Non-perturbative Supersymmetry Breaking and Finite Temperature Instabilities in N=4 Superstrings

Costas Kounnas*

Laboratoire de Physique Théorique, Ecole Normale Supérieure, 24 rue Lhomond, F–75231 Paris Cedex 05, France

E-mail: costas.kounnas@cern.ch

ABSTRACT: We obtain the non-perturbative effective potential for the dual five-dimensional $N = 4$ strings in the context of finite-temperature regarded as a breaking of supersymmetry into four space-time dimensions. Using the properties of gauged $N = 4$ supergravity we derive the universal thermal effective potential describing all possible high-temperature instabilities of the known $N = 4$ superstrings. These strings undergo a high-temperature transition to a new phase in which five-branes condense. This phase is described in detail, using both the effective supergravity and non-critical string theory in six dimensions. In the new phase, supersymmetry is perturbatively restored but broken at the non-perturbative level.

KEYWORDS: String Duality, Supersymmetry Breaking.

1. Introduction

A convenient way to analyse a D-dimensional theory at finite temperature is to identify the temperature with the inverse radius of a compactified Euclidean time on S^1, $R = 1/2\pi T$ and to modify the boundary conditions around the S^1 according to spin-statistics: periodic for bosons, antiperiodic for fermions. The modified boundary conditions shift the S^1 Kaluza-Klein charge by an amount proportional to the helicity of the state, $m \to m + Q$. In string theories this shift is generalised and includes a winding contribution: $m \to m + Q + \delta n/2$. This shift is dictated by the world-sheet modular invariance; $\delta = 1$ for the heterotic string and $\delta = 0$ for the type II strings [1, 2, 3]. Furthermore, the GSO projection in the odd winding number sector is reversed.

For an even winding number n, the thermal modification can be regarded as a shift of m and Q compatible with the (supersymmetric) GSO projection. As a consequence, the spectrum in even n sectors is not different in the thermal and supersymmetric cases, the mass formula for the (lightest) BPS fermions, gauge bosons and scalars with even windings n remains $\mathcal{M}^2 = P^2$, with m modified, and tachyonic states are not present. The situation is not the same for states with odd winding number n due to the reversion of the GSO projection. It follows that the only states that can become tachyonic are those with $n = \pm 1$ and correspond to $(D-1)$-dimensional scalars coming from the longitudinal components of the D-dimensional metric.

Tachyons cannot appear in a perturbative supersymmetric field theory, which behaves like the zero-winding sector of strings; all (squared) masses are increased by finite temperature corrections, $\mathcal{M}^2 = P^2$, and a thermal instability is never generated by a state becoming tachyonic at high temperature. However, as we will see below, in non-perturbative supersymmetric field theories such an instability can arise from thermal dyonic modes, which behave as the odd winding string states [4]. Indeed, in theories with $N = 4$ supersymmetries, the BPS mass formula is determined by the central extension of the corresponding superalgebra [5]–[7] and dyonic field theory states are mapped to string winding modes [8, 7]. Using heterotic–type II duality, one can ar-

*With I. Antoniadis (Ecole Polytechnique, Palaiseau) and J.-P. Derendinger (Neuchâtel University, Neuchâtel).

gue that the thermal shift of the BPS masses modifies only the perturbative momentum charge m. In both heterotic and type II perturbative strings, the thermal winding number n is not affected by the temperature shifts. Since, in dimensions lower than six, heterotic–type II duality exchanges the winding numbers n of the two theories, and since the winding number of the one theory is the magnetic charge of the other, it is inferred that field theory magnetic numbers are not shifted at finite temperature. This in turn indicates how to modify the BPS mass formula at finite temperature [4].

It turns out that string theories with D-dimensional space-time supersymmetry look at finite temperature as if supersymmetry were spontaneously broken in $D-1$ dimensions [1]–[4].

2. Thermal masses and string-string dualities

The non-perturbative four-dimensional thermal mass formula has been obtained in Ref. [4]. The procedure is to start with the $N=4$ four-dimensional BPS mass formula on a circle with radius R, which depends on an effective string tension

$$\begin{aligned} T_{p,q,r} &= \frac{p}{\alpha'_H} + \frac{q}{\lambda_H^2\alpha'_H} + \frac{rR_6^2}{\lambda_H^2(\alpha'_H)^2} \\ &= \frac{p}{\alpha'_H} + \frac{q}{\alpha'_{IIA}} + \frac{r}{\alpha'_{IIB}}. \end{aligned} \tag{2.1}$$

The modified finite-temperature formula reads then

$$\begin{aligned} \mathcal{M}_T^2 &= \left(\frac{m+Q'+\frac{kp}{2}}{R} + k\, T_{p,q,r}\, R\right)^2 \\ &\quad -2\, T_{p,q,r}\, \delta_{|k|,1}\, \delta_{Q',0}\,, \end{aligned} \tag{2.2}$$

In these expressions, kp is the winding number in the heterotic string representation with intercept scale α'_H, while kq is the magnetic Kaluza-Klein charge and kr is the magnetic winding charge. Still in the heterotic picture, kq is the wrapping number of the heterotic five-brane around $T^4 \times S^1_R$, while kr corresponds to the same wrapping number after performing a T-duality along the circle of the sixth dimension. The shift in the momentum Kaluza-Klein number m

$$m \quad \longrightarrow \quad m + Q' + \frac{kp}{2},$$

is dictated by the change of boundary conditions at finite temperature, compared to a simple circle compactification. The helicity charge Q' distinguishes (four-dimensional) bosons and fermions. The shift $kp/2$ is dictated then by modular invariance of the dual perturbative strings. Finally, the mass formula (2.2) includes a subtraction in the odd k winding sector of the effective $T_{p,q,r}$ string. We refer to Ref. [4] for a detailed discussion.

The mass formula (2.2) depends on three parameters: the six-dimensional heterotic string coupling λ_H, the circle compactification R_6 from six to five dimensions, and the radius R which will be identified with the inverse temperature. It also depends on a scale: the duality invariant scale is the (four-dimensional) Planck scale $\kappa = \sqrt{8\pi}M_P^{-1} = (2.4\times 10^{18}\text{ GeV})^{-1}$. It is convenient to introduce instead of λ_H, R_6 and R the (dimensionless) variables

$$t = \frac{RR_6}{\alpha'_H}, \qquad u = \frac{R}{R_6}, \qquad s = g_H^{-2} = \frac{t}{\lambda_H^2}, \tag{2.3}$$

which will be directly related to moduli of the effective supergravity theory; g_H is now the *four*-dimensional heterotic string coupling. The various α' scales in the effective tension (2.1) are

$$\alpha'_H = 2\kappa^2 s, \quad \alpha'_{IIA} = 2\kappa^2 t, \quad \alpha'_{IIB} = 2\kappa^2 u, \tag{2.4}$$

when expressed in Planck units. In addition, string–string dualities have a simple formulation using these variables. Before the temperature shift on m, the BPS mass formula is invariant under the exchanges $s \leftrightarrow t$, $s \leftrightarrow u$ and $t \leftrightarrow u$. These operations correspond respectively to heterotic–IIA, heterotic–IIB and IIA–IIB dualities in the undeformed (by temperature) $N=4$ supersymmetric theories. In terms of s, t and u, the temperature radius R is given by

$$R^2 = \alpha'_H tu = 2\kappa^2 stu \tag{2.5}$$

and R is by construction identical in all three string theories.

