Lepton Flavor Violation in SUSY models with $U(1)$–textures

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Abstract: $U(1)$ family symmetries have led to successful predictions of the fermion mass spectrum and the mixing angles of the hadronic sector. In the context of the supersymmetric unified theories, they further imply a non-trivial mass structure for the scalar partners, giving rise to new sources of flavour violation. While $\tau \to \mu\gamma$ decays are mostly expected to arise at rates significantly smaller than the current experimental limits, the $\mu \to e\gamma$ rare decays impose important bounds on the model parameters. Even if universal soft-terms are assumed at the GUT scale, when massive neutrinos are included in these theories, new mixings appear in the soft-terms. The predicted branching ratios for rare decays are in this case below the experimental bounds.

1. Introduction

The SM predicts conservation of lepton flavor in the limit of zero neutrino masses. In the case of massive non-degenerate neutrinos, the amount of lepton flavour violation (LFV) is proportional to the factor $\Delta m_\nu^2/M_W^2$ [1], highly suppressing all relevant processes. The current experimental limits are [2]:

$$\begin{aligned} BR(\mu \to e\gamma) &< 4.9 \cdot 10^{-11} \\ BR(\tau \to \mu\gamma) &< 3.0 \cdot 10^{-6} \\ BR(\tau \to e\gamma) &< 2.7 \cdot 10^{-6} \end{aligned} \tag{1.1}$$

SUSY theories assume a scalar(fermion) partner for every fermion (scalar) of the standard model. The new interactions introduced by these theories can generate LFV diagrams, as the ones in Fig.[1]. In the limit of flavor universal soft terms, leptons and sleptons can be simultaneously diagonalized and hence LFVB processes will be suppressed. Since experimental limits for these processes are very restrictive, some flavor dependence in the Soft-Breaking structure will be reflected in an important LFV at low energies.

In SUSY-GUT's theories even if universal sof-terms are universal at M_{Planck}, non universalities are radiatively generated at the GUT scale due to the evoution of quarks and leptons in the same multiplets [3]. When a phenomenologically aceptable *ansatze* about Yukawa structures is introduced in these theories, new sources of LFV appear due to a non-minimal Higgs sector and additional Symmetries [4].

When right-handed neutrinos enter in the model, Dirac mass matrices arise of the order of the up-quark masses. Charged leptons and neutrinos are no longer diagonal in the same basis and a leptonic mixing matrix, similar to the V_{CKM} one for the quarks, is unavoidable. Moreover, it enters in the construction of the 12×12-sneutrino mass matrix which in principle would have the potential to give rise to additional flavour-violating effects. Nevertheless, since only the effective light sneutrino mass matrix is relevant in the calculation, we will show in this work that the m_D effects are canceled at first order, when the see-saw mechanism is applied.

In this talk we summarize the results of some recent work [5], in which we analyze the branch-

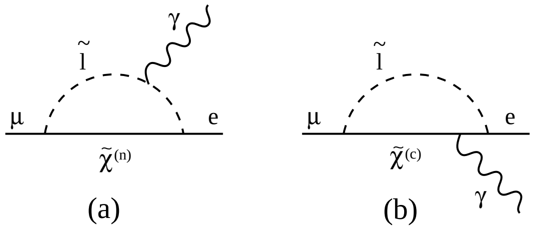

Figure 1: The generic Feynman diagrams for the $\mu \to e\gamma$ decay. $\tilde{l}$ stands for charged slepton (a) or sneutrino (b), while $\tilde{\chi}^{(n)}$ and $\tilde{\chi}^{(c)}$ represent neutralinos and charginos respectively.

ing ratios for lepton flavor violating decays in a class of models predicting fermion masses throught one simple $U(1)$-family symmetry.

2. The model

We consider a $SU(3)\times SU(2)\times U(1)$ SUSY model with an additional $U(1)$-family ($U(1)_f$) symmetry as the one proposed by Ibáñez and Ross [6]. Massive right-handed neutrinos are included in the model allowing neutrinos to obtain a small mass via a "see-saw" mechanism. The relevant superpotential terms are:

$$\mathcal{W}_{lep} = e^{cT}\lambda_e \ell H + N^c\lambda_D \ell H' + \lambda_N \chi N^c N^c \quad (2.1)$$

where ℓ is the left lepton doublet, e^c is the right singlet charged lepton, H and H' are higgs doublets, N^c is the right-handed (RH) neutrino field and $\lambda_{e,D}$ represent Yukawa coupling matrices in flavour space. In addition, soft supersymmetry breaking terms generate mass matrices for the charged slepton fields, denoted by $\tilde{m}_\ell$, $\tilde{m}_{e_R}$.

