of the light fields, but also to the masses of the heavy fields, and so $\delta m_{\rm B}^2$ can be of the order $(\alpha/4\pi)\Lambda^2$, where α is some generic coupling. Since the Higgs mass should not exceed few hundred GeV in the SM, the natural scale of the fundamental theory, which contains the standard model Higgs and also involves quadratic divergences, is at best few TeV. So according 't Hooft, ordinary Grand Unified Theories (GUTs), for instance, are unnatural [15].

Supersymmetry, thanks to its very renormalization property known as non-renormalization theorem [16, 17], can save the situation. The cancellation of the quadratic divergences, which was first observed by Wess and Zumino [16], is exact if the masses of the bosonic and fermionic superpartners are the same. However, supersymmetry is unfortunately broken in nature, so that the cancellation is not exact. The mass squared difference, $m_{\rm B}^2-m_{\rm F}^2$, characterizes the energy scale of supersymmetry breaking. To make compatible supersymmetry breaking with the naturalness notion of 't Hooft, we must impose the constraint on the supersymmetry-breaking scale $M_{\rm SUSY}$. A simple calculation yields that $M_{\rm SUSY}$ should be less than few TeV.

2.2 Soft supersymmetry-breaking parameters

Since the pioneering works by Fayet and Iliopoulos [18], and the others in late 70's, a lot of attempts to understand supersymmetry-breaking mechanism have been done. However, unfortunately, we still do not know how supersymmetry is really broken in nature. It, therefore, may be reasonable at this moment to pick up the common feature of supersymmetry breaking which effect the SM. The so-called minimal supersymmetric standard model (MSSM) is "defined" along this line of thought. The MSSM contains the ordinary gauge bosons and fermions together with their superpartners, and two supermultiplets for the Higgs sector. (With one supermultiplet in the Higgs sector, it is not possible to give masses to all the fermions of the MSSM.)

It is expected that the common effect of supersymmetry breaking is to add the so-called soft supersymmetry-breaking terms (SSB) to the symmetry theory. The SSB terms are defined as those which do not change the infinity structure of the parameters of the symmetric theory. So they are additional terms in the Lagrangian that do not change the RG functions such as the β- and γ-functions of the symmetric theory. (More precisely, there exists a renormalization scheme in which the RG functions are not altered by the SSB terms.) There exist four types of such terms [19].

$$\begin{aligned} &1.~~\text{Soft scalar mass terms}: && (m^2)^j_i\phi_j\phi^{*i}~,\\ &2.~~B-\text{terms}: && B^{ij}\phi_j\phi_i+~\text{H.C}~, \end{aligned} \qquad (2.3)$$

3. Gaugino mass terms : $M\lambda\lambda +$ H.C ,
4. Trilinear scalar couplings :
   $h^{ijk}\phi_i\phi_j\phi_k +$ H.C ,

where ϕ_j and λ denote the scalar component in a chiral supermultiplet and the gaugino (the fermionic component) in a gauge supermultiplet, respectively.

If one insists only renormalizability for the MSSM, the number of the SSB parameters amounts to about 100, which is about five times of that of the SM. The commonly made assumption to reduce this number is the assumption of universality of the SSB terms, which is often justified by saying that supersymmetry breaking occurs in a flavor blind sector [20]. That is, it is assumed that the soft scalar masses and the trilinear scalar couplings are universal or flavor blind at the scale where supersymmetry breaking takes place. The so-called constrained MSSM contains thus only four independent massive parameters. But we could easily imagine that nature might not be so universal as one wants. In fact it possible to construct a lot of models with non-universal SSB terms [21] (even in models in which supersymmetry-breaking occurs in the so-called hidden sector which does not interact directly with the observable sector), and once we deviate from the universality, there will be chaotic varieties.

The application of reduction of couplings in the SSB sector is based on the assumption that the SSB terms organize themselves into a most economic structure that is consistent with renormalizability. We will come to discuss this later. We have insisted in low energy supersymmetry, because we would like to argue that supersymmetric theories offer an ideal place where the reduction method, especially for massive parameters, can be applied and tested experimentally.

3. Supersymmetric Gauge-Yukawa Unification

Before we discuss the SSB sector, let us stay in the sector of the dimensionless couplings in realistic supersymmetric GUTs and discuss about certain phenomenological successes of reduction of parameters in these theories. We would like to emphasize that in contrast to the SM, supersymmetric GUTs can be asymptotically free or even finite.