As a consequence of the BPS conditions and the $s \leftrightarrow t \leftrightarrow u$ duality symmetry in the undeformed supersymmetric theory, the integers p, q, r are non-negative and relatively prime. Furthermore, $mk \geq -1$ because of the inversion of the GSO projection in the theory deformed by temperature. Using these constraints, it is straightforward to show that in general there are two potential tachyonic series with $m = -1$ and $p = 1, 2$:

$$\begin{aligned} &p = 1, \quad \forall (q, r) \text{ relat. primes :} \\ &R = \left(\frac{\sqrt{2} \pm 1}{\sqrt{2}}\right) \frac{1}{\sqrt{T_{1,q,r}}}, \\ &p = 2, \quad \forall (p, q, r) \text{ relat. primes :} \\ &R = \sqrt{\frac{2}{T_{2,q,r}}} \end{aligned} \tag{2.6}$$

One of the perturbative heterotic, type IIA or type IIB potential tachyons corresponds to a critical temperature that is always lower than the above two series. The perturbative Hagedorn temperatures are:

$$\text{heterotic tachyon :} \quad m = \mp 1, kp = \pm 1, Q' = 0$$

$$2\pi T = \left(\sqrt{2} - 1\right) \sqrt{\frac{2}{\alpha'_H}};$$

$$\text{type IIA tachyon :} \quad m = 0, kq = \pm 1, Q' = 0$$

$$2\pi T = \frac{1}{\sqrt{2\alpha'_{IIA}}};$$

$$\text{type IIB tachyon :} \quad m = 0, kr = \pm 1, Q' = 0$$

$$2\pi T = \frac{1}{\sqrt{2\alpha'_{IIB}}},$$

and $T = (2\pi R)^{-1}$.

This discussion shows that the temperature modification of the mass formula inferred from perturbative strings and applied to the non-perturbative BPS mass formula produces the appropriate instabilities in terms of Hagedorn temperature. We will now proceed to show that it is possible to go beyond the simple enumeration of Hagedorn temperatures. We will construct an effective supergravity Lagrangian that allows a study of the nature of the non-perturbative instabilities and the dynamics of the various thermal phases.

The above formula hold for supersymmetry broken by temperature effects in Euclidean space. They would similarly hold for a non-supersymmetric four-dimensional Minkowski theory in which supersymmetry would be broken by a particular Scherk-Schwarz compactification of the fifth dimension.

3. Effective supergravity in N=1 representation

In the previous section, we have studied the appearance of tachyonic states generating thermal instabilities at the level of the mass formula for $N = 4$ BPS states. To obtain information on dynamical aspects of these instabilities, we now construct the full temperature-dependent effective potential for the would-be tachyonic states.

Our procedure to construct the effective theory is as follows. We consider five-dimensional $N = 4$ theories at finite temperature. They can then effectively be described by four-dimensional theories, in which supersymmetry is spontaneously broken by thermal effects. Since we want to limit ourselves to the description of instabilities, it is sufficient to only retain, in the full $N = 4$ spectrum, the potentially massless and tachyonic states. This restriction will lead us to consider only spin 0 and 1/2 states, the graviton and the gravitino[1]. This sub-spectrum is described by an $N = 1$ supergravity with chiral multiplets.

The scalar manifold of a generic, *unbroken*, $N = 4$ theory is [9]–[12]

$$\begin{aligned} &\left(\frac{Sl(2,R)}{U(1)}\right)_S \times G/H, \\ &G/H = \left(\frac{SO(6, r+n)}{SO(6) \times SO(r+n)}\right)_{T_I, \phi_A}. \end{aligned} \tag{3.1}$$

The manifold G/H of the $N = 4$ vector multiplets naturally splits into a part that includes the $6r$ moduli T_I, and a second part which includes the infinite number $n \to \infty$ of BPS states ϕ_A.

In the manifold G/H, we are only interested in keeping the six BPS states $Z^{\pm}_A$, $A = 1, 2, 3$, which, according to our discussion in the previous section, generate thermal instabilities in heterotic, IIA and IIB strings. For consistency, these states must be supplemented by two moduli T and U among the T_I's. We consider heterotic

[1]The four gravitinos remain degenerate at finite temperature; it is then sufficient to retain only one of them.

and type II strings respectively on $T^4 \times S^1_6 \times S^1_5$ and $K_3 \times S^1_6 \times S^1_5$, where S^1_6 is a trivial circle and S^1_5 is the temperature circle. The moduli T and U describe the $T^2 \equiv S^1_5 \times S^2_6$ torus. Thus, $r+n=8$ in the $N=4$ manifold (3.1). To construct the appropriate truncation of the scalar manifold G/H, which only retains the desired states of $N=1$ chiral multiplets, we use a $Z_2 \times Z_2$ subgroup contained in the $SO(6)$ R-symmetry of the coset G/H. This symmetry can be used as the point group of an $N=1$ orbifold compactification, but we will only use it for projecting out non-invariant states of the $N=4$ theory[2] with $r+n=8$.

The $Z_2 \times Z_2$ projection splits $H = SO(6) \times SO(8)$ in $SO(2)^3 \times SO(2) \times SO(3)^2$ and the scalar manifold becomes

$$\left(\frac{Sl(2,R)}{U(1)}\right)_S \times \left(\frac{Sl(2,R)}{U(1)}\right)_T \times \left(\frac{Sl(2,R)}{U(1)}\right)_U \times \left(\frac{SO(2,3)}{SO(2)\times SO(3)}\right)_{Z^+_A} \times \left(\frac{SO(2,3)}{SO(2)\times SO(3)}\right)_{Z^-_A}, \tag{3.2}$$

$A=1,2,3$. The tachyonic instabilities will however be controlled by the diagonal sub-manifold,

$$\left(\frac{Sl(2,R)}{U(1)}\right)_S \times \left(\frac{Sl(2,R)}{U(1)}\right)_T \times \left(\frac{Sl(2,R)}{U(1)}\right)_U \times \left(\frac{SO(2,3)}{SO(2)\times SO(3)}\right)_{Z_A}, \tag{3.3}$$

identifying $Z^+_A = Z^-_A = Z_A$.

