Fermion Masses and Mixing resulting from Unimodular Complex Mass Matrices

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ABSTRACT: We discuss a simple ansatz for fermion masses based on unimodular complex mass matrices. This hypothesis can reproduce fermionic masses and mixing with success both in the quark and leptonic sector. Full calculability for the mixing matrix, i.e., the possibility of expressing the mixing matrices in terms of fermion mass ratios, can be achieved in both sectors with realistic results and high predictive power. In the leptonic sector we suggested an ansatz leading to quasi degenerate neutrino masses in agreement with the present experimental evidence.

1. Introduction

The standard $SU(3)\times SU(2)\times U(1)$ model (SM) accommodates the observed masses and mixing of the quark sector without predicting them. This is due to the fact that gauge invariance does not constrain the flavour structure of Yukawa couplings.

Some time ago it was suggested [1] that the quark mass matrices might be of the form

$$M_u = c_u \left[e^{i\theta^u_{ij}} \right]; \quad M_d = c_d \left[e^{i\theta^d_{ij}} \right] \tag{1.1}$$

where c_u and c_d are real numbers. In the framework of the standard Higgs mechanism these mass matrices are based on the hypothesis of universal strength for Yukawa couplings (USY). Taking into account the observed quark mass spectrum a minimum of two Higgs doublets is required with up and down quarks coupling to a different doublet, respectively ϕ_u and ϕ_d. In this case one would have

$$c_u = |g_Y v_u| \;\; and \;\; c_d = |g_Y v_d| \tag{1.2}$$

where $|g_Y|$ is the universal strength of Yukawa couplings and v_u, v_d stand for the vacuum expectation values of the neutral components of each doublet. Hence within the USY hypothesis the flavour dependence is fully contained in the phases. It has been shown [1] that for two generations the USY hypothesis does not imply any constraints since both the quark masses and the Cabibbo angle remain arbitrary whilst for three generations the situation is quite different since once the quark mass spectrum is fixed there are restrictions in the charged current mixing. Yet it is possible to find a range of parameters fitting the experimental values of quark masses and mixing [1, 2]. The success of the USY hypothesis in the quark sector prompted the analysis of the question of calculability of the CKM matrix elements in terms of quark mass ratios [3, 4]. In fact the matrices in eq. 1.1 still contain a large number of free parameters. A judicious choice of phases allows for the full prediction of the CKM matrix in agreement with the experimental results.

In the leptonic sector there is the possibility that both both charged leptons and neutrinos have mass. Recent experimental evidence points towards massive neutrinos with specific forms being allowed for the leptonic mixing matrix. If confirmed this is a clear sign of physics beyond the SM. The question of whether or not ansätze of the type of eq. 1.1 would lead to realistic re-

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sults in the leptonic sector was addressed [5, 6] with very interesting results.

This paper is organized as follows. In the next section we present in some detail the most important results obtained for the quark sector in references [3, 4]. In section 3 we analyse the leptonic sector in the light of simple USY type ansätze leading to quasi degenerate neutrinos [5, 6]. Our conclusions are presented in section 4.

The simplicity of the USY type ansätze suggests the existence of an underlying symmetry principle, still to be found, which together with the richness of the ansätze seems to indicate that this approach may provide the basis for a future deeper insight into the pattern of fermion masses and mixing.

2. USY in the quark sector [3, 4]

Not all phases appearing in M_u and M_d in eq. 1.1 have physical meaning since some of them can be eliminated by means of a weak-basis (WB) transformation of the the type

$$\begin{array}{l} M_u \longrightarrow M'_u = K_L^\dagger\ M_u\ K_R^u \\ M_d \longrightarrow M'_d = K_L^\dagger\ M_d\ K_R^d \end{array} \tag{2.1}$$

where K_L, K_R^u and K_R^d are diagonal and unitary matrices, i.e., of the form diag $(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$. As a result one can simplify M_u, M_d into