3.1 Unification of the gauge and Yukawa couplings based on the principle of reduction of couplings

Finite theories have attracted many theorists. By a finite theory we mean a theory with the vanishing β-functions and anomalous dimensions. As we know, the $N = 4$ supersymmetric Yang-Mills theory is a well-known example [24]. There have been made many attempts to construct $N = 1$ supersymmetric finite theories [23, 25, 26]. Sibold *et. al.* [27] gave an elegant existence proof of finite $N = 1$ supersymmetric theories, where we would like to recall that their proof is strongly based on the Adler-Bardeen non-renormalization theorem of chiral anomaly [28] [1]. The reduction of Yukawa couplings in favor of the gauge coupling is one of the necessary condition for a theory to be finite in perturbation theory. So in a finite theory, Gauge-Yukawa unification is achieved. In ref. [23] a top quark mass of about 180 GeV in a finite $SU(5)$ GUT has been obtained. Since Gauge-Yukawa unification results from the reduction of Yukawa couplings in favor of the gauge coupling, it can be achieved not only in finite theories but also in non-finite theories, as it has been shown in ref. [30]. Relations among the gauge and Yukawa couplings, which are missing in ordinary GUTs, could be a consequence of a further unification provided by a more fundamental theory. And so Gauge-Yukawa unification is a natural extension to the ordinary GUT idea. This idea of unification relies on a symmetry principle as well as on the principle of reduction of couplings. The latter principle requires the existence of RG invariant relations among couplings, which do not necessarily result from a symmetry, but nevertheless preserve perturbative renormalizability or even finiteness.

[1] It is currently studied how to extend their theorem; for instance a non-perturbative extension has also been proposed in [29].

3.2 The double-role of $\tan\beta$

Before we discuss Gauge-Yukawa unification more in details, we would like to discuss about an important parameter, $\tan\beta$, in the MSSM. It is a very popular parameter among SUSY physicists, but let us make few comments on this parameter, because it plays also an important role for Gauge-Yukawa unification. As already mentioned the MSSM contains two Higgs supermultiplets. The most general form of the Higgs potential which is consistent with renormalizability and with the softness of the SSB parameters can be written as

$$\begin{aligned} V &= (m_{H_d}^2 + |\mu_H|^2)\, \hat{H}_d^\dagger \hat{H}_d + (m_{H_u}^2 + |\mu_H|^2) \\ &\quad \times \hat{H}_u^\dagger \hat{H}_u + (\, B\hat{H}_d \hat{H}_u + \text{ H.C.}\,) \\ &\quad + \frac{\pi}{2}(3\alpha_1^2/5 + \alpha_2^2)(\, \hat{H}_d^\dagger \hat{H}_d - \hat{H}_u^\dagger \hat{H}_u\,)^2, \end{aligned} \tag{3.1}$$

where μ_H is the only massive parameter in the supersymmetric limit, while $m_{H_u}^2$, $m_{H_d}^2$ and B are the SSB parameters in this sector. ($m_{H_u}^2$, $m_{H_d}^2$ are real while μ_H and B may be complex parameters.) Here $\hat{H}_{u,d}$ denote the scalar components of the two Higgs supermultiplets. There are four independent massive parameters in this sector as we can see in (3.1). These parameters should give the only one independent mass parameter of the SM, for instance the mass of Z. Now instead of regarding these parameters as independent one can regard also the ratio of the vacuum expectation values [32]

$$\tan\beta \equiv \frac{< \hat{H}_u >}{< \hat{H}_d >} \tag{3.2}$$

as independent. ($\tan\beta$ can be assumed to be real.) Usually one regards $|\mu_H|$ and B as dependent [2]. So the Higgs sector in the tree approximation is characterized by the parameters

$$\tan\beta\ ,\ m_{H_1}^2\ ,\ m_{H_2}^2\ . \tag{3.3}$$

The crucial point for Gauge-Yukawa unification is that $\tan\beta$ plays a double-role. On one hand, it is a parameter in the Higgs potential as we have seen above, and on the other hand it it is a mixing parameter to define the standard model Higgs field out of the two Higgs fields of the MSSM. That is, $\tan\beta$ appears also in the dimensionless sector, and in fact it can be fixed through Gauge-Yukawa unification with the knowledge of the tau mass M_τ, as we would like to explain more in detail below.

[2] If $\tan\beta$ is real as we assume here, B can become complex starting in one-loop order [31].