¿From the structure of the truncated scalar manifold, we find that the Kähler potential is

$$\begin{aligned} K \;=\; & -\log[(S+S^*)(T+T^*)(U+U^*)] \\ & -2\log[1-2Z_AZ^*_A + (Z_AZ_A)(Z^*_BZ^*_B)]. \end{aligned} \tag{3.4}$$

The superpotential of the theory is obtained using the fact that at the level of $N=4$ supergravity, finite temperature corresponds to a particular Scherk-Schwarz gauging, breaking supersymmetry spontaneously. This gauging is defined by a set of generalized (field-dependent) structure constants, involving the compensating multiplets which are used to define the G/H manifold. The truncation to $N=1$ supergravity delivers then the following expression for the superpotential:

$$\begin{aligned} W \;=\; & \sqrt{2}\big[(1-Z_AZ_A)(1-Z_BZ_B) \\ & +2(TU-1)Z^2_1 + 2SUZ^2_2 + 2STZ^2_3\big]. \end{aligned} \tag{3.5}$$

¿From the Kähler potential and the superpotential we can then compute the full effective scalar potential and study its instabilities. Its complicated expression simplifies drastically in the directions relevant to instabilities. Introducing the variables

$$\begin{gathered} s = \mathrm{Re}\, S, \quad t = \mathrm{Re}\, T, \quad u = \mathrm{Re}\, U, \\ z_A = \mathrm{Re}\, Z_A, \quad x^2 = \textstyle\sum_A z^2_A, \\ H_A = \frac{z_A}{1-x^2}, \qquad A = 1,2,3, \\ \xi_1 = tu, \quad \xi_2 = su, \quad \xi_3 = st, \end{gathered} \tag{3.6}$$

the resulting scalar potential becomes

$$\begin{aligned} V \;=\; & V_1 + V_2 + V_3, \\ \kappa^4 V_1 \;=\; & \frac{4}{s}\Big[(\xi_1+\xi_1^{-1})H^4_1 \\ & + \frac{1}{4}(\xi_1 - 6 + \xi_1^{-1})H^2_1\Big], \\ \kappa^4 V_2 \;=\; & \frac{4}{t}\Big[\xi_2 H^4_2 + \frac{1}{4}(\xi_2-4)H^2_2\Big], \\ \kappa^4 V_3 \;=\; & \frac{4}{u}\Big[\xi_3 H^4_3 + \frac{1}{4}(\xi_3-4)H^2_3\Big]. \end{aligned} \tag{3.7}$$

This expression displays the duality properties

$\xi_1 \;\to\; \xi_1^{-1}$: heterotic temperature duality;

$t \;\leftrightarrow\; u$, $H_2 \;\leftrightarrow\; H_3$: IIA–IIB duality.

3.1 Phase structure of the thermal effective theory

The scalar potential (3.7) derived from our effective supergravity possesses four different phases corresponding to specific regions of the s, t and u moduli space. Their boundaries are defined by critical values of the moduli s, t, and u (or of ξ_i, $i=1,2,3$), or equivalently by critical values of the temperature, the (four-dimensional) string coupling and the compactification radius R_6. These four phases are:

1. The *low-temperature* phase: $T < (\sqrt{2}-1)^{1/2}/(4\pi\kappa)$;

[2] Only untwisted states would contribute to thermal instabilities.

2. The *high-temperature heterotic* phase:
$T > (\sqrt{2}-1)^{1/2}/(4\pi\kappa)$, $g_H^2 < (2+\sqrt{2})/4$;

3. The *high-temperature type IIA* phase:
$T > (\sqrt{2}-1)^{1/2}/(4\pi\kappa)$, $g_H^2 > (2+\sqrt{2})/4$ and $R_6 > \sqrt{\alpha'_H}$;

4. The *high-temperature type IIB* phase:
$T > (\sqrt{2}-1)^{1/2}/(4\pi\kappa)$, $g_H^2 > (2+\sqrt{2})/4$ and $R_6 < \sqrt{\alpha'_H}$.

The distinction between phases 3 and 4 is, however, somewhat academic, since there is no phase boundary at $R_6 = \sqrt{\alpha'_H}$.

3.1.1 Low-temperature phase

This phase, which is *common* to all three strings, is characterized by

$$H_1 = H_2 = H_3 = 0, \qquad V_1 = V_2 = V_3 = 0. \tag{3.8}$$

The potential vanishes for all values of the moduli s, t and u, which are then restricted only by the stability of the phase, namely the absence of tachyons in the mass spectrum of the scalars H_i. This mass spectrum is analysed in Ref. [4]. Stability requires then:

$$\xi_1 > \xi_H = (\sqrt{2}+1)^2, \qquad \xi_2 > 4, \qquad \xi_3 > 4. \tag{3.9}$$

¿From the above conditions, it follows in particular that the temperature must verify

$$T = \frac{1}{2\pi\kappa}\left(\frac{1}{\xi_1\xi_2\xi_3}\right)^{1/4} < \frac{(\sqrt{2}-1)^{1/2}}{4\pi\kappa}. \tag{3.10}$$

Since the (four-dimensional) string couplings are

$$s = \sqrt{2}g_H^{-2}, \qquad t = \sqrt{2}g_A^{-2}, \qquad u = \sqrt{2}g_B^{-2},$$

this phase exists in the perturbative regime of all three strings. The relevant light thermal states are just the massless modes of the five-dimensional $N = 4$ supergravity, with thermal mass scaling like $1/R \sim T$.

Alternatively, if this effective theory is considered as a six-dimensional Minkowski model compactified on $S^1_R \times S^1_{R_6}$, with spontaneously broken supersymmetry, then the lowest S^1_R Kaluza-Klein modes have masses shifted by a quantity proportional to the gravitino mass scale,

$$m_{3/2}^2 = \frac{1}{4R^2},$$

which then controls both the Kaluza-Klein mass shifts and the splitting of supersymmetric multiplets.

3.1.2 High-temperature heterotic phase

This phase is defined by

$$\xi_H > \xi_1 > \frac{1}{\xi_H}, \quad \xi_2 > 4, \quad \xi_3 > 4, \tag{3.11}$$

with $\xi_H = (\sqrt{2}+1)^2$, as in Eq. (3.9). The inequalities on ξ_2 and ξ_3 eliminate type II instabilities. In this region of the moduli, and after minimization with respect to H_1, H_2 and H_3, the potential becomes

$$\kappa^4 V = -\frac{1}{s}\frac{(\xi_1+\xi_1^{-1}-6)^2}{16(\xi_1+\xi_1^{-1})}.$$

It has a stable minimum for fixed s (for fixed α'_H) at the minimum of the self-dual[3] quantity $\xi_1 + \xi_1^{-1}$:

$$\begin{gathered} \xi_1 = 1, \quad H_1 = \tfrac{1}{2}, \quad H_2 = H_3 = 0, \\ \kappa^4 V = -\frac{1}{2s}. \end{gathered} \tag{3.12}$$

The transition from the low-temperature vacuum is due to a condensation of the heterotic thermal winding mode H_1, or equivalently by a condensation of type IIA NS five-brane in the type IIA picture.