$$\begin{array}{l} M_u = c_u \begin{bmatrix} e^{ip_u} & e^{ir_u} & 1 \\ e^{iq_u} & 1 & e^{it_u} \\ 1 & 1 & 1 \end{bmatrix} \\ M_d = c_d\ K^\dagger \begin{bmatrix} e^{ip_d} & e^{ir_d} & 1 \\ e^{iq_d} & 1 & e^{it_d} \\ 1 & 1 & 1 \end{bmatrix} K \end{array} \tag{2.2}$$

where $K = \text{diag}\ (1, e^{i\alpha_1}, e^{i\alpha_2})$. It is clear from eq. 2.2 that the phases α_1, α_2 can only influence the CKM matrix whilst the phases $p_{u,d}$; $q_{u,d}$; $r_{u,d}$; $t_{u,d}$ will affect both the quark masses and the CKM matrix.

In the USY framework all solutions with $m_u = m_d = 0$ can be classified into two classes which correspond to simple choices of the parameters {p, q, r, t}:

$$\begin{array}{ll} \text{Class I} \left\{ \begin{array}{lll} \text{a)}\ p = 0,\ t = q & & \text{q, r free} \\ \text{b)}\ r = 0,\ t = 0 & & \text{p, q free} \\ \text{c)}\ r = p,\ q = 0 & & \text{p, t free} \end{array} \right. \\ \text{Class II} \left\{ \begin{array}{lll} \text{a)}\ q = 0, & t = 0 & \text{p, r free} \\ \text{b)}\ p = 0, & r = 0 & \text{q, t free} \\ \text{c)}\ p = q + r, & t = -r & \text{p, r free} \end{array} \right. \end{array} \tag{2.3}$$

Any two solutions within the same class can be transformed into one another through the multiplication by a diagonal unitary matrix combined with permutations. In this limit, i.e., the chiral limit, and for $K = \mathbf{1}$ in eq. 2.2, solutions of class II cannot generate a realistic CKM matrix while solutions of class I can. It is possible to obtain full calculability of the CKM, i.e., to have the CKM matrix entirely expressed in terms of quark mass ratios with no free parameters, and at the same time obtain realistic quark masses by breaking the chiral symmetry by means of a small perturbation, still within USY, in the neighbourhood of interesting chiral solutions. This procedure motivates the construction of ansätze with only three independent parameters for each of the mass matrices M_u and M_d. Without relaxing the condition of full calculability only one realistic solution with M_u and M_d of the same form was obtained:

$$M_{u,d} = c_{u,d} \begin{bmatrix} 1 & e^{ir} & 1 \\ e^{iq} & 1 & e^{i(q-r)} \\ 1 & 1 & 1 \end{bmatrix}_{u,d} \tag{2.4}$$

corresponding to $p = 0$, $t = q - r$, $\alpha_1 = \alpha_2 = 0$, with the parameters fixed by

$$\begin{array}{l} c^2 = \frac{1}{9}(m_1^2 + m_2^2 + m_3^2) \\ \sin^2\left(\frac{r}{2}\right) = \frac{3}{4}\sqrt{3\delta}, \quad \delta = \det MM^\dagger \frac{1}{3c^2} \\ \sin^2\left(\frac{q}{2}\right) = \frac{9\chi - 18\sqrt{3\delta}}{16 - 12\sqrt{3\delta}} \\ \chi = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 \\ \lambda_i = 3m_i^2/(m_1^2 + m_2^2 + m_3^2) \end{array} \tag{2.5}$$

In this case the CKM matrix is correctly predicted without free parameters. Starting with Input:
$m_t^{\text{physical}} = 174$ Gev,
$m_u(1\ \text{Gev}) = 1.0$ Mev,
$m_c(1\ \text{Gev}) = 1.35$ Gev,
$m_d(1\ \text{Gev}) = 6.5$ Mev,

