N=1 Super-Yang-Mills on the Lattice: Weak and Strong Coupling Limits

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Abstract: We present a general review about the N=1 supersymmetric $SU(N_c)$ Yang-Mills on the lattice focusing our attention on the quenched approximation in supersymmetry. Finally we analyse and discuss the recent results obtained at strong coupling and large N_c for the mesonic and fermionic propagators and spectrum.

1. Introduction

In this paper we consider the non-perturbative aspects of the strongly interacting supersymmetric gauge theories. [1]. In particular we concentrate our attention on the pure N=1 Supersymmetric Yang-Mills (SYM) theory.

The fundamental question of the supersymmetry (SUSY) breaking of the N=1 SYM was addressed in Refs.[2], [3]. According to the general argument of the Witten index [2] or the Veneziano-Yankielowicz (VY) low energy effective theory [3] one can conclude that:

- the spontaneous breaking of chiral symmetry occurs: the gluino condensate $\langle\bar{\lambda}\lambda\rangle \neq 0$
- the low energy supermultiplet is given by the scalar, pseudoscalar and fermion colourless bound states.
- SUSY is not broken
- no goldstone boson (or pion) associated with the chiral symmetry breaking is present, as the latter is broken by the anomaly.

A primary tool for a direct study of strongly coupled field theories is the space-time lattice regularization. In QCD the non perturbative phenomena such as confinement, chiral symmetry breaking and spectra can be studied on the lattice by using numerical Monte Carlo simulation in the weak coupling limit. Nevertheless, when the supersymmetric gauge theories are formulated on the lattice, the following problems arise

- The lattice breaks the Poincaré group and so there are no continuum SUSY algebra transformations.
- The naive lattice formulation breaks the balance n^0 *bosons*=n^0 *fermions* due to the extra poles in the gluino propagator. The Wilson term cures this problem, but generates a bare mass term for the gluinos which breaks explicitely both the chiral symmetry and supersymmetry.

The basic idea to circumvent these problems was proposed some time ago by Curci and Veneziano [4]. They suggested to leave that the lattice regularization spoil SUSY and chiral symmetries. Then the Ward Identities (WI) in the continuum limit should be recovered by using appropriate renormalized operators for the SUSY and chiral currents [4], such as the case of chiral symmetry in QCD [5]. The main result is that in the continuum limit the chiral limit defines the SUSY point and viceversa [4].

Recently two different collaborations [6]-[9] studied non perturbatively on the lattice the spec-

*Works made in collaboration with A. Donini and B. Gavela for *Quenched Supersymmetry* and A. González-Arroyo and C. Pena for *Strong Coupling Limit* trum of the N=1 SYM theory following the guide lines of Ref. [4]. In Ref. [8], because of the limitation deriving from the use of computing resources, the quenched approximation was used to study the spectrum. This approximation is implemented by neglecting the internal gluino loops or in other words by setting to unity the fermion determinant in the correlation functions of composite operators. In the SYM theory the quenched approximation badly breaks SUSY and it cannot be a good approximation on the basis of large N_c dominance since gluinos are in the adjoint representation of the colour group. However the quenched numerical results for the low energy spectrum show no deviations from the supersymmetry expectation within the statistical errors [8].

In connection with this result, in Ref. [10] a qualitative and quantitative understanding of the effects of quenching in N=1 SYM theory has been analysed in the framework of low energy effective theory. The result is that the splitting in the mass spectrum of the low energy supermultiplet is connected to the changing of the anomalies structure induced by quenching.

Recently it has been analysed the strong coupling limit of the N=1 SYM on the lattice in the large N_c limit [11]. The method used in Ref. [11] is based on the hopping parameter (k) expansion in terms of random walks which have been resummed for any value of the Wilson parameter (r) in the small hopping parameter region. Analytical results have been obtained for the propagator and spectra of the mesonic 2-gluino and fermionic 3-gluino operators in terms of r and k. Moreover the critical lines in k and r space, where the chiral symmetry and supersymmertry can be recovered in the continuum limit, have been analysed for any dimension [11].

