Confining N=1 SUSY gauge theories from Seiberg duality

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ABSTRACT: In this talk I review and generalize an idea of Seiberg that an $N = 1$ supersymmetric gauge theory shows confinement without breaking of chiral symmetry when the gauge symmetry of its magnetic dual is completely broken by the Higgs effect. It is shown how the confining spectrum of a supersymmetric gauge theory can easily be derived when a magnetic dual is known and this method is applied to many models containing fields in second rank tensor representations and an appropriate tree-level superpotential.

1. Introduction

Due to holomorphicity properties and non-renormalization theorems valid in $N = 1$ supersymmetric theories it has become possible to argue that some supersymmetric gauge theories with special matter content confine at low energies. The first example is due to Seiberg [1] who found that supersymmetric quantum chromodynamics (SQCD) with gauge group $SU(N_c)$ and N_f quark flavors shows confinement when $N_f = N_c$ or $N_f = N_c + 1$. This has been generalized to more complicated models. All $N = 1$ supersymmetric gauge theories with vanishing tree-level superpotential which confine at low energies could be classified [2, 3, 4] because they are constrained by an index argument. When a tree-level superpotential is present the index argument is no longer valid. Because of the lower symmetry the non-perturbative superpotential is less constrained and one expects more confining models to exist. Indeed, Csáki and Murayama [5] showed that many of the Kutasov-like [6] models exhibit confinement for special values of the number of quark flavors N_f.[1] These models contain fields in tensor representations of the gauge group and an appropriate superpotential for these tensor fields.

[1]Some further confining models with non-vanishing tree-level superpotential are discussed in [7].

For all of the models considered in [5] a dual description in terms of magnetic variables is known [6, 8, 9, 10] and the authors of [5] used the fact that the electric gauge theory confines when its magnetic dual is completely higgsed.

Seiberg already used this idea as an additional consistency check in his original paper establishing electric-magnetic duality for non-Abelian $N = 1$ supersymmetric gauge theories [11]. He showed how the confining superpotential of SQCD with $N_f = N_c + 1$ could be obtained by a perturbative calculation in the completely broken magnetic gauge theory. Under duality the fields of the magnetic theory (which are gauge singlets as the gauge symmetry is completely broken) are mapped to the mesons and baryons of the electric theory and the confining superpotential is easily shown to be the image of the magnetic superpotential under this mapping [11]. This a realization in $N = 1$ supersymmetric gauge theories of an old idea of 't Hooft and Mandelstam [12] that confinement is driven by condensation of magnetic monopoles.

Now, many gauge theory models have been found that possess a dual description in terms of magnetic variables in the infrared. This allows us to predict many new examples of confining gauge theories. The idea described in the previous paragraph was first used by the authors of [13] to determine the confining spectrum of the model proposed by Kutasov [6] and has been applied by Csáki and Murayama [5] to six further models that confine in the presence of an appropriate superpotential.

In this talk I review the original example of Seiberg [11] and explain how electric-magnetic duality is used to obtain the low-energy spectrum and the form of the non-perturbative superpotential of confining gauge theories [14]. One finds that all of the gauge theory models based on simple gauge groups considered in [10, 15] confine when the gauge groups of their magnetic duals are completely broken by the Higgs effect. For nine of these theories the confining phase has not been discussed before.

2. Phase structure of SQCD

Let us briefly review the well-known phase structure of SQCD [16]. By this we mean an $N = 1$ supersymmetric $SU(N_c)$ gauge theory with N_f quark flavors, i.e. N_f chiral matter supermultiplets Q transforming in the fundamental representation of the gauge group and the same amount of matter multiplets $\bar{Q}$ transforming in the antifundamental representation. Consider first the case of vanishing tree-level superpotential. Depending on the relative values of N_f and N_c the the low-energy theory resides in different phases.

$N_f = 0$: This is pure super Yang-Mills theory. It is believed to show confinement. According to an index argument by Witten [17] there are N_c distinct supersymmetric vacua.