At the level of the potential only, this phase exhibits a runaway behaviour in s. We will show in the next section the existence of a stable solution to the effective action with non-trivial metric and/or dilaton.

In heterotic language, s, t and u are particular combinations of the four-dimensional gauge coupling g_H, the temperature $T = (2\pi R)^{-1}$ and the compactification radius from six to five dimensions R_6. The relations are

$$\begin{gathered} s = \sqrt{2}g_H^{-2}, \quad t = \sqrt{2}\frac{RR_6}{\alpha'_H}, \quad u = \sqrt{2}\frac{R}{R_6}, \\ \xi_1 = tu = \frac{2R^2}{\alpha'_H}, \quad \xi_2 = \frac{2R}{g_H^2 R_6}, \quad \xi_3 = \frac{2RR_6}{\alpha'_H g_H^2}. \end{gathered} \tag{3.13}$$

[3] With respect to heterotic temperature duality.

As expected, ξ_2 and ξ_3 are related by radius inversion, $R_6 \to \alpha'_H R_6^{-1}$. Then, in Planck units,

$$\begin{aligned} R &= \frac{1}{2\pi T} = \kappa\sqrt{stu} = \kappa[\xi_1\xi_2\xi_3]^{1/4}, \\ R_6 &= \kappa\left(\frac{2st}{u}\right)^{1/2} = \frac{\sqrt{2}\kappa\xi_3}{[\xi_1\xi_2\xi_3]^{1/4}}. \end{aligned} \tag{3.14}$$

The first equation indicates that the temperature, when expressed in units of the *four-dimensional* gravitational coupling constant κ is invariant under string–string dualities.

In terms of heterotic variables, the critical temperatures (3.11) separating the heterotic phases are

$$\begin{aligned} \xi_1 &= \xi_H: \quad 2\pi T_H^< = \frac{g_H}{2^{1/4}\kappa}(\sqrt{2}-1), \\ \xi_1 &= \frac{1}{\xi_H}: \quad 2\pi T_H^> = \frac{g_H}{2^{1/4}\kappa}(\sqrt{2}+1). \end{aligned} \tag{3.15}$$

In addition, heterotic phases are separated from type II instabilities by the following critical temperatures:

$$\begin{aligned} \text{IIA}: \quad \xi_2 &= 4, \quad 2\pi T_A = \frac{R_6}{4\sqrt{2}\kappa^2}, \\ \text{IIB}: \quad \xi_3 &= 4, \quad 2\pi T_B = \frac{1}{2g_H^2 R_6}. \end{aligned} \tag{3.16}$$

Then the domain of the moduli space that avoids type II instabilities is defined by the inequalities $\xi_{2,3} > 4$. In heterotic variables,

$$\begin{aligned} 2\pi T &< \frac{1}{2\alpha'_H g_H^2}\min\left(R_6\ ;\ \alpha'_H/R_6\right) \\ &= \frac{1}{4\sqrt{2}\kappa^2}\min\left(R_6\ ;\ \alpha'_H/R_6\right). \end{aligned} \tag{3.17}$$

Type II instabilities are unavoidable when $T > T_{\text{self-dual}}$, with

$$2\pi T_{\text{self-dual}} = \frac{1}{2g_H^2\sqrt{\alpha'_H}} = \frac{2^{1/4}}{4\kappa g_H}.$$

The high-temperature heterotic phase cannot be reached[4] for any value of the radius R_6 if

$$T_H^< > T_{\text{self-dual}},$$

or

$$g_H^2 > \frac{\sqrt{2}+1}{2\sqrt{2}} \sim 0.8536. \tag{3.18}$$

[4]From low heterotic temperature.

In this case, $T_H^<$ always exceeds T_A and T_B. Only type II thermal instabilities exist in this strong-coupling regime and the value of $R_6/\sqrt{\alpha'_H}$ decides whether the type IIA or IIB instability will have the lowest critical temperature, following Eq. (3.16).

If on the other hand the heterotic string is weakly coupled,

$$g_H^2 < \frac{\sqrt{2}+1}{2\sqrt{2}}, \tag{3.19}$$

the high-temperature heterotic phase is reached for values of the radius R_6 verifying $T_H^< < T_A$ and $T_H^< < T_B$, or

$$2\sqrt{2}g_H^2(\sqrt{2}-1) < \frac{R_6}{\sqrt{\alpha'_H}} < \frac{1}{2\sqrt{2}g_H^2(\sqrt{2}-1)}. \tag{3.20}$$

The large and small R_6 limits, with fixed coupling g_H, again lead to either type IIA or type IIB instability.

3.1.3 High-temperature type IIA and IIB phases

These phases are defined by inequalities:

$$\xi_2 < 4 \qquad \text{and/or} \qquad \xi_3 < 4. \tag{3.21}$$

In this region of the parameter space, either H_2 or H_3 become tachyonic and acquire a vacuum value:

$$H_2^2 = \frac{4-\xi_2}{8\xi_2}, \qquad \kappa^4 V_2 = -\frac{1}{t}\frac{(4-\xi_2)^2}{16\xi_2}, \tag{3.22}$$

and/or

$$H_3^2 = \frac{4-\xi_3}{8\xi_3}, \qquad \kappa^4 V_3 = -\frac{1}{u}\frac{(4-\xi_3)^2}{16\xi_3}. \tag{3.23}$$

In contrast with the high-temperature heterotic phase, the potential does not possess stationary values of ξ_2 and/or ξ_3, besides the critical $\xi_{2,3} = 4$.

Suppose for instance that $\xi_2 < 4$ and $\xi_3 > 4$. The resulting potential is then V_2 only and ξ_2 slides to zero. In this limit,

$$V = -\frac{1}{stu\kappa^4},$$

and the dynamics of $\phi \equiv -\log(stu)$ is described by the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{e}{2\kappa^2}\left[R + \frac{1}{6}(\partial_\mu\phi)^2 - \frac{2}{\kappa^2}e^{\phi}\right].$$

Other scalar components $\log(t/u)$ and $\log(s/u)$ have only derivative couplings, since the potential only depends on ϕ. They can be taken to be constant and arbitrary. The dynamics only restricts the temperature radius $\kappa^{-2}R^2 = e^{-\phi}$, R_6 and the string coupling are not constrained, besides inequalities (3.21).