The paper is organized as follows. In the next section we discuss the weak coupling limit on the lattice and summarize the approach of Ref. [4]. In section 3 we present the results on the SUSY spectrum induced by the quenched approximation by means of a low energy effective lagrangian. In section 4 we discuss the strong coupling limit at large N_c and give the main results for the correlation functions and spectra for the 2-gluino and 3-gluino operators. Finally in the last section we summarize our conclusions.

2. Weak coupling limit

The lattice chiral WI can be obtained by applying the chiral transformations to the $N=1$ SYM lattice. The result is given by [1]

$$\nabla_\mu A_\mu = 2m_0 P + X_A \tag{2.1}$$

where m_0 is the gluino bare mass. The operator X_A comes from the chiral symmetry-breaking due the lattice spacing and it vanishes in the continuum limit ($a \to 0$) since it is of order $O(a)$. Nevertheless when we take the matrix elements of Eq.(2.1) between external states, the contribution of the operator X_A could not vanish in the continuum limit. Indeed X_A can induce divergencies of order $O(1/a)$ that compensate the explicit factor a in X_A and spoil the WI in the continuum limit.

However it is possible to define a renormalized operator $\hat{X}_A$ whose matrix elements are still of order $O(a)$ [5],[4]. Due to the symmetries of the action, $\hat{X}_A$ can mix only with the following operators

$$\begin{aligned}\hat{X}_A &= X_A + (Z_A - 1)\nabla_\mu A_\mu - \tilde{Z}_A \nabla_\mu A_\mu \\ &- Z_Q P_{\mu\nu}\tilde{P}_{\mu\nu} + 2\bar{m}P \end{aligned}\tag{2.2}$$

where $P_{\mu\nu}$ is the lattice transcription of the field strength $F_{\mu\nu}$ and $\tilde{P}_{\mu\nu}$ is the dual. Finally, by inserting Eq.(2.2) inside Eq.(2.1), we obtain the renormalized chiral WI which has the good continuum limit [4]

$$\nabla_\mu \hat{A}_\mu = 2(m_0 - \bar{m})Z_P^{-1}\hat{P} + \hat{Q} + O(a) \tag{2.3}$$

provided that

$$\begin{aligned}\hat{A}_\mu &= Z_A(g_0)A_\mu, \quad \hat{P} = Z_P(g_0)P \\ \hat{Q} &= Z_Q(g_0)P_{\mu\nu}\tilde{P}_{\mu\nu} + \tilde{Z}_A(g_0)\nabla_\mu A_\mu\end{aligned}$$

where the $\hat{Q}$ term reproduces the usual chiral anomaly. It is important to note that the Eq.(2.3) has the same form as the continuum one provided that we identify on the lattice the renormalized gluino mass $\hat{m}_\lambda$ as follows [4]

$$\hat{m}_\lambda = (m_0 - \bar{m})Z_P^{-1}. \tag{2.4}$$

[1] the expression for the chiral currents A_μ and the pseudoscalar density P, together with the N=1 Super-Yang-Mills action on the lattice, can be found in Refs. [4], [8]

Finally the chiral symmetry on the lattice is recovered by tuning m_0 to a critical value m_0^{crit}

$$m_0^{crit} - \bar{m}_\lambda(m_0^{crit}, g_0, r) = 0 \qquad (2.5)$$

where the $\bar{m}$ term depends in general on m_0, g_0 and r. Note that Z_A is of order $Z_A = 1 + O(g_0)$ and in the continuum limit ($g_0 \to 0$) we obtain $Z_A = 1$, as expected by the current non renormalization theorem.