3.3 How to predict M_t from Gauge-Yukawa Unification

The consequence of a Gauge-Yukawa unification in a GUT is that the gauge and Yukawa couplings are related above the GUT scale M_{GUT}. In the following discussions we consider only the Gauge-Yukawa unification in the third generation sector[3]:

$$\begin{aligned} g_i &= \kappa_i\, g\, \sum_{n=1}(\, 1 + \kappa_i^{(n)} g^{2n}\,)\ , \\ i &= 1,2,3,t,b,\tau\ , \end{aligned} \tag{3.4}$$

where g denotes the unified gauge coupling, g_i denote the gauge and Yukawa couplings of the MSSM. Note that the constants κ_i's can be explicitly calculated from the principle of reduction of couplings. Once $\tan\beta$ and the Yukawa couplings are known, the fermion masses can be calculated as one can easily see from the tree level mass formulae

$$\begin{aligned} M_t &= \sqrt{2}\frac{M_{\text{Z}}}{g_2}\sin\beta\cos\theta_{\text{W}}\ g_t\ , \\ M_{b,\tau} &= \sqrt{2}\frac{M_{\text{Z}}}{g_2}\cos\beta\cos\theta_{\text{W}}\ g_{b,\tau}\ , \end{aligned} \tag{3.5}$$

where M_t, M_b and M_τ are the masses of the top and bottom quarks and tau, respectively. Assume that we use the tau mass M_τ as input and also that below M_{SUSY} ($> M_t$) the effective theory of the GUT is the SM. At M_{SUSY} the couplings of the SM and MSSM have to satisfy the matching conditions [4]

$$\begin{aligned} \alpha_t^{\text{SM}} &= \alpha_t \sin^2\beta\ ,\ \alpha_b^{\text{SM}} \ =\ \alpha_b \cos^2\beta\ , \\ \alpha_\tau^{\text{SM}} &= \alpha_\tau \cos^2\beta\ ,\ \alpha_\lambda = \frac{1}{4}(\frac{3}{5}\alpha_1 + \alpha_2)\cos^2 2\beta\ , \end{aligned}$$

where α_i^{SM} ($i = t,b,\tau$) are the SM Yukawa couplings and α_λ is the Higgs coupling. It is now easy to see that there is no longer freedom for

[3] A naive extension to include other generations into this scheme fails phenomenologically.

[4] There are MSSM threshold corrections to the matching conditions [33, 34], which are ignored here.

$\tan\beta$ because with a given set of the input parameters, especially $M_\tau = 1.777$ GeV and $M_Z = 91.187$ GeV, the matching conditions (3.6) at $M_{\rm SUSY}$ and the Gauge-Yukawa unification boundary condition (3.5) at $M_{\rm GUT}$ can be simultaneously satisfied only if we have a specific value of $\tan\beta$. In this way Gauge-Yukawa unification enables us to predict the top and bottom masses in supersymmetric GUTs.

Table 1 shows the predictions in the case of a finite $SU(5)$ GUT [26], in which the one-loop reduction solution is given by

$$g_t^2 = \frac{4}{5}\,g^2 \ , \ g_b^2 = g_\tau^2 = \frac{3}{5}\,g^2 \ . \qquad (3.6)$$

The experimental values of M_t, M_b and $\alpha_3(M_Z)$ are [14]

$$\begin{aligned} \alpha_3(M_Z) &= 0.119 \pm 0.002 \ , \\ M_t &= 173.8 \pm 5.2 \ \ \text{GeV} \ , \\ M_b &= 5.2 \pm 0.2 \ \ \text{GeV} \ . \end{aligned} \qquad (3.7)$$

We see that the predictions of the model reasonably agree with the experimental values[5]. This means, among other things, that the top-bottom hierarchy could be explained to a certain extent in this Gauge-Yukawa unified model, which should be compared with how the hierarchy of the gauge couplings of the SM can be explained if one assumes the existence of a unifying gauge symmetry at $M_{\rm GUT}$ [35]. More details on the different gauge-Yukawa unified models and their predictions can be found in [7, 36, 37].

4. Reduction of Massive Parameters: Application to the Soft Supersymmetry-Breaking Sector

To formulate reduction of massive parameters, one first has to formulate reduction of dimensionless parameters in a massive theory, which was initiated in ref. [38]. To keep the generality of the formulation in the massive case is much more involved than in the massless case, because the RG functions now can depend on the ratios of mass parameters in a complicated way. In the massless case they are just power series in coupling constants (at least in perturbation theory). For phenomenological and also practical applications of the reduction method, it is therefore most convenient to work in a mass independent renormalization scheme, such as the dimensional renormalization scheme. There exists a transformation of one scheme to another one [39, 22]. Consequently, there exist a transformation of a set of the reduction solutions in a mass-dependent renormalization scheme into a set of the reduction solutions in a mass-independent renormalization scheme. Moreover, the renormalization scheme independence of the reduction method has been proved in [22]. Thus, the naive treatment on the massive parameters (which was performed in phenomenological analyses [40, 41]) can now be justified by his theorem [6].