In conformally flat gravity background, the equation of motion of the scalar ϕ is

$$\Box\phi = -\frac{6}{\kappa^2}e^{\phi}.$$

The solution of the above and the Einstein equations defines a non-trivial gravitational ϕ-background. This solution will correspond to the high-temperature type II vacuum. We will not study this solution further here.

4. High-temperature heterotic phase

The thermal phase relevant to weakly-coupled, high-temperature heterotic strings at intermediate values of the radius R_6 [see inequalities (3.19) and (3.20)] has an interesting interpretation; we study this here, using the information contained in its effective theory, which is characterized by Eqs. (3.12):

$$tu = 1, \qquad H_1 = \frac{1}{2}, \qquad H_2 = H_3 = 0. \quad (4.1)$$

These values solve the equations of motion of all scalar fields with the exception of s. The resulting bosonic effective Lagrangian describing the dynamics of s and $g_{\mu\nu}$ is

$$\mathcal{L}_{\rm bos} = -\frac{1}{2\kappa^2}eR - \frac{e}{4\kappa^2}(\partial_\mu \ln s)^2 + \frac{e}{2\kappa^4 s}. \quad (4.2)$$

For all (fixed) values of s, the cosmological constant is negative since $e^{-1}V = -(2\kappa^4 s)^{-1}$ and the apparent geometry is anti-de Sitter. But the effective theory (4.1) does not stabilize s.

To study the bosonic Lagrangian, we first rewrite it in the string frame. Defining the dilaton as

$$e^{-2\phi} = s, \quad (4.3)$$

and rescaling the metric according to

$$g_{\mu\nu} \quad\longrightarrow\quad \frac{2\kappa^2}{\alpha'_H}e^{-2\phi}g_{\mu\nu}, \quad (4.4)$$

one obtains[5]

$$\mathcal{L}_{\text{string frame}} = \frac{e^{-2\phi}}{\alpha'_H}\Big[-eR + 4e(\partial_\mu\phi)(\partial^\mu\phi) + \frac{2e}{\alpha'_H}\Big]. \quad (4.5)$$

The equation of motion for the dilaton then is

$$R + 4(\partial_\mu\phi)(\partial^\mu\phi) - 4\Box\phi = \frac{2}{\alpha'_H}. \quad (4.6)$$

Comparing with the two-dimensional sigma-model dilaton β-function [13] with central charge deficit $\delta c = D - 26$, which leads to

$$R + 4(\partial_\mu\phi)(\partial^\mu\phi) - 4\Box\phi = -\frac{\delta c}{3\alpha'_H}, \quad (4.7)$$

we find a central charge deficit $\delta c = -6$, or, for a superstring,

$$\delta\hat{c} = \frac{2}{3}\delta c = -4. \quad (4.8)$$

In the string frame, a background for theory (4.5) has flat (sigma-model) metric $g_{\mu\nu} = \eta_{\mu\nu}$ and linear dilaton dependence [14] on a spatial coordinate, say x^1:

$$\phi = \phi_0 + Qx^1, \qquad Q^2 = \frac{\delta\hat{c}}{8\alpha'_H} = \frac{1}{2\alpha'_H} \quad (4.9)$$

(ϕ_0 is a constant).

The linear dilaton background breaks both four-dimensional Lorentz symmetry and four-dimensional Poincaré supersymmetry. Since supersymmetry breaks spontaneously, one expects to find goldstino states in the fermionic mass spectrum and massive spin 3/2 states. And, because of the non-trivial background, the theory in the high-temperature heterotic phase is effectively a three-dimensional supergravity.

To discuss the pattern of goldstino states, observe first that the supergravity extension of the bosonic Lagrangian (4.2) includes a non-zero gravitino mass term for all values of s since

$$m^2_{3/2} = \kappa^{-2}\, e^{\mathcal{G}} = \frac{1}{4\kappa^2 s} = \frac{1}{2\alpha'_H} = Q^2. \quad (4.10)$$

Notice also that the potential at the vacuum verifies

$$V = -\frac{2}{\kappa^4}e^{\mathcal{G}} = -\frac{1}{2\kappa^4 s} = -\frac{2}{\kappa^2}\, m^2_{3/2}. \quad (4.11)$$

[5] Since the rescaling $g_{\mu\nu} \to e^{-2\sigma}g_{\mu\nu}$ leads to $e[R + 6(\partial_\mu\sigma)^2] \to e^{-2\sigma}eR$.

Consider then the transformation of fermions in the chiral multiplet (z^i, χ^i) [6]:

$$\delta\chi_{Li} = \frac{1}{2}\kappa(\partial\!\!\!/ z_i)\epsilon_R - \frac{1}{2}e^{\mathcal{G}/2}\,(\mathcal{G}^{-1})_i^j\mathcal{G}_j\,\epsilon_L + \ldots, \tag{4.12}$$

omitting fermion contributions. In the high-temperature heterotic phase,

$$\mathcal{G}_S = \frac{\partial}{\partial S}\mathcal{G} = -\frac{1}{2s}, \qquad \mathcal{G}_a = \frac{\partial}{\partial z^a}\mathcal{G} = 0, \tag{4.13}$$

and the Kähler metric is diagonal with $\mathcal{G}_S^S = (2s)^{-2}$. Since also

$$\partial\!\!\!/ s = -2Qs\gamma^1, \qquad e^{\mathcal{G}/2} = \kappa Q,$$

only the fermionic partner χ_s of the dilaton s participates in supersymmetry breaking, with the transformation

$$\delta\chi_s = \frac{\sqrt{s}}{2}(1-\gamma^1)\epsilon. \tag{4.14}$$

Supersymmetries generated by $(1-\gamma^1)\epsilon$ are then broken in the linear dilaton background in the x_1 direction while those with parameters $(1+\gamma^1)\epsilon$ remain unbroken. Starting then from sixteen supercharges ($N = 4$ supersymmetry) at zero temperature, the high-temperature heterotic vacuum has eight unbroken supercharges. Since the effective space-time symmetry is three-dimensional, the high-temperature phase has $N_3 = 4$ supersymmetry: the linear dilaton background acts identically with respect to the $N = 4$ spinorial charges. It simply breaks one half of the charges in each spinor. Thus, the high-temperature phase is expected to be stable because of supersymmetry of its effective field theory and because of its superconformal content.

The mass spectrum of the effective supergravity theory in the linear dilaton background is analyzed in Ref. [4]. One first observes that the Kähler potential does not induce any mixing between the dilaton multiplet and other chiral multiplets. Then, the dilaton multiplet only plays an active role in the breaking of supersymmetry.