$0 < N_f < N_c$: There is a non-perturbative superpotential generated by gluino condensation (for $N_f < N_c - 1$) or by instantons (for $N_f = N_c - 1$), as was shown by Affleck, Dine and Seiberg [18]:

$$W_{\rm np} = (N_c - N_f)\left(\frac{\Lambda^{3N_c-N_f}}{\det M}\right)^{\frac{1}{N_c-N_f}} , \quad (2.1)$$

where Λ is the dynamically generated scale of the theory and the meson matrix M is defined by $M^{ij} = Q^{\alpha i}\bar{Q}^j_\alpha$, $i,j = 1,\ldots,N_f$, $\alpha = 1,\ldots,N_c$. The minimum of the potential lies at infinite field expectation values and therefore the theory has no stable vacuum for this range of parameters.

$N_f = N_c$: In this case the superpotential vanishes even at the non-perturbative level [1]. As a consequence the flat directions that parametrize the moduli space of vacua are not lifted in the quantum theory. The low-energy spectrum is given by the mesons M^{ij} defined above and the baryons $B = \det Q$, $\bar{B} = \det \bar{Q}$ (the quarks Q, $\bar{Q}$ are viewed as $(N_f \times N_c)$-matrices). The physical degrees of freedom at low energies being gauge invariant means that the theory confines. The classical constraint $\det M = B\bar{B}$ is modified in the quantum theory [1] to

$$\det M - B\bar{B} = \Lambda^{2N_c} . \quad (2.2)$$

The expectation values of the mesons and baryons that satisfy this constraint span the quantum moduli space. The observation that the expectation values of $\det M$ and $B\bar{B}$ cannot vanish simultaneously tells us that the chiral symmetry is spontaneously broken.

$N_f = N_c + 1$: There is again a quantum moduli space, but now the classical constraints are not modified by quantum effects. In the low-energy theory they can be derived from the non-perturbative superpotential [1]

$$W_{\rm np} = \frac{\bar{B}MB - \det M}{\Lambda^{2N_f-3}} , \quad (2.3)$$

where the baryons B^i are defined as the determinant of the quark matrix Q with the i-th line omitted. This describes confinement without breaking of the chiral symmetry.

$N_c + 2 \le N_f \le \frac{3}{2}N_c$: The low-energy energy theory is rather complicated and more appropriately described in terms of dual magnetic variables. The dual magnetic theory is infrared free for this range of parameters.

$\frac{3}{2}N_c < N_f < 3N_c$: At low energies the theory is driven to an infrared fixed point of the renormalization group [11] and resides in a non-Abelian Coulomb phase. Seiberg found a dual description of this model [11] by an $SU(N_f - N_c)$ gauge theory with N_f (magnetic) quark flavors q, $\bar{q}$ and N_f^2 additional singlets $M^{ij}_{\rm mag}$ which couple to the magnetic quarks via the superpotential

$$W_{\rm mag} = M_{\rm mag} q\bar{q} . \quad (2.4)$$

This magnetic theory flows to the same infrared fixed point. The gauge invariant operators of both theories are in one-to-one correspondence:

$$M \longleftrightarrow \mu M_{\rm mag} , \qquad (2.5)$$
$$B \longleftrightarrow \sqrt{-(-\mu)^{N_c-N_f}\Lambda^{3N_c-N_f}}\, B_{\rm mag} .$$

The mass scale μ had to be introduced by dimensional analysis. The three scales Λ, $\Lambda_{\rm mag}$ and μ are related by

$$\Lambda^{3N_c-N_f}\Lambda_{\rm mag}^{3(N_f-N_c)-N_f} = (-1)^{N_f-N_c}\mu^{N_f} . \qquad (2.6)$$

$N_f > 3N_c$: In the infrared the theory flows to the trivial fixed point of free quarks and gluons.