4.1 Application to the minimal model

Now let us come to the SSB sector of a supersymmetric GUT. Recall that the Higgs potential (3.1) (in the tree approximation) is completely characterized by the soft scalar masses $m^2_{H_u}$, $m^2_{H_d}$ and $\tan\beta$, where $\tan\beta$ is fixed through Gauge-Yukawa unification as we have seen before. We [41] applied the the reduction method of massive parameters to the SSB sector of the minimal supersymmetric $SU(5)$ GUT with Gauge-Yukawa unification in the third generation ($g_t^2 = (2533/2605)g^2$, $g_b^2 = g_\tau^2 = (1491/2605)g^2$) [30], and obtained the reduction solution

$$h_t = -g_t\,M \ , \ h_b = -g_b\,M \ , \qquad (4.1)$$

$$\begin{aligned} m^2_{H_u} &= -\frac{569}{521}M^2 \ , \ m^2_{H_d} = -\frac{460}{521}M^2 \ , \\ m^2_{b_R} &= m^2_{\tau_L} = m^2_{\nu_\tau} = \frac{436}{521}M^2 \ , \\ m^2_{d_R} &= m^2_{e_L} = m^2_{\nu_e} = m^2_{s_R} \\ &= m^2_{\mu_L} = m^2_{\nu_\mu} = \frac{8}{5}M^2 \ , \qquad (4.2) \\ m^2_{t_L} &= m^2_{b_L} = m^2_{t_R} = m^2_{\tau_R} = \frac{545}{521}M^2 \ , \\ m^2_{u_L} &= m^2_{d_L} = m^2_{u_R} = m^2_{e_R} \end{aligned}$$

[5] The correction to M_b coming from the MSSM superpartners can be as large as 50% for very large values of $\tan\beta$ [33, 34]. In Table 1 we have not included these corrections because they depend on the SSB parameters. The GUT threshold corrections are ignored too.

[6] It is assumed in the theorem that the β-functions in a mass-dependent renormalization scheme have a sufficiently smooth behavior in the massless limit [22].

M [GeV]	$\alpha_3(M_Z)$	$\tan\beta$	$M_{\rm GUT}$ [GeV]	M_b [GeV]	M_t [GeV]
800	0.118	48.2	1.3×10^{16}	5.4	173
10^3	0.117	48.1	1.2×10^{16}	5.4	173
1.2×10^3	0.117	48.1	1.1×10^{16}	5.4	173

Table 1: The predictions for different $M_{\rm SUSY}$ for the finite $SU(5)$ model.

$$= m^2_{c_L} = m^2_{s_L} = m^2_{c_R} = m^2_{\mu_R} = \frac{12}{5}M^2$$

in the one-loop approximation, where h_i's are the trilinear scalar couplings, m_i's are the soft scalar masses, and M is the unified gaugino mass. We found moreover that we can consistently regard μ_H and B as free parameters. As we can see from (4.1) and (4.2) the unified gaugino mass parameter M plays a similar role as the gravitino mass $m_{2/3}$ in supergravity coupled to a GUT and characterizes the scale of the supersymmetry-breaking [7]. Note that the reduction solution for the soft scalar masses (4.2) is not of the universal form while those for the trilinear couplings (4.1) are universal in the one-loop approximation.

Regarding the reduction solutions (4.1) and (4.2) as boundary conditions at $M_{\rm GUT}$ in the minimal supersymmetric GUT with Gauge-Yukawa unification in the third generation, we can compute the spectrum of the superpartners of the MSSM, which is shown in Table 2, where we have used the unified gaugino mass $M = 1$ TeV. The prediction above depends basically only on the unified gaugino mass M, and so the model has an extremely strong predictive power. Note also that $m^2_{H_u}$, $m^2_{H_d}$ and $\tan\beta$ (see the Higgs potential (3.1) and the definition (3.2)) are now fixed outside of the Higgs sector, so that there is no guaranty that the Higgs potential (3.1) yields the desired symmetry breaking of $SU(2)_{\rm L}\times U(1)_{\rm Y}$ gauge symmetry. Surprisingly, in the case at hand it does! (If the sign of $m^2_{H_u}$ in (4.2) were different, for instance, it would not do.) In Table 3 I give the predictions from the dimensionless sector of the model. At last but not least we would like to emphasize that the reduction solutions (4.1) and (4.2) do *not* lead to the flavor changing neural current (FCNC) problem. This is not something put ad hoc by hand; it is a consequence of the principle of reduction of couplings.