$m_s(1 \text{ Gev}) = 165$ Mev,
$m_b(1 \text{ Gev}) = 5.4$ Gev
the resulting CKM matrix is:
Output:

$$|V| = \begin{bmatrix} 0.9752 \; 0.2213 \; 0.0032 \\ 0.2210 \; 0.9745 \; 0.0391 \\ 0.0117 \; 0.0375 \; 0.9992 \end{bmatrix} \quad (2.6)$$

Other realistic examples can be obtained provided that the condition of full calculability is relaxed by allowing for K different from **1** or, alternatively, chosing different ansätze for M_u and M_d still within calculability (for more examples see [3, 4]). In fact, there is no fundamental reason for choosing the same ansatz for M_u and M_d, various of the viable Yukawa textures classified in ref [7] correspond to taking different forms for M_u and M_d.

A renormalization group analysis of the CKM matrix was performed. Starting from quark masses at a unification scale of 10^{16} Gev the corresponding CKM matrix was computed for the ansatz given here explicitly. This CKM matrix was then evaluated at the scale of 1 Gev and compared to the CKM matrix computed directly from 1 Gev quark masses.

3. USY type matrices in the leptonic sector [5, 6].

3.1 Introduction

In the recent past several different experiments measuring solar [8] and neutrino fluxes [9] provided evidence pointing towards neutrino oscillations thus implying nonzero neutrino masses and leptonic mixing. The most recent results of the Superkamiokande (SK) collaboration [10], [11] strengthen the possibility of nearly maximal mixing angle for atmospheric neutrino oscillations with the experimental parameters within the range [12] $\Delta m^2_{atm} = (1.5 - 8) \times 10^{-3}\; eV^2$, $\sin^2(2\theta_{atm}) > 0.8$. In the absence of sterile neutrinos the dominant mode is $\nu_\mu \longleftrightarrow \nu_\tau$ oscillations while the sub-leading mode $\nu_\mu \longleftrightarrow \nu_e$ is severely restricted by the SK and CHOOZ [13] data which require $V_{31} \leq 0.2$ for the range given above, with V defined in the next subsection.

The interpretation of the present solar neutrino data leads to oscillations of the electron neutrino into some other neutrino species, with three different ranges of parameters still allowed. In the framework of the MSW mechanism [14] there are two sets of solutions, the adiabatic branch (AMSW) requiring a large mixing, ($\Delta m^2_{sol} = (2{-}20) \times 10^{-5}\; eV^2$, $\sin^2(2\theta_{sol}) = 0.65{-}0.95$) [11] [15], and the non-adiabatic branch (NAMSW) requiring small mixing ($\Delta m^2_{sol} = 5.4 \times 10^{-6}\; eV^2$, $\sin^2(2\theta_{sol}) \sim 6.0 \times 10^{-3}$, for the best fit) [15]. In the framework of vacuum oscillations again large mixing is required ($\Delta m^2_{sol} = 8.0 \times 10^{-11}\; eV^2$, $\sin^2(2\theta_{sol}) = 0.75$, for the best fit) [15].

In our work we did not take into consideration the results of the LSND collaboration [16] which have not yet been confirmed by other experiments and the KARMEN data [17] already excludes a sizable region of the allowed parameter space.

Astrophysical considerations, in particular the possibility that neutrinos constitute the hot dark matter, favour neutrino masses of the order of a few eV [18], which combined with the above constraints leads to a set of highly degenerate neutrinos. In our work we considered three neutrino families without additional sterile neutrinos.