3. Confinement from duality

Let us consider deformations [11] of the theory described in the previous section by mass terms $W = {\rm Tr}(mM)$, where m is an $(N_f \times N_f)$-matrix of rank p. By the duality mapping (2.5) this corresponds to adding a term $\mu\,{\rm Tr}(mM_{\rm mag})$ to the superpotential (2.4) in the magnetic theory (cf figure 1). As our treatment of SQCD is restricted to the (Wilsonian) low-energy effective action we have to integrate out the massive modes from the deformed model. In the electric theory this just leads to a reduction of the number of quark flavors by p. Therefore the low-energy theory is an $SU(N_c)$ gauge theory with $N_f - p$ quark flavors and vanishing superpotential $\hat{W} = 0$. In the magnetic theory, integrating out the massive components of $M_{\rm mag}$ leads to non-vanishing expectation values for q, $\bar{q}$ and thus the gauge symmetry is broken spontaneously. The low-energy theory is an $SU(N_f - N_c - p)$ gauge theory with $N_f - p$ quark flavors and superpotential $\hat{W}_{\rm mag} = \hat{M}_{\rm mag}\hat{q}\hat{\bar{q}}$, where hats denote the low-energy fields. One finds that the two effective theories are again dual to each other [11], as shown in figure 1.

It is interesting to consider the special case $p = N_f - N_c - 1$. Then one has an effective $SU(N_c)$ gauge theory with $N_c + 1$ quark flavors on the electric side. The magnetic gauge symmetry is completely broken by the Higgs effect. However, one color component of each of the N_c+1 quark flavors stays massless after the symmetry breaking. These $2(N_c + 1)$ gauge singlets are denoted by $\hat{q}$, $\hat{\bar{q}}$. They couple to the meson singlets $\hat{M}_{\rm mag}$ via the tree-level superpotential (2.4). Due to non-renormalization theorems this is not corrected in perturbation theory. But there are non-perturbative corrections generated by instantons. The full superpotential of the low-energy magnetic theory reads [11]

$$\hat{W}_{\rm mag} = \hat{M}_{\rm mag}\hat{q}\hat{\bar{q}} + \Lambda_{\rm mag}^{3-N_f}\det\hat{M}_{\rm mag}. \qquad (3.1)$$

From the fact that all physical degrees of freedom of the effective magnetic theory are gauge invariant one expects that, as a consequence of the duality mapping, the degrees of freedom of the electric theory are gauge singlets as well. We will see that this intuition is right. The mapping (2.5) gives

$$M \longleftrightarrow \mu M_{\rm mag} , \qquad (3.2)$$
$$B \longleftrightarrow \sqrt{\mu^{-1}\Lambda^{2N_f-3}}\,\hat{q} ,$$

and the scale matching (2.6) now reads

$$\Lambda^{2N_f-3}\Lambda_{\rm mag}^{3-N_f} = -\mu^{N_f} . \qquad (3.3)$$

The effective electric theory is described by the mesons M and the baryons B, $\bar{B}$. One can check that the 't Hooft anomaly matching conditions [19] are satisfied for this low-energy spectrum. These conditions require that if one gauges the global symmetries of the theory then the values of the various triangle anomalies (which in general will not vanish) must coincide for the microscopic description in terms of quarks and gluons and the macroscopic description in terms of mesons and baryons. Performing this calculation one finds that the mesons and baryons obtained from the duality mapping (3.2) are just the right degrees of freedom to match the global anomalies of the microscopic theory. This means that the theory is in the confining phase [1], in agreement with the result for $N_f = N_c+1$ of the previous section. It is easy to determine the full superpotential of the effective electric theory by applying the duality mapping (3.2) to the magnetic superpotential (3.1). Using the scale relation (3.3) one finds

$$\hat{W} = \frac{\bar{B}MB - \det M}{\Lambda^{2N_f-3}} , \qquad (3.4)$$

which coincides with the superpotential (2.3) found in the previous section for $N_f = N_c + 1$.

$$
\begin{array}{ccc}
SU(N_c), N_f \text{ flavors} & \xleftrightarrow{\text{duality}} & SU(N_f - N_c), N_f \text{ flavors} \\
W = \mathrm{Tr}(mM) & & W_{\mathrm{mag}} = M_{\mathrm{mag}} q \bar{q} + \mu \, \mathrm{Tr}(m M_{\mathrm{mag}}) \\
\downarrow & & \downarrow \\
SU(N_c), N_f - p \text{ flavors} & \xleftrightarrow{\text{duality}} & SU(N_f - N_c - p), N_f - p \text{ flavors} \\
\hat{W} = 0 & & \hat{W}_{\mathrm{mag}} = \hat{M}_{\mathrm{mag}} \hat{q} \hat{\bar{q}}
\end{array}
$$

Figure 1: The electric theory is deformed by adding mass terms for some of the quarks and the magnetic theory is deformed correspondingly. After having integrated out the massive modes one finds two effective theories that are again dual to each other. Displayed are the tree-level contributions to the superpotentials.