[7]See for instance [20].

5. Sum Rules for the Soft Super–symmetry Breaking Parameters

5.1 Renormalization group invariant sum rules

Now let us to come the next topic. To proceed recall the result of the reduction of the SSB parameters in favor of the unified gaugino mass M in the minimal SUSY $SU(5)$ model which I have discussed just above. As we have seen, the reduction solutions for the trilinear couplings are universal while those for the soft scalar masses are not (see (4.1) and 4.2)). However, if one adds the soft scalar mass squared in an appropriate way, one finds something interesting [43]. For instance,

$$\begin{aligned} M^2 &= m^2_{t_L} + m^2_{t_R} + m^2_{H_u} \\ &= m^2_{b_L} + m^2_{b_R} + m^2_{H_d}\ . \end{aligned} \tag{5.1}$$

This is not an accidental coincidence. One can in fact show that the sum rules in this form are RG invariant at one-loop [43].

In last years there have been continues developments [44]–[48] in computing the RG functions in softly broken supersymmetric Yang-Mills theories, and the well-known result on the QCD β-function obtained by Novikov *et. al.* [49] has been generalized so as to include to the SSB sector [44]–[48], which is based on a clever spurion superfield technique along with power counting [8]. Using this result, it is possible to find a closed form of the sum rules that are RG invariant to all orders in perturbation theory [45]-[48].

To be specific, we consider a softly broken supersymmetric theory described by the super-

[8]It is not clear at the moment in which class of renormalization schemes exactly the result is valid; a renormalization scheme independent investigation of this result is certainly desirable.

$m_\chi = m_{\chi_1}$ (TeV)	0.45	$m_{\tilde{b}_2}$ (TeV)	1.88
m_{χ_2} (TeV)	0.84	$m_{\tilde{\tau}} = m_{\tilde{\tau}_1}$ (TeV)	0.92
m_{χ_3} (TeV)	1.73	$m_{\tilde{\tau}_2}$ (TeV)	1.10
m_{χ_4} (TeV)	1.73	$m_{\tilde{\nu}_1}$ (TeV)	1.43
$m_{\chi_1^\pm}$ (TeV)	0.84	m_A (TeV)	0.70
$m_{\chi_2^\pm}$ (TeV)	1.73	$m_{H^\pm}$ (TeV)	0.70
$m_{\tilde{t}_1}$ (TeV)	1.69	m_H (TeV)	0.70
$m_{\tilde{t}_2}$ (TeV)	1.89	m_h (TeV)	0.120
$m_{\tilde{b}_1}$ (TeV)	1.70		

Table 2: The prediction of the superpartner spectrum for $M = 1$ TeV in the minimal gauge-Yukawa unified model. The mass unit is TeV.

$\alpha_3(M_Z)$	$\tan\beta$	$M_{\rm GUT}$ [GeV]	M_b [GeV]	M_t [GeV]
0.119	48.8	1.47×10^{16}	5.4	177

Table 3: The predictions from the dimensionless sector of the minimal model ($M = 0.5$ TeV).

potential

$$W = \frac{1}{6} Y^{ijk}\, \Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{ij}\,\Phi_i\Phi_j\ , \quad (5.2)$$

along with the Lagrangian for the SSB terms,

$$\begin{aligned} -\mathcal{L}_{\rm SB} &= \frac{1}{6} h^{ijk}\,\phi_i\phi_j\phi_k + \frac{1}{2} b^{ij}\,\phi_i\phi_j \quad (5.3)\\ &+ \frac{1}{2}(m^2)^j_i\,\phi^{*\,i}\phi_j + \frac{1}{2} M\,\lambda\lambda + \text{H.c.}\ , \end{aligned}$$

where Φ_i stands for a chiral superfield with its scalar component ϕ_i, and λ is the gaugino field. It has been found [46] that the expressions [9]