3.2 Choice of ansatz

Within the USY framework, meaning with mass matrices of the form given by eq. 1.1, it can be shown that for matrices M_ν having at least two degenerate eigenvalues, there is a weak-basis where M_ν has one of the following forms (modulo trivial permutations)

$$\begin{aligned} M_\nu^{\mathrm{I}} &= c_\nu\; K \cdot \begin{bmatrix} e^{i\alpha} & 1 & 1 \\ 1 & e^{i\alpha} & 1 \\ 1 & 1 & e^{i\alpha} \end{bmatrix} ; \\ M_\nu^{\mathrm{II}} &= c_\nu\; K \cdot \begin{bmatrix} e^{i\alpha} & 1 & 1 \\ 1 & e^{i\alpha} & 1 \\ 1 & 1 & e^{-2i\alpha} \end{bmatrix} ; \\ M_\nu^{\mathrm{III}} &= c_\nu\; K \cdot \begin{bmatrix} e^{i\alpha} & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \end{aligned} \quad (3.1)$$

where K is a diagonal unitary matrix.

The first two cases given in eq. 3.1 have the special property that for $\alpha = 2\pi/3$ they lead to three degenerate neutrino masses, rather than only two.

Inspired by these observations we suggested the following ansatz within USY:

$$M_\ell = c_\ell \begin{bmatrix} e^{-ia} & 1 & 1 \\ 1 & e^{ia} & 1 \\ 1 & 1 & e^{ib} \end{bmatrix}; \qquad M_\nu = c_\nu \begin{bmatrix} e^{i\alpha} & 1 & 1 \\ 1 & e^{i\alpha} & 1 \\ 1 & 1 & e^{i\beta} \end{bmatrix} \tag{3.2}$$

the resulting lepton mixing matrix is given by

$$V = U_\nu^\dagger \cdot U_\ell \tag{3.3}$$

with U_ℓ the matrix which diagonalizes $M_\ell \ M_\ell^\dagger$ and U_ν the matrix which diagonalizes $M_\nu \ M_\nu^\dagger$ in the case of Dirac neutrinos. [1]

The parameters a and b in M_ℓ can be expressed in terms of mass ratios of charged leptons. Due to the observed strong hierarchy of masses in this sector these parameters will be close to zero. The limit of a and b equal to zero corresponds to the well known democratic limit [20] where all the entries of the mass matrix are equal to one. In leading order one obtains:

$$a = 3\sqrt{3}\ \frac{\sqrt{m_e m_\mu}}{m_\tau}; \quad b = \frac{9}{2}\ \frac{m_\mu}{m_\tau} \tag{3.4}$$

In the neutrino sector α and β are also determined by mass ratios. In order to obtain highly degenerate neutrino masses α and β should be close to $2\pi/3$, the limit of exact degeneracy. This justifies the introduction of two small parameters ϵ_{32} and ϵ_{21}, defined by:

$$\alpha = \frac{2\pi}{3} - \epsilon_{32} - \epsilon_{21}; \quad \beta = \frac{2\pi}{3} + 2\epsilon_{32} \tag{3.5}$$

and, in leading order, one has

$$\epsilon_{32} = \frac{1}{\sqrt{3}} \frac{\Delta m^2_{32}}{m_3^2}; \quad \epsilon_{21} = \frac{3\sqrt{3}}{4} \frac{\Delta m^2_{21}}{m_3^2} \tag{3.6}$$

where

$$\Delta m^2_{ji} = |m_j^2 - m_i^2| \tag{3.7}$$

[1] If both Dirac mass terms (M_D) and Majorana mass terms for the righthanded neutrinos (M_R) are present, with the light Majorana neutrinos acquiring mass through the seesaw mechanism [19], we identify M_ν of eq. 3.2 with the effective mass matrix for the light neutrinos given by $M_\nu = M_D M_R^{-1} M_D^T$ and, in this case, U_ν is very approximately the matrix verifying $M_\nu = U_\nu \cdot \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \cdot U_\nu^T$

The resulting mixing matrix V is thus completely fixed by charged lepton mass ratios and neutrino mass ratios and can be computed exactly. In leading order V is given by:

$$|V| = \begin{bmatrix} 1 & \sqrt{\frac{m_e}{m_\mu}} & \frac{\sqrt{2 m_e m_\mu}}{m_\tau} \\ \sqrt{\frac{m_e}{3 m_\mu}} & \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} \\ \sqrt{\frac{2 m_e}{3 m_\mu}} & \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}. \tag{3.8}$$

The dependence on neutrino mass ratios does not appear in leading order.