This splitting of chiral multiplets does not exist in the low-temperature phase $H_1 = H_2 = H_3 = 0$, in which

$$\mathcal{G}_S = -(2s)^{-1}, \quad \mathcal{G}_T = -(2t)^{-1}, \quad \mathcal{G}_U = -(2u)^{-1}, \tag{4.15}$$

with

$$\psi_G = \frac{1}{2s}\chi_s + \frac{1}{2t}\chi_t + \frac{1}{2u}\chi_u$$

as goldstino state[7]. The low-temperature phase is symmetric in the moduli s, t and u: it is common to the three dual strings, in their perturbative and non-perturbative domains. In contrast, the high-temperature heterotic phase only exists in the perturbative domain of the heterotic string, where s is the dilaton, and, by duality, in non-perturbative type II regimes.

In the computation of the mass spectrum, one needs then to isolate the contributions from the non-zero $\mathcal{G}_S$ in the mass matrices. Because of the existence of couplings SUZ_2^2 and STZ_3^2 in the superpotential, there will be mass splittings of the O'Raifeartaigh type in the sectors Z_2 and Z_3. It turns out that all supersymmetry breaking contributions to the mass matrices are due to these superpotential couplings. We then conclude that the spectrum is supersymmetric in the perturbative heterotic and moduli sector (T, U, Z_1), and with O'Raifeartaigh pattern in the non-perturbative sectors:

$$Z_2: m^2_{bosons} = m^2_{fermions} \pm 2su\, m^2_{3/2},$$

$$Z_3: m^2_{bosons} = m^2_{fermions} \pm 2st\, m^2_{3/2}.$$

As already observed in Ref. [3], a similar analysis applied to the perturbative heterotic string only would have led to a supersymmetric spectrum.

In the special infinite heterotic temperature limit discussed in Ref. [4], in which $\alpha'_H \to 0$, all massive states decouple and consequently one recovers $N = 2$ unbroken (rigid) supersymmetry in the effective (topological) field theory of the remaining massless hypermultiplets.

5. The high-temperature heterotic phase transition

As we already discussed, the high-temperature phase of $N = 4$ strings is described by a non-

[6] The notation is as in Ref. [15], with sign-reversed $\mathcal{G}$ and $\sigma^{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu]$. Indices $i, j, \ldots$, enumerate all chiral multiplets (z^i, χ^i).

[7] Expressed using non-normalized fermions. Canonical normalization of the spinors would lead to $\psi_G = \chi_s + \chi_t + \chi_u$.

critical string with central charge deficit $\delta\hat{c} = -4$, provided the heterotic string is in the weakly-coupled regime with $g_H^2 < g_c^2 = \frac{\sqrt{2}+1}{2\sqrt{2}}$. One possible description is in terms of the (5+1) super-Liouville theory compactified (at least) on the temperature circle with radius fixed at the fermionic point $R = \sqrt{\alpha'_H/2}$. The perturbative stability of this ground state is guaranteed when there is at least $\mathcal{N}_{\rm sc} = 2$ superconformal symmetry on the world-sheet, implying at least $N = 1$ supersymmetry in space-time. However, our analysis of the previous section shows that the boson–fermion degeneracy is lost at the non-perturbative level, even though the ground state remains supersymmetric.

An explicit example with $\mathcal{N}_{\rm sc} = 4$ superconformal was given in Ref. [16, 17]. It is obtained when together with the temperature circle there is an additional compactified coordinate on S^1 with radius $R_6 = \sqrt{\alpha'_H/2}$. These two circles are equivalent to a compactification on $[SU(2)\times SU(2)]_k$ at the limiting value of level $k = 0$. Indeed, at $k = 0$, only the 6 world-sheet fermionic $SU(2)\times SU(2)$ coordinates survive, describing a $\hat{c} = 2$ system instead of $\hat{c} = 6$ of $k \to \infty$, consistently with the decoupling of four supercoordinates, $\delta\hat{c} = -4$. The central charge deficit is compensated by the linear motion of the dilaton associated to the Liouville field, $\phi = Q^\mu x_\mu$ with $Q^2 = 1/(2\alpha'_H)$ so that $\delta\hat{c}_L = 8\alpha'_H Q^2 = 4$.

Using the techniques developed in Refs. [18, 17], one can derive the one-loop (perturbative) partition function in terms of the left- and right-moving degrees of freedom on the world-sheet [4]:

$$
\begin{aligned}
Z^{\rm Liouv}[SU(2)\times SU(2)]_{k=0} &= \frac{{\rm Im}\,\tau^{-1}}{\eta^6\ \bar{\eta}^{18}} \\
&\times \frac{1}{8}\sum_{\alpha,\beta,\bar{\alpha},\bar{\beta},h,g} (-)^{\alpha+\beta+\alpha\beta}\theta^2\begin{bmatrix}\alpha\\ \beta\end{bmatrix}\theta\begin{bmatrix}\alpha+h\\ \beta+g\end{bmatrix} \qquad (5.1)\\
&\times \theta\begin{bmatrix}\alpha-h\\ \beta-g\end{bmatrix}\bar{\theta}\begin{bmatrix}\bar{\alpha}+h\\ \bar{\beta}+g\end{bmatrix}\bar{\theta}\begin{bmatrix}\bar{\alpha}-h\\ \bar{\beta}-g\end{bmatrix}\bar{\theta}^{14}\begin{bmatrix}\bar{\alpha}\\ \bar{\beta}\end{bmatrix}.
\end{aligned}
$$

This partition function encodes a number of properties, which deserve some comments:

- The initial $N = 4$ supersymmetry is reduced to $N = 2$ (or $N_3 = 4$) because of the Z_2 projection generated by (h, g). This agrees with our effective field theory analysis of the high-temperature phase given previously. The (perturbative) bosonic and fermionic mass fluctuations are degenerate due to the remaining $N = 2$ supersymmetry.

- The $h = 0$ sector does not have any massless fluctuation due to the linear dilaton background or to the coupling to temperature. The linear dilaton background shifts the bosonic masses (squared) by $m_{3/2}^2$, so that all bosons in this sector have masses larger than or equal to $m_{3/2}$. This is again in agreement with our effective theory analysis. Similarly, fermion masses are shifted by the same amount because of the S_R^1 temperature modification.

- In the $h = 1$, "twisted", sector there are massless excitations as expected from the (5+1) super-Liouville theory [19, 20, 17].

- The $5 + 1$ Liouville background can be regarded as a Euclidean five-brane solution wrapped on $S^1\times S^1$ preserving one-half of the space-time supersymmetries ($N = 2$).

- The massless space-time fermions in the $h = 1$ sector are six-dimensional spinors constructed with the left-moving supercoordinates Ψ_μ and β,γ superghosts. They are also spinors under the $SO(4)_{\rm right}$ constructed using four right-moving fermions $\overline{\chi}_I$ which parametrize the fifth and sixth compactified coordinates, with $R = R_6 = \sqrt{\alpha'_H/2}$. They are also vectors under the $SO(28)$ constructed with 28 right-moving fermions $\overline{\Psi}_A$ ($c_R = 14$).