$$\begin{aligned} b^{ij} &= -M\mu^{ij}\frac{d\ln\mu^{ij}(g)}{d\ln g}\ ,\\ h^{ijk} &= -M\frac{dY^{ijk}(g)}{d\ln g}\ , \quad (5.4)\\ m_i^2 &= \frac{1}{2}|M|^2(g/\beta_g)\frac{d\gamma_i(g)}{d\ln g} \quad (5.5) \end{aligned}$$

are RG invariant to all orders in perturbation theory in a certain class of renormalization schemes, which are the higher order results for the one-loop reduction solutions (4.1) and (4.2). Similarly, the sum rule (5.1) in higher orders becomes [47]

$$\begin{aligned} & m_i^2 + m_j^2 + m_k^2 = |M|^2\\ &\times\left\{ \frac{1}{1-g^2C(G)/(8\pi^2)}\frac{d\ln Y^{ijk}}{d\ln g} + \frac{1}{2}\frac{d^2\ln Y^{ijk}}{d(\ln g)^2} \right\}\\ &+\sum_l \frac{m_l^2 T(R_l)}{C(G)-8\pi^2/g^2}\frac{d\ln Y^{ijk}}{d\ln g}\ , \quad (5.6) \end{aligned}$$

in the renormalization scheme which corresponds to that of [49]. Here $C(G)$ is the quardratic Casimir in the adjoint representation, $T(R)$ stands for the Dynkin index of the representation R, β_g is the β-function of the gauge coupling g, γ_i is the anomalous dimension of Φ_i. These expressions look slightly complicated. But if one uses the freedom of reparametrization [3], they can be transformed into a more simple form $((d\ln Y^{ijk}/d\ln g) = 1)$ to obtain

$$\begin{aligned} h^{ijk} &= -Y^{ijk}(g)M\ , \quad (5.7)\\ m_i^2 + m_j^2 + m_k^2 &= |M|^2\ \frac{1}{1-g^2C(G)/(8\pi^2)}\\ &+\sum_l \frac{m_l^2 T(R_l)}{C(G)-8\pi^2/g^2} \quad (5.8) \end{aligned}$$

It is exactly this form which coincides with the results obtained in certain orbifold models of superstrings [26] [10]. We believe that this coincidence is not accidental, and we also believe that target-space duality invariance [52], which is supposed to be an exact symmetry of compactified superstring theories [11], is mostly responsible for the coincidence. In fact there exist already some indications for that. Hopefully we shall report

[9] The Yukawa couplings Y^{ijk} and μ^{ij} are assumed to be functions of the gauge coupling g.

[10] Tree-level sum rules (like (5.1)) in string theories are found in [43], [50]-[51]

[11] See [53], for instance, for target-space duality.

on the true reason of this interesting coincidence in near future.

5.2 Finiteness and sum rules

At this stage it may be worthwhile to mention that the reduction solution (5.4) and the sum rules (5.6) ensure the finiteness of the SSB sector in a finite theory [12]. For the $N = 4$ supersymmetric Yang Mills theory in terms of $N = 1$ superfields, for instance, we have $\sum_l m_l^2 T(R_l) = (m_i^2 + m_j^2 + m_k^2) C(G)$ so that the all order sum rule (5.8) assumes the tree level form $m_i^2 + m_j^2 + m_k^2 = |M|^2$. Applied to the finite $SU(5)$ model [26] which has been discussed in the previous section (Table 1 presents the prediction from the dimensionless sector), it means that the sum rules [26]

$$\begin{aligned} m_{H_u}^2 + 2m_{\mathbf{10}}^2 &= M^2 \ , \ m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3} \ , \\ m_{\overline{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 &= \frac{4M^2}{3} \end{aligned} \tag{5.9}$$

should be satisfied at and above $M_{\rm GUT}$ for the two-loop finiteness of the SSB sector requires that, where

$$\begin{aligned} m_{\mathbf{10}} &= m_{t_L} = m_{b_L} = m_{t_R} = m_{\tau_R} \ , \\ m_{\overline{\mathbf{5}}} &= m_{b_R} = m_{\tau_L} = m_{\nu_\tau} \ . \end{aligned} \tag{5.10}$$

In this case we have an additional free parameter, $m_{\mathbf{10}}$, in the SSB sector. It turned out that the mass of a superpartner of the tau (s-tau) tends to become very light in this model. Consequently, in order to obtain a neutral lightest superparticle (LSP) (because we assume that R-parity is intact), we have to have a large unified gravitino mass $M \gtrsim 0.8$ TeV. For $M = 1$ TeV, only the window $0.62 \text{ TeV} < m_{\mathbf{10}} < 0.66 \text{ TeV}$ is allowed. In Table 4 we give the prediction of the superpartner spectrum of the model for $m_{\mathbf{10}} = 0.62/0.66$ TeV and $M = 1$ TeV. We have assumed the universal soft masses for the first two generations. But this assumption does not change practically our prediction of the spectrum expect for those that are directly of the first two generations.