3.3 Confronting the data

In the context of three left-handed neutrinos and neglecting CP violation the probability for a neutrino ν_α to oscillate into other neutrinos is given by:

$$\begin{aligned} 1 - \mathrm{P}(\nu_\alpha \to \nu_\alpha) \ &= \\ = 4 \textstyle\sum_{i<j} \ |V_{\alpha i}|^2 \ |V_{\alpha j}|^2 \ \sin^2 \left[\frac{\Delta m^2_{ij}}{4} \ \frac{L}{E}\right] \end{aligned} \tag{3.9}$$

where E is the neutrino energy, and L denotes the distance travelled between the source and the detector.

The translation of the experimental bounds, which are given in terms of only two flavour mixing, into the three flavour mixing is simple, since for V_{31} sufficiently small and also for $\Delta m^2_{32} >> \Delta m^2_{21}$, we may safely identify

$$\sin^2 2\theta_{\rm atm} = 4(V_{23}V_{23}V_{21}V_{21} + V_{22}V_{22}V_{23}V_{23}) \tag{3.10}$$

$$\sin^2 2\theta_{\rm sol} = 4V_{11}V_{11}V_{12}V_{12} \tag{3.11}$$

With the ansatz chosen in the previous section, fixing the parameters (a, b, ϵ_{32},ϵ_{21}) through the input of the leptonic masses, we can fully predict the leptonic mixing matrix in agreement with the small mixing solution for the solar neutrino deficit (NAMSW).

For the charged leptons we fixed

$$m_e = 0.511\ MeV, \quad m_\mu = 105.7\ MeV, \quad m_\tau = 1777\ MeV \tag{3.12}$$

corresponding to the phases $a = 0.0214$ and $b = 0.2662$. For the neutrino sector we chose

$$m_{\nu_3} = 2\ eV, \quad \Delta m^2_{21} = 9.2 \times 10^{-6}\ eV^2, \quad \Delta m^2_{32} = 5.0 \times 10^{-3}\ eV^2 \tag{3.13}$$

corresponding to $\epsilon_{21} = 3 \times 10^{-6}$ and $\epsilon_{32} = 7.2 \times 10^{-4}$. The resulting mixing matrix V computed beyond leading order is given by

$$|V| = \begin{bmatrix} 0.9976 & 0.0692 & 0.0058 \\ 0.0463 & 0.6068 & 0.7935 \\ 0.0518 & 0.7918 & 0.6085 \end{bmatrix} \quad (3.14)$$

which translates into

$$\sin^2 2\theta_{\rm atm} = 0.933 \quad (3.15)$$

and

$$\sin^2 2\theta_{\rm sol} = 0.019 \quad (3.16)$$

in agreement with the present experimental data.

The numerical values given here were computed in ref [5] and the input chosen for the neutrino masses does not exactly coincide with the best fit given in subsection 3.1, this is due to the fact that at the time ref[5] was written the experimental values differed slightly from the present ones. Yet, the difference is not meaningfull and in particular does not alter our conclusions.

We have shown afterwards [6] that it is also possible to reproduce the experimental data for the large mixing solar solution with a different USY type ansatz for the leptonic sector, still with full calculability of the mixing matrix.

4. Conclusions

The discussion of fermion masses and mixing throughout this work was done strictly in the context of USY-type mass matrices.

Fermion masses and mixing can be accommodated with success in ansätze based on unimodular complex matrices both in the quark and the leptonic sector with the possibility of achieving full calculability for the respective mixing matrices.

We have an ansatz that leads in a natural way to small mixing involving neutrinos of quasi degenerate masses.

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