4. Generalizations

The results on SQCD described in the previous sections have been generalized to other gauge theory models involving different gauge groups and/or matter fields transforming in representations other than the fundamental. For each of these models the electric theory shows confinement without breaking of the chiral symmetry when the gauge symmetry of its magnetic dual is completely broken.

4.1 other gauge groups

The simplest extension of the results on SQCD described above consists in gauge theories with orthogonal or symplectic gauge groups. Let us first consider an $SO(N_c)$ gauge theory with N_f matter fields Q (quarks) transforming in the vector representation of the gauge group and vanishing tree-level superpotential. This has a dual description [11, 20] in terms of a magnetic $SO(N_f + 4 - N_c)$ gauge theory with N_f quarks q transforming in the vector representation and $\frac{1}{2}N_f(N_f+1)$ meson singlets M_{mag} that couple to the magnetic quarks via $W_{\mathrm{mag}} = M_{\mathrm{mag}}qq$. The gauge invariant operators of both theories are in one-to-one correspondence to each other by a mapping very similar to (2.5). For $N_f = N_c - 3$ the magnetic theory is completely higgsed, and one finds that the electric theory confines. The confining spectrum (mesons M and baryons B) as well as the correct confining superpotential

$$W = \frac{MBB}{\Lambda^{2N_f+3}} \tag{4.1}$$

can be obtained from the effective magnetic theory via the duality mapping.

An $Sp(2N_c)$ gauge theory with $2N_f$ quarks Q transforming in the fundamental representation and vanishing tree-level superpotential can equivalently be described [22] by an $Sp(2(N_f - 2 - N_c))$ gauge theory with $2N_f$ quarks q and $N_f(2N_f-1)$ meson singlets M_{mag} that couple to the magnetic quarks via $W_{\mathrm{mag}} = M_{\mathrm{mag}}qq$. The duality mapping between the gauge invariant operators of the two theories is simply given by $M \leftrightarrow \mu\, M_{\mathrm{mag}}$; there are no baryons in symplectic gauge theories. For $N_f = N_c+2$ the magnetic theory is completely higgsed, and one finds that the electric theory confines. The confining spectrum (mesons M) can be obtained from the effective magnetic theory via the duality mapping. To obtain the correct confining superpotential

$$W = \frac{\mathrm{Pf}\, M}{\Lambda^{2N_f-3}} \tag{4.2}$$

more care is needed, because it is due to instanton corrections in the effective magnetic gauge theory.

4.2 gauge theories containing tensor fields

For any $N = 1$ supersymmetric model with vanishing tree-level superpotential the form of the most general superpotential that can possibly be generated by non-perturbative effects is completely fixed by the requirement that it be invariant under all symmetries of the considered model [18, 23, 2]. For a theory with gauge group G and chiral matter fields ϕ_l in representations r_l of G and dynamically generated scale Λ one finds

$$W \propto \left(\frac{\prod_l (\phi_l)^{\mu_l}}{\Lambda^b} \right)^{\frac{2}{\Delta}}, \tag{4.3}$$

where μ_l is the (quadratic) Dynkin index of the representation r_l, μ_G denotes the index of the adjoint representation, $\Delta = \sum_l \mu_l - \mu_G$ and $b = \frac{1}{2}(3\mu_G - \sum_l \mu_l)$ is the coefficient of the 1-loop β-function. In general the complete non-perturbative superpotential consists of a sum of terms of the form (4.3) with different possible contractions of all gauge and flavor indices. The relative coefficients of these terms cannot be fixed by symmetry arguments but must be inferred from a different reasoning.

If the theory is confining at every point of the moduli space then the superpotential must either vanish or be a smooth function of the confined degrees of freedom [2]. All such models with a smooth confining superpotential could be classified [2] as they have to verify the constraint $\Delta = 2$. But only for some of these smoothly confining gauge theories containing tensor fields a dual description in terms of magnetic variables is known.