[12]There exists a fine difference in the opinions about this point. See, for instance, [45, 46].

5.3 Sum rules in the superpartner spectrum

The sum rules (5.1) or (5.8) can be translated into the sum rules of the superpartner spectrum of the MSSM [54] as we will show now. To be specific we assume an $SU(5)$ type Gauge-Yukawa unification in the third generation of the form (3.5). For a given model, the constants κ's are fixed, but here we consider them as free parameters. As before we use the tau mass M_τ as an input parameter, and we go from the parameter space $(\kappa_t \ , \ \kappa_b)$ to another one $(\kappa_t \ , \ \tan\beta)$, because in this analysis we use the physical top quark M_t, too, as an input parameter. Then the unification conditions of the gauge and Yukawa couplings of the MSSM (i.e., $g = g_1 = g_2 = g_3 \ , \ g_b = g_\tau$) fixes the allowed region (line) in the $\kappa_t - \tan\beta$ space for a given value of the unified gaugino mass M. The parameter space in the SSB sector at $M_{\rm GUT}$ is constrained due to unification:

$$\begin{aligned} M &= M_1 = M_2 = M_3 \ , \\ m_{t_R}^2 &= m_{t_L}^2 = m_{b_L}^2 = m_{\tau_R}^2 \ , \\ m_{b_R}^2 &= m_{\tau_L}^2 = m_{\nu_\tau}^2 \ , \end{aligned} \tag{5.11}$$

where M_i $(i = 1, 2, 3)$ are the gaugino masses for $U(1)_{\rm Y}$ (bino), $SU(2)_{\rm L}$ (wino) and $SU(3)_{\rm C}$ (gluino). And the one-loop sum rules at $M_{\rm GUT}$ yield

$$\begin{aligned} h_t &= -M \ , \ h_b = h_\tau = -M\, g_b \ , \\ M^2 &= m_{\Sigma(t)}^2 = m_{\Sigma(b)}^2 = m_{\Sigma(\tau)}^2 \ , \end{aligned} \tag{5.12}$$

where

$$\begin{aligned} m_{\Sigma(t)}^2 &\equiv m_{t_R}^2 + m_{t_L}^2 + m_{H_u}^2 \ , \\ m_{\Sigma(b,\tau)}^2 &\equiv m_{b_R,\tau_R}^2 + m_{b_L,\tau_L}^2 + m_{H_d}^2 \ . \end{aligned} \tag{5.13}$$

(The above equations are the same as (4.1) and (5.1), respectively.) We would like to emphasize that in the one-loop RG evolution of m_Σ^2's in the MSSM only the same combinations of the sum of m_i^2's enter. Therefore, as far as we are interested in the evolution of m_Σ^2's, we have only one additional parameter $M_{\rm SUSY}$.

To derive the announced sum rules for the superpartner spectrum, we define

$$s_i \equiv m_{\Sigma(i)}^2 / M_3^2 \ \ (i = t, b, \tau) \quad \text{at} \ \ Q = M_{\rm SUSY} \ . \tag{5.14}$$

m_{χ_1}	0.45/0.45	$m_{\tilde{s}_1} = m_{\tilde{d}_1}$	1.95/1.95
m_{χ_2}	0.84/0.84	$m_{\tilde{s}_2} = m_{\tilde{d}_2}$	2.06/2.05
m_{χ_3}	1.29/1.32	$m_{\tilde{\tau}_1}$	0.46/0.46
m_{χ_4}	1.29/1.32	$m_{\tilde{\tau}_2}$	0.73/0.66
$m_{\chi_1^\pm}$	0.84/0.84	$m_{\tilde{\nu}_\tau}$	0.70/0.57
$m_{\chi_2^\pm}$	1.29/1.32	$m_{\tilde{\mu}_1} = m_{\tilde{e}_1}$	0.70/0.71
$m_{\tilde{t}_1}$	1.50/1.51	$m_{\tilde{\mu}_2} = m_{\tilde{e}_2}$	0.89/0.89
$m_{\tilde{t}_2}$	1.72/1.74	$m_{\tilde{\nu}_\mu} = m_{\tilde{\nu}_e}$	0.89/0.88
$m_{\tilde{b}_1}$	1.51/1.46	m_A	0.63/0.77
$m_{\tilde{b}_2}$	1.70/1.71	$m_{H^\pm}$	0.63/0.77
$m_{\tilde{c}_1} = m_{\tilde{u}_1}$	1.96/1.96	m_H	0.63/0.77
$m_{\tilde{c}_2} = m_{\tilde{u}_2}$	2.05/2.05	m_h	0.120/0.120
M_3	2.21/2.21		