The fermion mass hierarchy is successfully obtained using an additional singlet field θ and which develops a vev an order of magnitude below the string scale M_U. Throughout our calculations, we will further assume the existence of a grand unified symmetry M_{GUT} without specifying the gauge group. Below the unification scale, only the MSSM spectrum is assumed, therefore the unification point will be taken at $\sim 10^{16}$ GeV. When the $U(1)_f$ symmetry is exact, only the third generation has a Yukawa term in the superpotential, whilst all mixing angles are zero and lighter generations remain massless and uncoupled. When ϕ acquires a vev, the $U(1)_f$-symmetry is broken and mass terms fill in the rest of the mass matrix entries with Yukawa terms suppressed by powers of the ratio ϕ/M_U.

The fermion mass matrix for charged leptons predicted in this theory is:

$$m_\ell = \begin{pmatrix} c_{11}\tilde{\epsilon}^{2|a+b|} & c_{12}\tilde{\epsilon}^{|a|} & c_{13}\tilde{\epsilon}^{|a+b|} \\ c_{21}\tilde{\epsilon}^{|a|} & c_{22}\tilde{\epsilon}^{2|b|} & c_{23}\tilde{\epsilon}^{|b|} \\ c_{31}\tilde{\epsilon}^{|a+b|} & c_{32}\tilde{\epsilon}^{|b|} & 1 \end{pmatrix} m_\tau \quad (2.2)$$

where the parameter $\tilde{\epsilon}$ is some power of the singlet vev scaled by the unification mass, while a, b are certain combinations of the lepton and quark $U(1)_f$-charges. The parameters c_{ij} in front of the various entries (not calculable in this simple model) are assumed to reproduce the fermion mass relations after renormalization group running. These parameters are usually left unspecified, here however their exact values are necessary for a reliable calculation of the lepton violating processes. A successful lepton mass hierarchy in this case is obtained for the choice $a = 3, b = 1$ and $\tilde{\epsilon} = 0.23$. In this case, a possible choice is given by $c_{12} = c_{21} = 0.4, c_{22} = 2.2$, with the rest of the coefficients being unity.

The scalar mass matrices of this model are also affected by the $U(1)_f$-symmetry [7]. In particular, for the sleptons we obtain at the GUT scale

$$\tilde{m}^2_{\ell,e_R} \approx \begin{pmatrix} 1 & \tilde{\epsilon}^{|a+6b|} & \tilde{\epsilon}^{|a+b|} \\ \tilde{\epsilon}^{|a+6b|} & 1 & \tilde{\epsilon}^{|b|} \\ \tilde{\epsilon}^{|a+b|} & \tilde{\epsilon}^{|b|} & 1 \end{pmatrix} m^2_{3/2} = m^2_{3/2} I + \Delta \quad (2.3)$$

The Dirac mass matrix in the above model has a similar structure. Due to the simple $U(1)$ structure of the theory, the powers appearing in its entries are the same as the lepton mass matrix, however, the expansion parameter is in general different[8]. Thus, its form is given by

$$m_{\nu_D} \approx \begin{pmatrix} \epsilon^{2|a+b|} & \epsilon^{|a|} & \epsilon^{|a+b|} \\ \epsilon^{|a|} & \epsilon^{2|b|} & \epsilon^{|b|} \\ \epsilon^{|a+b|} & \epsilon^{|b|} & 1 \end{pmatrix} m_{top} \quad (2.4)$$

The choice of charges a=3, b=1 allows to identify the Dirac mass matrix with the up-quark mass matrix. A choice of coefficients leading to correct up-quark masses is obtained for $c_{12} = c_{21} = .5, c_{32} = c_{23} = 1.5$, with the rest of the coefficients being unity and $\epsilon = .053$.

3. Sneutrino mass matrix

The s-neutrino mass matrix is a 12×12 matrix, its structure is given in terms of the 3×3 Dirac, Majorana and slepton mass matrices. In the absence of scalar mixing effects at the unification scale where any other source of flavour violation is rather irrelevant in the calculation of the branching ratios, it is generally expected that –as in the case of charged sleptons– the Dirac term induces considerable mixing effects. We will show here that this is *not* the case in the sneutrino mass matrix.