On the other hand many dualities for models including tensor fields have been found once an appropriate tree-level superpotential for the tensors is added. Let us review one example first studied by the authors of [6]. They considered an $SU(N_c)$ gauge theory with N_f quark flavors Q, $\bar{Q}$, an additional matter field X in the adjoint representation and a tree-level superpotential $W_{\rm tree} = {\rm Tr}\, X^{k+1}$, where $k > 1$ is some integer. This model has a dual description in terms of an $SU(kN_f - N_c)$ gauge theory with N_f quark flavors q, $\bar{q}$, an adjoint tensor Y, kN_f^2 singlets $M_{{\rm mag},j}$, $j = 0, \ldots, k-1$ and tree-level superpotential

$$W_{\rm mag} = {\rm Tr}\, Y^{k+1} + \sum_{j=0}^{k-1} M_{{\rm mag},k-1-j}\, q Y^j \bar{q}\,. \quad (4.4)$$

For $N_c = kN_f - 1$ the magnetic theory is completely higgsed and one expects the electric theory to confine. Indeed, one finds that the 't Hooft anomaly matching conditions are satisfied if the confined spectrum of the electric theory is given by [13]

$$\begin{aligned} M_j &= Q X^j \bar{Q}\,, \quad j = 0, \ldots, k-1\,, \quad (4.5) \\ B &= (Q)^{N_f} \cdots (X^{k-1}Q)^{N_f} (X^k Q)^{N_f - 1}\,, \\ \bar{B} &= (\bar{Q})^{N_f} \cdots (X^{k-1}\bar{Q})^{N_f} (X^k \bar{Q})^{N_f - 1}\,, \end{aligned}$$

where the gauge indices are contracted with a Kronecker delta for the mesons and with an epsilon tensor of rank N_c for the baryons. In addition the flavor indices of the baryons are contracted with an epsilon tensor of rank kN_f leaving $2N_f$ independent baryons. It is easy to see [14] that these are exactly the degrees of freedom that are mapped under duality on the magnetic singlets $\hat{M}_{{\rm mag},j}$, $\hat{q}$, $\hat{\bar{q}}$ that stay massless after the Higgs effect. The matching of the 't Hooft anomalies between the microscopic (i.e. quarks and gluons) and the macroscopic (i.e. confined) description of the electric theory can thus be seen as a consequence of the anomaly matching between the electric and the magnetic theory. In this sense confinement can be derived from duality.

It is straightforward to apply this idea to all gauge theory models of [10, 15] based on simple gauge groups. One first builds the gauge invariant composite operators whose expectation values span the moduli space, then finds the duality mapping between the electric and the magnetic theory for these operators and finally applies this mapping to the completely higgsed effective magnetic theory to obtain the confined spectrum of the electric theory. In addition, in most cases at least some of the terms of the confining superpotential can be determined from the magnetic tree-level superpotential. To constrain the possible form of the confining superpotential we would like to generalize the formula (4.3) to the case of a non-vanishing tree-level superpotential. Therefore divide the matter fields into two subsets $\{\phi_l\} = \{\bar{\phi}_{\bar{l}}\} \cup \{\hat{\phi}_{\hat{l}}\}$, with $\{\bar{\phi}_{\bar{l}}\} \cap \{\hat{\phi}_{\hat{l}}\} = \emptyset$, and add a tree-level term for the hatted fields:

$$W_{\rm tree} = h \prod_{\hat{l}} \left(\hat{\phi}_{\hat{l}}\right)^{n_{\hat{l}}}\,, \quad (4.6)$$

where h is a dimensionful coupling parameter and the $n_{\hat{l}}$ are positive integers. To be invariant under all global symmetries the full superpotential must be of the form [14]

$$W \propto \left(\frac{\prod_{\bar{l}} (\bar{\phi}_{\bar{l}})^{\mu_{\bar{l}}}}{\Lambda^b} \right)^{\alpha} \prod_{\hat{l}} \left(\hat{\phi}_{\hat{l}}\right)^{\beta_{\hat{l}}} h^{\gamma}\,, \quad (4.7)$$

where the powers α, $\beta_{\hat{l}}$, γ must verify the following relations:

$$\begin{aligned} \gamma &= 1 - \tfrac{1}{2}\alpha\Delta\,, \\ \beta_{\hat{l}} &= \mu_{\hat{l}}\, \alpha + \gamma n_{\hat{l}}\,. \quad (4.8) \end{aligned}$$