- Similarly, the massless space-time bosons are $SO(4)_{\rm right}$ spinors and $SO(28)$ vectors. They are also spinors under the $SO(4)_{\rm left}$ constructed with the left-moving fermions χ_I for the fifth and sixth coordinates compactified at the fermionic point. Together with the massless fermions, they form 28 $N = 2$ hypermultiplets.

These 28 massless hypermultiplets are the only states that survive in the zero-slope limit and their effective field theory is described by a $N = 2$ sigma-model on a hyper-Kähler manifold. This topological theory arises in the infinite temperature limit of the $N = 4$ strings after the heterotic Hagedorn phase transition.

Although the $5 + 1$ Liouville background is perturbatively stable due to the $\mathcal{N}_{sc} = 4$ superconformal symmetry, its stability is not ensured at the non-perturbative level when the heterotic coupling is large:

$$g_H^2(x_\mu) = e^{2(\phi_0 - Q^\mu x_\mu)} > \frac{\sqrt{2}+1}{2\sqrt{2}} \sim 0.8536. \qquad (5.2)$$

Indeed, the high-temperature heterotic phase only exists if $g_H^2(x_\mu)$ is lower than a critical value separating the heterotic and Type II high-temperature phases. Thus one expects a domain wall in space-time, at $x_\mu^0 = 0$, separating these two phases: $g_H^2(Q^\mu x_\mu^0) \sim 0.8536$. This domain wall problem can be avoided by replacing the $(5 + 1)$ super-Liouville background with a more appropriate one with the same superconformal properties, $\mathcal{N}_{\rm sc} = 4$, obeying however the additional perturbative constraint $g_H^2(x_\mu) << 1$ in the entire space-time.

Exact superstring solutions based on gauged WZW two-dimensional models with $\mathcal{N}_{\rm sc} = 4$ superconformal symmetries have been studied in the literature [21, 22, 16, 17, 23]. We now consider the relevant candidates with $\delta\hat{c} = -4$.

The first one is the $5+1$ super-Liouville with $\delta\hat{c} = 4$, already examined above. It is based on the $2d$-current algebra:

$$\begin{aligned} &U(1)_{\delta\hat{c}=4} \times U(1)^3 \times U(1)_{R^2=\alpha'_H/2} \\ &\quad\times U(1)_{R_6^2=\alpha'_H/2} \\ &\equiv U(1)_{\delta\hat{c}=4} \times U(1)^3 \times SO(4)_{k=1}. \end{aligned} \qquad (5.3)$$

Another class of candidate background consists of the non-compact parafermionic spaces described by gauged WZW models:

$$\begin{aligned} &\left[\frac{SL(2,R)}{U(1)_{V,A}}\right]_{k=4} \times \left[\frac{SL(2,R)}{U(1)_{V,A}}\right]_{k=4} \\ &\times U(1)_{R^2=\alpha'_H/2} \times U(1)_{R_6^2=\alpha'_H/2} \\ \equiv &\left[\frac{SL(2,R)}{U(1)_{V,A}}\right]_{k=4} \times \left[\frac{SL(2,R)}{U(1)_{V,A}}\right]_{k=4} \\ &\times SO(4)_{k=1}, \end{aligned} \qquad (5.4)$$

where indices A and B stand for the "axial" and "vector" WZW $U(1)$ gaugings.

Then, many backgrounds can be obtained by marginal deformations of the above, preserving at least $\mathcal{N}_{\rm sc} = 2$, or also by acting with S- or T-dualities on them.

As already explained, the appropriate background must verify the weak-coupling constraint:

$$g_H^2(x_\mu) = e^{2\phi} << \sim 0.8536\,, \qquad (5.5)$$

in order to avoid the domain-wall problem, and in order to trust the perturbative validity of the heterotic string background. This weak-coupling limitation is realized in the "axial" parafermionic space. In this background, $g_H^2(x_\mu)$ is bounded in the entire non-compact four-dimensional space, with coordinates $\{z, z^*, w, w^*\}$, provided the initial value of $g_0^2 = g_H^2(x_\mu = 0)$ is small.

$$\frac{1}{g_H^2(x_\mu)} = e^{-2\phi} = \frac{1}{g_0^2}\,(1+zz*)\,(1+ww*) \;\geq\; \frac{1}{g_0^2}. \qquad (5.6)$$

The metric of this background is everywhere regular:

$$ds^2 = \frac{4dzdz^*}{1+zz^*} + \frac{4dwdw^*}{1+ww^*}\,. \qquad (5.7)$$

The Ricci tensor is

$$R_{z\,z^*} = \frac{1}{(1+zz^*)^2}\,, \quad R_{w\,w^*} = \frac{1}{(1+ww^*)^2}\,. \qquad (5.8)$$

The scalar curvature

$$R = \frac{1}{4(1+zz^*)} + \frac{1}{4(1+ww^*)}$$

vanishes for asymptotically large values of $|z|$ and $|w|$ (asymptotically flat space). This space has maximal curvature when $|z| = |w| = 0$. This solution has a behaviour similar to that of the Liouville solution in the asymptotic regime $|z|$, $|w| \to \infty$. In this limit, the dilaton ϕ becomes linear when expressed in terms of the flat coordinates x_i:

$$\phi = -\,\mathrm{Re}[\log z] - \mathrm{Re}[\log w] = -Q^1|x_1| - Q^2|x_2|, \qquad (5.9)$$

where

$$\begin{aligned} x_1 &= -\mathrm{Re}[\log z], \quad & x_2 &= -\mathrm{Re}[\log w], \\ x_3 &= \mathrm{Im}[\log z], \quad & x_4 &= \mathrm{Im}[\log w], \end{aligned}$$

and the line element is $ds^2 = 4(dx_i)^2$. The important point here is that, for large values of $|x_1|$ and $|x_2|$, $\phi \ll 0$, in contrast to the Liouville background in which $\phi = Q^1x_1+Q^2x_2$, the dilaton becomes positive and arbitrarily large in one half of the space, violating the weak-coupling constraint (5.5).

We then conclude that the high-temperature phase is described by the above parafermionic space, which is stable because of $N = 2$ supersymmetry. Since it is perturbative everywhere, the perturbative massive bosonic and fermionic fluctuations are always degenerate. On the other hand, the non-perturbative ones are superheavy and decouple in the limit of vanishing coupling.