This 12×12 matrix is rather complicated and not easy to handle. Vastly different scales are involved and numerical investigations should be carried out with great care. Its form is as follows:

$$\begin{array}{c|cc|cc} & \tilde{\nu} & \tilde{\nu}^* & \tilde{N}^c & \tilde{N}^{c^*} \\ \hline \tilde{\nu}^* & & & & \\ & M^2_{\tilde{\nu}LL} & & M^2_{\tilde{\nu}LR} & \\ \tilde{\nu} & & & & \\ \hline \tilde{N}^{c^*} & & & & \\ & M^2_{\tilde{\nu}RL} & & M^2_{\tilde{\nu}RR} & \\ \tilde{N}^c & & & & \end{array} \tag{3.1}$$

Where all entries are 6×6 matrices given by:

$$M^2_{\tilde{\nu}LL} = \begin{pmatrix} m^2_{\tilde{l}} + m^*_D m^T_D & 0 \\ 0 & m^2_{\tilde{l}} + m_D m^+_D \end{pmatrix}$$

$$M^2_{\tilde{\nu}LR} = \begin{pmatrix} m^*_D M^T & (A^*_\nu + \mu cot\beta)\, m^*_D \\ (A_\nu + \mu cot\beta)\, m_D & m_D M^+ \end{pmatrix}$$

$$M^2_{\tilde{\nu}RL} = \begin{pmatrix} M^* m^T_D & m^+_D\,(A^*_\nu + \mu cot\beta) \\ m^T_D\,(A_\nu + \mu cot\beta) & M m^+_D \end{pmatrix}$$

$$M^2_{\tilde{\nu}RR} = \begin{pmatrix} m^2_N + M^* M^T & A_N{}^* M^* \\ +m^*_D m^+_D & \\ A_N M & m^2_N + M M^+ \\ & +m_D m^+_D \end{pmatrix}$$

One can construct an effective 6×6 matrix for the light sector, by applying matrix perturbation theory, similar to the see-saw mechanism. The result up to second order has the form:

$$\begin{pmatrix} (M^2_{\tilde{\nu}eff})_{LL} & (M^2_{\tilde{\nu}eff})_{LR} \\ (M^2_{\tilde{\nu}eff})_{RL} & (M^2_{\tilde{\nu}eff})_{RR} \end{pmatrix} \tag{3.2}$$

Where all entries are 3×3 matrices given by:

$$\begin{aligned} (M^2_{\tilde{\nu}eff})_{LL} &= m^2_{\tilde{\ell}} - (A_\nu + \mu\cot\beta)\cdot \\ &\qquad (A_\nu - 2A_N)(m_D M^{-2} m^\dagger_D) \\ (M^2_{\tilde{\nu}eff})_{LR} &= ((2A_\nu + A_N) + 2\mu\cot\beta)\cdot \\ &\qquad (m_D M^{-1} m^\dagger_D) \\ (M^2_{\tilde{\nu}eff})_{RL} &= ((2A_\nu + A_N) + 2\mu\cot\beta)\cdot \\ &\qquad (m_D M^{-1} m^\dagger_D)^* \\ (M^2_{\tilde{\nu}eff})_{LL} &= m^2_{\tilde{\ell}} - (A_\nu + \mu\cot\beta)\cdot \\ &\qquad (A_\nu - 2A_N)(m_D M^{-2} m^\dagger_D) \end{aligned}$$

The first and second order terms are obtained asuming all parameters as real and the A-matrices proportional to the identity. Notice that the second order term in the LL and RR pieces can be neglected. However the first order in the LR and RR pieces must be retained, since they lead to complete mixing of the pairwise degenerate states. This however, does not affect the flavor-violating branching ratio.

The simplicity of this result is rather astonishing. Moreover, there is an additional benefit, since the complication of the initial 12×12 mass matrix can now be avoided. A direct numerical calculation of mass eigenstates and mixing angles would be a hard task, due to the vastly different scales.

4. Scalar mass matrices

If we consider common scalar masses and trilinear terms at the GUT scale, leptons and sleptons will be diagonal in the same superfield basis. However, due to the presence of (a) the non diagonal GUT terms Δ at the GUT scale, and (b) the appearance of λ_D in the RG equations, the lepton Yukawa matrix and the slepton mass matrix can not be brought simultaneously to a diagonal form at the scale of the heavy Majorana masses. Therefore, lepton number will be violated by the one loop diagrams of Fig. 1.