In the case of SUSY WI we have an analogous result of Eq.(2.1)

$$\nabla_\mu S_\mu = 2m_0\chi + X_S$$

where now $S_\mu(x)$ is the bare SUSY current on the lattice and $\chi = 1/2 P^a_{\mu\nu}\sigma_{\mu\nu}\lambda^a$ where the λ^a is the gluino field. The expressions for X_S and S_μ can be found in Ref.[4]. The operator X_S, which is of order $O(a)$, spoil the continuum SUSY WI, like X_A in the case of chiral symmetry.
By applying the same method used for the chiral WI one obtains the following renormalized SUSY WI [4]

$$\nabla_\mu \hat{S}_\mu = 2(m_0 - \bar{m})Z_\chi^{-1}\hat{\chi} + O(a)$$

where

$$\hat{\chi} = Z_\chi \chi, \quad \hat{S}_\mu = Z_S S_\mu + Z_T T_\mu$$
$$T_\mu(x) \equiv \gamma_\nu P^a_{\nu\mu}(x)\lambda^a(x)$$

This result coincides with the corresponding one in the continuum, provided that the renormalized gluino mass $\hat{m}_\lambda$ is identified with

$$\hat{m}_\lambda = (m_0 - \bar{m})Z_\chi^{-1} \qquad (2.6)$$

Then the relevant conclusion is that in the continuum limit the chiral limit of Eq.(2.4) defines the SUSY point and viceversa [4].

The present numerical analysis implement these guidelines for studing the spectrum of the N=1 SYM with SU(2) gauge group. According to Veneziano-Yankielowicz [3], the low-energy SUSY supermultiplet is given by the following colourless composite fields

$$S(x) = \bar{\lambda}^a(x)\lambda^a(x), \qquad P(x) = \bar{\lambda}^a(x)\gamma_5\lambda^a(x),$$
$$\chi(x) = G^a_{\mu\nu}(x)\sigma_{\mu\nu}\lambda^a(x) \qquad (2.7)$$

where the sum on the colours is assumed. As usual the masses are extracted from the large Euclidean-time behaviour of the lattice correlation functions for the corresponding operators in Eq.(2.7).

In Ref. [8] the quenched approximation is used in which dynamical gluino loops are neglected, or the fermion determinant $\det(K)$ is setted to 1 in the correlation functions.[2] In QCD the quenched approximation is a good one because the $\det(K) = O(1/N_c)$ in the large N_c limit. In the present case the gluinos are in the adjoint representation of the colour group (like the gluons) and by using naive arguments based on perturbation theory one should expect that the quenched approximation badly breaks SUSY. [3] Nevertheless the quenched results of Ref. [8] show a dynamical chiral symmetry breaking and a quite degenerate spectrum in low energy supermultiplet. Moreover in Ref.[8] the OZI approximation has been used. In this approximation the diagrams which contribute to the chiral anomaly are neglected and by using general arguments [4] one should expect a massless pseudo-goldstone boson or pion in the spectrum.

In the next section we will show how to implement the quenching in the fundamental theory. Then we will give an estimation of the systematic error induced by the quenching on the spectrum, by means of a low energy lagrangian approach.

3. Quenched Supersymmetry

In the continuum theory the on-shell action of the N=1 SYM theory is given by

$$S_{SYM} = \int d^4x \left\{ -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{i}{2}\bar{\lambda}^a\gamma^\mu D^{ab}_\mu\lambda^b \right\} \qquad (3.1)$$

where D^a_μ is the covariant derivative acting on the gluino field λ^a. At the classical level this action is $U(1)_A$ invariant, as well as scale invariant. At the quantum level these symmetries are broken by the corresponding anomalies and the anomalous WI are given by

$$\partial^\mu J_\mu = -c(g)F^a_{\mu\nu}\tilde{F}^{a\mu\nu}, \quad \Theta^\mu_\mu = c(g)F^a_{\mu\nu}F^{a\mu\nu}$$

[2] Really in the present case one has the Pfaffian instead of $\det(K)$ since the gluinos are Majorana fields.

[3] These arguments, based on perturbation theory, do not apply in the strong coupling limit (see section 4), where indeed this approximation is exact at large N_c

$$\gamma^\mu S_\mu = 2c(g)\sigma_{\mu\nu}F^a_{\mu\nu}\lambda^a \tag{3.2}$$

where J_μ and S_μ are the chiral and SUSY currents respectivley, Θ^ν_μ the energy momentum tensor, with $c(g) = \beta(g)/2g$ and $\beta(g)$ is the beta function of N=1 SYM. The above two anomalies and the SUSY trace anomaly belong to the same supermultiplet.