The asymptotic solution of the parafermionic space suggests an alternative super-Liouville solution with

$$\begin{aligned} \phi &= \phi_0 - \mathrm{Re}[\log z] - \mathrm{Re}[\log w] \\ &= \phi_0 - Q^1|x_1| - Q^2|x_2|. \end{aligned} \tag{5.10}$$

The appearance of the absolute value of $|x_i|$ gives an upper bound on the coupling constant provided Q^i are positive. However, the conical singularity at $x_i = 0$ implies, via the dynamical equation (4.6), the presence of curvature singularities at these points,

$$R_{z\,z^*} = -\pi\delta^{(2)}(z), \quad R_{w\,w^*} = -\pi\delta^{(2)}(w). \tag{5.11}$$

In the above modified Liouville background, the $g_H^2(x_\mu)$ is bounded in the entire non-compact four-dimensional space, provided the initial value $g_0^2 = g_H^2(x_\mu = 0) = e^{2\phi_0}$ is small.

6. Conclusions

$N = 4$ superstring theories at finite temperature T correspond to a particular gauging of the $N = 4$ supergravity. Using techniques of $N = 4$ gauged supergravity, we were able to compute the exact effective potential of all potential tachyonic modes, describing all three perturbative instabilities of $N = 4$ strings (heterotic, type IIA and type IIB) simultaneously. Hagedorn instabilities of different perturbative string descriptions appear as thermal dyonic 1/2-BPS modes that become massless (and then tachyonic) at (above) the corresponding Hagedorn temperature.

We find that the $N = 4$ thermal potential has a global stable minimum in a region where the heterotic string is weakly-coupled, so that the four-dimensional string coupling $g_H^2 < \frac{\sqrt{2}+1}{2\sqrt{2}}$. At the minimum, the temperature is fixed in terms of the heterotic string tension, the four internal supercoordinates decouple, and the system is described by a non-critical superstring in six dimensions. Supersymmetry, although restored in perturbation theory, appears to be broken at the non-perturbative level.

On the heterotic or type IIA side, the high-temperature limit corresponds to a topological theory described by an $N = 2$ supersymmetric sigma-model on a non-trivial hyper-Kähler manifold.

Acknowledgements

This research has been partially supported by the EEC under the contracts TMR–ERBFMRX–CT96–0045 and TMR–ERBFMRX–CT96–0090, by the Swiss National Science Foundation and the Swiss Office for Education and Science.

References

[1] J. J. Atick and E. Witten, *Nucl. Phys.* **B 310** (1988) 291.

[2] C. Kounnas and B. Rostand, *Nucl. Phys.* **B 341** (1990) 641.

[3] I. Antoniadis and C. Kounnas, *Phys. Lett.* **B 261** (1991) 369.

[4] I. Antoniadis, J.-P. Derendinger and C. Kounnas, Non-perturbative Temperature Instabilities in N=4 Strings, *Nucl. Phys.* **B 551** (99) 41; `hep-th/9902032`;
C. Kounnas, `hep-th/9902072`;
J.-P. Derendinger, `hep-th/9902107`, `hep-th/9903064`.

[5] J. H. Schwarz and A. Sen, *Phys. Lett.* **B 312** (1993) 105; *Nucl. Phys.* **B 411** (1994) 35;
A. Sen, *Phys. Lett.* **B 303** (1993) 22; *Phys. Lett.* **B 329** (1994) 217; *Nucl. Phys.* **B 329** (1994) 217;
M. J. Duff, J. T. Liu and J. Rahmfeld, *Nucl. Phys.* **B 459** (1996) 125;
M. Cvetic and D. Youm, *Phys. Rev.* **D 53** (1996) R584; *Phys. Lett.* **B 359** (1995) 87;

M. Cvetic and A. Tseytlin, *Phys. Lett.* **B 366** (1996) 95; *Phys. Rev.* **D 53** (1996) 5619;
G. L. Cardoso, G. Curio, D. Lüst, T. Mohaupt and S.-J. Rey, *Nucl. Phys.* **B 464** (1996) 18.

[6] R. Dijkgraaf, E. Verlinde and H. Verlinde, *Nucl. Phys.* **B 484** (1997) 543; *Nucl. Phys.* **B 486** (1997) 77; *Nucl. Phys.* **B 486** (1997) 89.

[7] C. Kounnas, *Nucl. Phys.* **B 58** [Proc. Suppl.] (1997) 57;
E. Kiritsis and C. Kounnas, *Nucl. Phys.* **B 503** (1997) 117.

[8] A. Sen, *Nucl. Phys.* **B 450** (1995) 103.

[9] J.-P. Derendinger and S. Ferrara, in: *Supersymmetry and Supergravity '84*, ed. B. de Wit, P. Fayet and P. van Nieuwenhuizen (World Scientific, Singapore, 1984), p. 159.

[10] A. H. Chamseddine, *Nucl. Phys.* **B 185** (1981) 403.

[11] M. de Roo, *Nucl. Phys.* **B 255** (1985) 515; *Phys. Lett.* **B 156** (1985) 331;
E. Bergshoeff, I.G. Koh and E. Sezgin, *Phys. Lett.* **B 155** (1985) 71.

[12] S. Ferrara and C. Kounnas, *Nucl. Phys.* **B 328** (1989) 406.

[13] E. Fradkin and A. Tseytlin, *Nucl. Phys.* **B 261** (1985) 1;
C. G. Callan, D. Friedan, E. Martinec and M. Perry, *Nucl. Phys.* **B 262** (1985) 593.

[14] I. Antoniadis, C. Bachas, J. Ellis and D.V. Nanopoulos, *Phys. Lett.* **B 211** (1988) 393; *Nucl. Phys.* **B 328** (1989) 117.

[15] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, *Nucl. Phys.* **B 212** (1983) 413.

[16] C. Kounnas, *Phys. Lett.* **B 321** (1994) 26.

[17] I. Antoniadis, S. Ferrara and C. Kounnas, *Nucl. Phys.* **B 421** (1994) 34.

[18] I. Antoniadis, C. P. Bachas and C. Kounnas, *Nucl. Phys.* **B 289** (1987) 87;
H. Kawai, D. C. Lewellen and S. H.-H. Tye, *Nucl. Phys.* **B 288** (1987) 1.

[19] A. Bilal and J.-L. Gervais, *Phys. Lett.* **B 177** (1986) 313; *Phys. Lett.* **B 187** (1987) 39; *Nucl. Phys.* **B 293** (1987) 1.

[20] D. Kutasov and N. Seiberg, *Phys. Lett.* **B 251** (1990) 67.

[21] I. Antoniadis, C. Bachas and A. Sagnotti, *Phys. Lett.* **B 235** (1990) 255.

[22] C. Kounnas, M. Porrati and B. Rostand, *Phys. Lett.* **B 258** (1991) 61.

[23] E. Kiritsis, C. Kounnas and D. Lüst, *Phys. Lett.* **B 331** (1994) 321; *Int. J. Mod. Phys.* **A 9** (1994) 1361.