We define the unitary matrices diagonalizing the Yukawa mass textures λ_D and λ_e, as follows

$$\lambda^\delta_D = T^T_R \lambda_D T_L \tag{4.1}$$

$$\lambda^\delta_e = V^T_R \lambda_e V_L \tag{4.2}$$

Here, the index δ indicates a diagonal form. Then, the mixing matrix K in the lepton sector, defined in analogy to V_{CKM} is given by the product

$$K = T_L^\dagger V_L \tag{4.3}$$

The charged slepton masses are obtained by numerical diagonalization of the 6×6 matrix

$$\tilde{m}_e^2 = \begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{RL}^2 & m_{RR}^2 \end{pmatrix} \tag{4.4}$$

where all entries are 3×3 matrices in the flavour space. In the superfield basis where λ_e is diagonal, it is convenient for later use to write the 3×3 entries of (4.4) in the form:

$$\begin{aligned} m_{LL}^2 &= (m_{\tilde{l}}^\delta)^2 + \delta m_N^2 + \Delta_L + m_l^2 \\ &\quad + M_Z^2(\frac{1}{2} - sin^2\theta_W)cos2\beta \quad (4.5) \\ m_{RL}^2 &= (A_e^\delta + \delta A_e + \mu tan\beta)m_l \quad (4.6) \\ m_{LR}^2 &= m_{RL}^{2\dagger} \quad (4.7) \end{aligned}$$

Each component above has a different origin and gives an independent contribution in the Branching Ratios.

We further wish to emphasize the following:

- $(m_{\tilde{l}}^\delta)^2, (m_{\tilde{e_R}}^\delta)^2, A_e^\delta$ denote the scalar diagonal contribution of the corresponding matrices; their entries are obtained by numerical integration of the RG. We consider $m_{3/2}^2$ as the common initial condition for the masses at the GUT scale, while the trilinear terms scale as $am_{3/2}$. Since in the RGEs we consider only third generation Yukawa couplings and common initial conditions at the GUT scale for the soft masses, our treatment is equivalent to working in superfield basis, such that: (i) λ_D is diagonal from the GUT scale to the intermediate scale and (ii) λ_e is diagonal from the intermediate scale to low energies. The change of bases will produce a shift in the diagonal elements of the soft mass matrices at the GUT and at the intermediate scale. This effect is negligible (less than one percent).

- δm_N^2 and δA_l stand for the off-diagonal terms which appear due to the fact that λ_D and λ_e may not be diagonalised simultaneously.

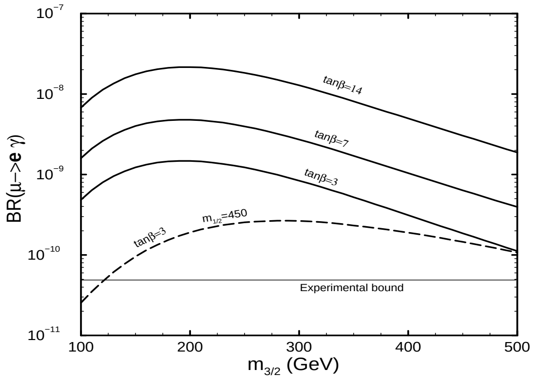

Figure 2: $BR(\mu \to e\gamma)$ for a range of values of $\tan\beta$ (labeled above). Non universal soft masses at the GUT scale are considered ($\Delta \neq 0$). Solid lines are obtained using, $m_{1/2} = 300$ GeV, and $A_0 = -1.5m_{3/2}$ as input parameters.

The intermediate scale that enters in the calculation (which is the mass scale for the neutral Majorana field M_N) is defined by demanding that neutrino masses $\approx 1eV$ are generated via the "see-saw" mechanism. This sets the M_N scale to be around the value 10^{13} GeV. Then, the following values are obtained:

$$\begin{aligned} \delta m_N^2 &= K^\dagger \left[m_{\tilde{l}}^2(m_N)\right] K|_{nondiag.} \quad (4.8) \\ \delta A_l &= V_L A_l(m_N) V_L^\dagger|_{nondiag.} \quad (4.9) \end{aligned}$$

- The following values for Δ_L and Δ_R are defined at the GUT scale:

$$\begin{aligned} \Delta_L &= V_L^\dagger \Delta V_L \quad (4.10) \\ \Delta_R &= V_R^\dagger \Delta V_R \quad (4.11) \end{aligned}$$

The effective 3×3 sneutrino mass matrix squared has the same form as the m_{LL}^2 part of the 6×6 charged slepton one, with the difference that now Dirac masses are absent (in consistency with what we have shown in the analysis of the 12×12 sneutrino matrix). Thus,

$$\tilde{m}_\nu^2 = (m_{\tilde{l}}^\delta)^2 + \delta m_N^2 + \Delta_L + \frac{1}{2}M_Z^2 cos2\beta \tag{4.12}$$

5. Results and Conclusions

The branching ratio formulae for the $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays involve the masses of most of the supersymmetric particles. It is important therefore for any given set of GUT parameters to know precisely all masses and the other low energy parameters. In the present work, this is obtained by numerical integration of the renormalization group equations of the MSSM with right handed neutrinos. We evaluate the coupling constants, using renormalisation-group equations at two loops. Threshold effects are also taken into account, by decoupling every sparticle at the scale of its running mass $Q = m_i(Q)$. Below the scale m_t, we use the SM beta functions..