	$SU(N_c)$			
tensors	*adj*	$\begin{smallmatrix}\square\\\square\end{smallmatrix}+\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\square\square+\overline{\square\square}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}+\overline{\square\square}$
W_{tree}	X^{k+1}	$(X\bar{X})^{k+1}$	$(X\bar{X})^{k+1}$	$(X\bar{X})^{2(k+1)}$
N_c	kN_f-1	$(2k+1)N_f-4k-1$	$(2k+1)N_f+4k-1$	$(4k+3)(N_f+4)-1$
α	k	1	$2(k+1)$	$2(k+1)$
β_X	$(k-1)N_c$	$k(N_f-1)$	$(k+1)(2k(N_f+2)-1)$	$2(k+1)((2k+1)(N_f+4)-3)$
$\beta_{\bar{X}}$		$k(N_f-1)$	$(k+1)(2k(N_f+2)-1)$	$2(k+1)((2k+1)(N_f+4)+1)$
γ	$-N_c$	$3-N_f$	$-(N_c+N_f+4)$	$-2(k+1)(N_f+4)$

	$Sp(2N_c)$		$SO(N_c)$	
tensors	$\square\square$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square\square$
W_{tree}	$X^{2(k+1)}$	X^{k+1}	$X^{2(k+1)}$	X^{k+1}
N_c	$(2k+1)N_f-2$	$k(N_f-2)$	$(2k+1)N_f+3$	$k(N_f+4)-1$
α	$2k+1$	1	1	k
β	$2k(N_c+1)$	$(k-1)(N_f-1)$	$2k(N_f+1)+4$	$(k-1)(N_c-2k)$
γ	$-(N_c+1)$	$3-N_f$	$1-N_f$	$-(N_c+2k)$

Table 1: Gauge theories that confine in the presence of a tree-level superpotential. The microscopic spectrum consists of N_f quark flavors and additional fields transforming in tensor representations represented by their Young tableaux. The coefficients α, β, γ refer to the powers in the non-perturbative superpotential (4.7).

The calculation of the confining spectra and the confining superpotentials for all simple group models of [10, 15] is performed in [14]. The results are displayed in tables 1 and 2.

5. A new confining model

To illustrate the ideas presented above I would like to treat one of the models of [14] in more detail. Consider an $SU(N_c)$ gauge theory with N_f+8 quarks Q, N_f antiquarks $\bar{Q}$, an antisymmetric tensor X and a conjugate symmetric tensor $\bar{X}$ and tree-level superpotential $W_{\text{tree}} = h\,\text{Tr}(X\bar{X})^{2(k+1)}$. This is a chiral theory and the difference between the number of quarks and antiquarks is required by anomaly freedom. The gauge invariant composite operators are given by

$$\begin{aligned}
&M_j = Q\bar{Q}_{(j)}\,, \quad P_r = Q\bar{X}Q_{(r)}\,,\ \bar{P}_r = \bar{Q}X\bar{Q}_{(r)}\,,\\
&\quad\text{with } Q_{(j)} = (X\bar{X})^j Q\,,\ \bar{Q}_{(j)} = (\bar{X}X)^j\bar{Q}\,,\\
&\quad j = 0,\ldots,2k+1\,, \quad r = 0,\ldots,2k\,,\\
&\mathcal{B}^{(\bar{n}_0,\ldots,\bar{n}_{2k},n_0,\ldots,n_{2k+1})} = (\bar{X}(X\bar{X})^k W_\alpha)^2\\
&\qquad \cdot(\bar{X}Q)^{\bar{n}_0}(\bar{X}Q_{(1)})^{\bar{n}_1}\cdots(\bar{X}Q_{(2k)})^{\bar{n}_{2k}}\\
&\qquad \cdot\bar{Q}^{n_0}\bar{Q}_{(1)}^{n_1}\cdots\bar{Q}_{(2k+1)}^{n_{2k+1}}\,,
\end{aligned}$$