Table 4: The predictions of the superpartner spectrum for the finite $SU(5)$ model. $M = 1$ TeV and $m_{\mathbf{10}} = 0.62/0.66$ TeV.

The parameters s_i's do not depend on the value of the unified gaugino mass M, but they do on $\tan\beta$. This dependence is shown in Fig. 3. We then express the masses of the superpartners in terms of the soft scalar masses and the masses of the ordinary particles to obtain the sum rules [54],

$$\begin{aligned} & -\cos 2\beta\; m_A^2 && (5.15)\\ &= (s_b - s_t)M_3^2 + 2(\hat{m}_t^2 - m_t^2) - 2(\hat{m}_b^2 - m_b^2) \\ &= (s_\tau - s_t)M_3^2 + 2(\hat{m}_t^2 - m_t^2) - 2(\hat{m}_\tau^2 - m_\tau^2), \end{aligned}$$

where m_A^2 is the neutral pseudoscalar Higgs mass squared, and $\hat{m}_i^2$ stands for the arithmetic mean of the two corresponding scalar superparticle mass squared.

Since we have assumed an $SU(5)$-type supersymmetric GUT with a gauge-Yukawa unification in the third generation, the result (5.16) is not a direct consequence of a superstring model, although the form of the sum rules in both kinds of unification schemes might coincide with each other as I mentioned (see footnote 11). However, under the following circumstances (only rough), the sum rules (5.16) could be a consequence of a superstring model: (i) The Yukawa coupling of the third generation is field-independent in the corresponding effective $N = 1$ supergravity. (ii) Below the string scale an $SU(5)$-type gauge-Yukawa unification is realized so that the sum rules are RG invariant below the string scale and are satisfied down to $M_{\rm GUT}$. (iii) Below $M_{\rm GUT}$ the effective theory is the MSSM.

The sum rules (5.16) could be experimentally tested if the superpartners are found in future experiments, e.g., at LHC. In any event, an experimental verification of the sum rules of the SSB parameters would give an interesting information on physics beyond the GUT scale.

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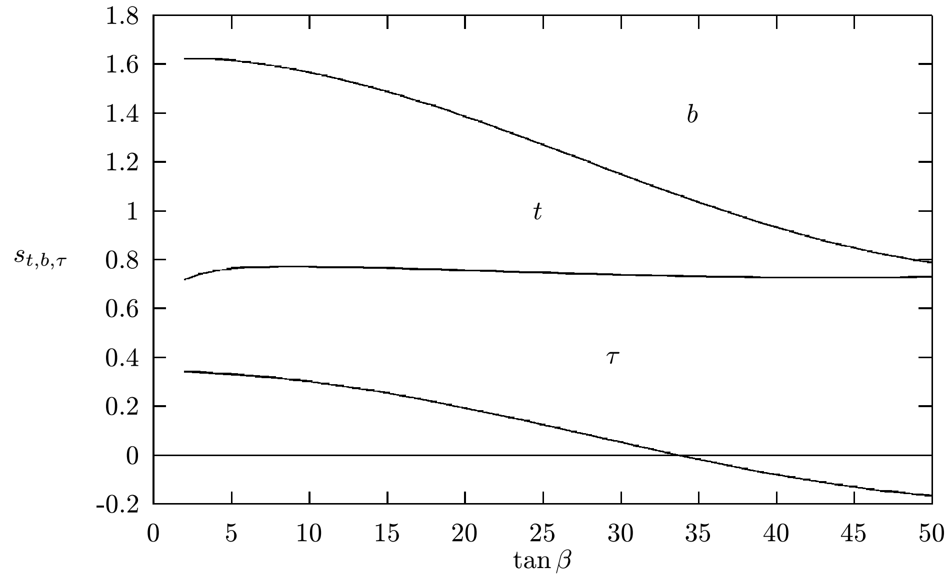

Figure 3: The dependence of the s_i's on $\tan\beta$.

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