Our analysis uses as input values the unified coupling constant α_G, at the GUT scale, these are determined at the meeting point of the tree couplings. The third generation Yukawa couplings λ_t, $\lambda_{b-\tau}$, determined by using the experimental values of m_t and m_τ respectively. The (effective) Higgs bilinear coupling μ (up to its sign) is determined by using the radiative electroweak breaking condition, and the ratio of the Higgs vev's described by $tan\beta$ and the flavour-symmetric soft-breaking parameter A_0. We then explore the values of the BR for all significant values of the input parameters $m_{1/2}, m_{3/2}$, A_0 and $tan\beta$ and sign of μ.

If we consider common scalar masses and trilinear terms at the GUT scale, leptons and sleptons will be diagonal in the same superfield basis. However, due to the presence of (a) the non diagonal GUT terms Δ at the GUT scale, and (b) the appearance of λ_D in the RG equations, the lepton Yukawa matrix and the slepton mass matrix can not be brought simultaneously to a diagonal form at the scale of the heavy Majorana masses.

In presenting our results we must consider that the mixing terms introduced in the the scalar matrices by the $U(1)_f$ symmetry (Δ in eq.2.3) are one order of magnitude higher than the ones due to the presence of right-handed neutrinos in the theory. We consider separately two distinct cases:

- a) Case with scalar mass mixing at the GUT scale, $\Delta \neq 0$. Terms $\Delta_{L,R}$, are independet of $\tan\beta$ and $m_{1/2}$, since they are much bigger than δm_N^2 and δA_e, the effects of due to the presence of right-handed neutrinos in the theory will be erased in this case. Fig.[2] shows that parameter space is very resticted in this kind of models, predictions for $\mu \to e\gamma$ are inside the experimental limits for low hight values of the gaugino masses combined with low values of $\tan\beta$ and scalar masses (dashed line). An alternative choice of the $U(1)_f$ charges can lead to a succesful prediction for the lepton masses. In ref. [7] the parameter b of eq. 2.2 is set to 1/2, however this choice of $U(1)_f$ charges increase the size of the mixings in the scalar matrices leading to a $BR(\mu \to e\gamma)$ one order of magnitude bigger than the results presented here.

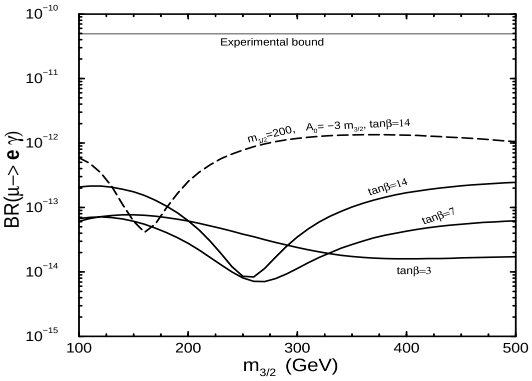

Figure 3: $BR(\mu \to e\gamma)$ for a range of values of $\tan\beta$ (labeled above). Universal soft masses at the GUT scale are considered ($\Delta = 0$). Solid lines are obtained using $m_{1/2} = 300$ GeV and $A_0 = -1.5 m_{3/2}$ as input parameters.

- b Case without scalar mass mixing at the GUT scale, $\Delta = 0$. We consider the case where the scalar mass matrices are protected from mixing effects by some kind of symmetry not affecting the fermion mass sector. Our results can be summarized in Fig.[3]. Branching ratios increases as $\tan\beta$ and A_0 increase and decrease with $m_{1/2}$. The sign of A_0 has little influence on the final result. In the dashed line we consider the set of input parameters leading to higher values for the branching ratio, the obtained values are two orders of magnitude below the current experimental bounds.

We can summarize our results: A simple $U(1)$ symetry can explain the hierarchy of the fermion masses to a good approximation. However when this symmetry is implemented to the scalar sector the amount of LFV introduced by the theory exceeds to experimental limits for most of the parameter space.

Assuming an additional symmetry, such that lepton scalar masses remain universal at the GUT scale, LFV arises from the presence of right-handed neutrino masses in the theory. The full effects of the 12×12 sneutrino matrix has been taken into account. In this case the calculated branching ratios are below the experimental limits, but still interesting for a future improvement in the experimental bounds.

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