$$\text{with } \sum_{j=0}^{2k+1} n_j + \sum_{j=0}^{2k}\bar{n}_j = N_c - 4\,, \tag{5.1}$$

$$\begin{aligned}
&B_n = X^n Q^{N_c-2n}\,, \quad n = 0,\ldots,\left\lceil\frac{N_c}{2}\right\rceil\,,\\
&\bar{B}_{\bar{n}} = \bar{X}^{\bar{n}}\bar{Q}^{N_c-\bar{n}}\bar{Q}^{N_c-\bar{n}}\,, \quad \bar{n} = 0,\ldots,N_c\,,\\
&T_i = \text{Tr}(X\bar{X})^i\,, \quad i = 1,\ldots,2k+1\,,
\end{aligned}$$

where the gauge indices are contracted with one epsilon tensor for the $\mathcal{B}^{(\cdots)}$, B_n and with two epsilon tensors for the $\bar{B}_{\bar{n}}$.

The authors of [10] found a dual description of this model in terms of a magnetic

$$SU((4k+3)(N_f+4)-N_c)$$

gauge theory, with N_f+8 quarks q, N_f antiquarks $\bar{q}$, an antisymmetric tensor Y, a conjugate symmetric tensor $\bar{Y}$ and singlets $M_{\text{mag},j}$, $P_{\text{mag},r}$, $\bar{P}_{\text{mag},r}$ and tree-level superpotential

$$W_{\text{mag}} = -h\,\text{Tr}(X\bar{X})^{2(k+1)} + \ldots\,,$$

where the dots indicate terms involving M_{mag}, P_{mag}, $\bar{P}_{\text{mag}}$. The duality mapping for the gauge invariant operators is given by

$$M_j,\ P_r,\ \bar{P}_r \ \leftrightarrow\ M_{\text{mag},j},\ P_{\text{mag},r},\ \bar{P}_{\text{mag},r}\,,$$

	$SU(N_c)$			
tensors	$2\,adj$	$adj + \begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	$adj + \Box\!\Box + \overline{\Box\!\Box}$	$adj + \begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \overline{\Box\!\Box}$
W_{tree}	$X^{k+1} + XY^2$	$X^{k+1} + XY\bar{Y}$	$X^{k+1} + XY\bar{Y}$	$X^{k+1} + XY\bar{Y}$
N_c	$3kN_f - 1$	$3kN_f - 5$	$3kN_f + 3$	$3k(N_f + 4) - 1$
α	$3k$	k	$4k$	$2k$

	$Sp(2N_c)$		$SO(N_c)$	
tensors	$2\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \Box\!\Box$	$2\Box\!\Box$	$\Box\!\Box + \begin{smallmatrix}\Box\\\Box\end{smallmatrix}$
W_{tree}	$X^{k+1} + XY^2$	$X^{k+1} + XY^2$	$X^{k+1} + XY^2$	$X^{k+1} + XY^2$
N_c	$3kN_f - 4k - 2$	$3kN_f - 4k + 2$	$3kN_f + 8k + 3$	$3kN_f + 8k - 5$
α	(k)	$(3k)$	$3k$	k

Table 2: Gauge theories that confine in the presence of a tree-level superpotential. The microscopic spectrum consists of N_f quark flavors and additional fields transforming in tensor representations represented by their Young tableaux. The coefficient α refers to the power in the non-perturbative superpotential (4.7).

$$
\begin{aligned}
\bar{\mathcal{B}}^{(\bar{n}_i, n_j)} &\leftrightarrow \bar{\mathcal{B}}^{(\bar{m}_i, m_j)}_{\text{mag}}\,, \\
\text{with} \quad & m_j = N_f - n_{2k+1-j}\,, \\
& \bar{m}_j = N_f + 8 - \bar{n}_{2k-j}\,, \qquad (5.2) \\
B_n &\leftrightarrow B_{\text{mag},m}\,, \\
\text{with} \quad & m = (2k+1)(N_f+4) - 2 - n\,, \\
\bar{B}_{\bar{n}} &\leftrightarrow \bar{B}_{\text{mag},\bar{m}}\,, \\
\text{with} \quad & \bar{m} = 2(2k+1)(N_f+4) + 4 - \bar{n}\,.
\end{aligned}
$$

The dependence on the mass scale μ has been suppressed.

For $N_c = (4k+3)(N_f+4) - 1$ the magnetic theory is completely higgsed and the electric theory confines with low-energy spectrum given by the composite fields

$$
\begin{aligned}
& M_j\,,\ P_r\,,\ \bar{P}_r\,, \\
& \quad j = 0, \ldots, 2k+1\,, \quad r = 0, \ldots, 2k\,, \\
& B \equiv B_{(2k+1)(N_f+4)-2}\,, \\
& \bar{B} \equiv \bar{\mathcal{B}}^{(N_f+8, \ldots, N_f+8, N_f, \ldots, N_f, N_f-1)}\,, \qquad (5.3) \\
& \bar{b} \equiv \bar{B}_{2(2k+1)(N_f+4)+3}\,,
\end{aligned}
$$

of eqs. (5.1). From (5.2) one finds the mappings $B, \bar{B} \leftrightarrow q, \bar{q}$ and $\bar{b} \leftrightarrow \bar{Y}$. One color component of each of the fields q, $\bar{q}$, $\bar{Y}$ together with the meson singlets are exactly the degrees of freedom that stay massless after breaking the magnetic gauge group.

As a further consistency check let us consider deformations of the theory along the flat directions corresponding to large expectation values of the baryons B, $\bar{b}$. A large VEV of B breaks the gauge symmetry to $Sp(2((2k+1)(N_f+4)-2))$ [10]. The low-energy theory contains $2(N_f+4)$ quarks Q, a symmetric tensor X and tree-level superpotential $\operatorname{Tr} X^{2(k+1)}$. This model is known to show confinement [5]. A large VEV of $\bar{b}$ breaks the gauge symmetry to $SO(2(2k+1)(N_f+4)+3)$ [10]. The low-energy theory contains $2(N_f+4)$ quarks Q, an antisymmetric tensor X and tree-level superpotential $\operatorname{Tr} X^{2(k+1)}$. This model is known to show confinement [5].

The effective low-energy superpotential of the magnetic theory contains the terms $M_{2k+1} q\bar{q} + P_{2k} q\bar{Y} q$. We thus expect that the confining superpotential of the electric theory has terms proportional to $\bar{B} M_{2k+1} B$, $BBP_{2k}\bar{b}$. The detailed analysis gives [14]

$$
\begin{aligned}
W = & \frac{\bar{B} M_{2k+1} B}{h^{2(k+1)(N_f+4)}\,\Lambda^{2(k+1)((8k+5)(N_f+4)-2)}} \\
& + \frac{BBP_{2k}\bar{b}}{h^{2(N_f+4)-1}\,\Lambda^{2((8k+5)(N_f+4)-2)}} \qquad (5.4) \\
& + \ldots\,,
\end{aligned}
$$

where the dots stand for possible further terms that could be generated by instanton effects in the completely broken magnetic gauge group.

6. Conclusion

I have shown how the non-Abelian duality of $N = 1$ supersymmetric gauge theories discovered by Seiberg can be used to find new models that confine in the presence of an appropriate superpotential. This is a very interesting application of the proposed duality because it enables us to obtain non-perturbative results for the electric theory by a perturbative calculation in its magnetic dual. Confinement in the electric theory can be understood from the Higgs phase of the magnetic theory. The confining spectrum can easily be derived from the duality mappings of gauge invariant operators. For SU and SO gauge groups one also obtains the form of the confining superpotential by applying these mappings to the magnetic tree-level superpotential. To determine the full confining superpotential one needs to include instanton corrections in the completely broken magnetic gauge group. For Sp gauge groups the tree-level superpotential of the completely higgsed magnetic theory vanishes. In this case the whole magnetic superpotential is non-perturbative and therefore more difficult to obtain.

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