The corresponding low energy theory of VY [3] was obtained by considering the chiral superfield S whose components are given by a complex scalar field ϕ, a Dirac fermion χ and complex auxiliary field M. In terms of the fundamental fields, they are described by [3]

$$\begin{aligned} \phi &= c(g)\bar\lambda^a_R\lambda^a_L\,, \\ \chi &= \frac{ic(g)}{2}\sigma_{\mu\nu}F^{a\mu\nu}\lambda^a\,, \\ M &= -\frac{c(g)}{2}\left(F^a_{\mu\nu}F^{a\mu\nu} + iF^a_{\mu\nu}\tilde F^{a\mu\nu}\right)\,, \end{aligned} \tag{3.3}$$

where $c(g)$ is the same factor appearing in the anomalies in Eq.(3.2).

The expression of the VY action in terms of the superfield S is given by [3]

$$S_{VY} = \int d^4x\left\{\frac{9}{\alpha}(S^\dagger S)^{1/3}_D + \left[\frac{1}{3}\left(S\log(\frac{S}{\mu^3}) - S\right)_F + h.c.\right]\right\}$$

where α and μ are two free parameters. Note that the request to reproduce the correct anomalies of the fundamental action in Eq.(3.1) fixes completely the form of the superpotential. The spectrum can be easily analysed by looking at the minimum of the scalar potential (V_{VY}) in the exponential representation for the scalar field $\phi \equiv \rho e^{i\theta}$

$$V_{VY} = \frac{\alpha^3}{81}\frac{\rho^4}{4}\left[\log^2(\frac{\sqrt{\alpha}}{3\sqrt{2}}\frac{\rho}{\mu}) + \theta^2\right].$$

Then the following conclusions are drawn [3]

- $\min(V_{VY})$ is obtained at a non-zero value of ρ: spontaneous chiral symmetry breaking occurs.
- The would-be goldstone boson, θ, is not a massless field: the anomaly term in the lagrangian explicitly breaks the chiral symmetry, providing a mass scale for the supermultiplet.
- SUSY is unbroken: mass degeneracy $m_\theta = m_\rho = m_\chi = \frac{1}{3}\alpha\mu$.

Now we explain how to implement the quenching in the N=1 SYM theory [10]. We extend the method proposed by Bernard and Golterman [12] for quenched QCD to the Majorana fermions in the adjoint representation of the colour group. In order to cancel the fermion determinant, we introduce a ghost Majorana field η^a which has the same quantum numbers as the gluino λ^a, but "wrong" (bosonic) spin-statistics. Then the quenched action S^q_{SYM} is given by

$$S^q_{SYM} = \int d^4x\left\{-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{i}{2}\bar\lambda^a\gamma^\mu D^{ab}_\mu\lambda^b + \frac{i}{2}\bar\eta^a(i\gamma^\mu\gamma_5)D^{ab}_\mu\eta^b\right\}$$

Note that (due to the wrong statistic and Majorana nature) $\bar\eta^a\gamma^\mu D^{ab}_\mu\eta^b = 0$ (up to total derivatives), in the same way as $\bar\lambda^a\gamma^\mu\gamma_5 D^{ab}_\mu\lambda^b = 0$. It is important to stress that the S^q_{SYM} is no longer supersymmetric, but it acquires a new $U(1\mid 1)$ symmetry [10]. Note that S^q_{SYM} violates unitarity due to the ghost η field.[4]

The $U(1\mid 1)$ group is a Z_2 graded Lie group with both bosonic and fermionic generators (the supersymmetric algebra itself obeys a Z_2 graded Lie group) [13]. In a more compact form:

$$S^q_{SYM} = \int d^4x\left\{-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\bar Q^a_R\gamma^\mu D^{ab}_\mu Q^b_R\right\}$$

where Q is the doublet $Q^a = (\lambda^a, \eta^a)$. Then S^q_{SYM} is invariant under chiral $U(1\mid 1)$ transformations, defined as follows:

$$\begin{aligned} Q_R &\to UQ_R = \exp\left\{i\frac{\alpha_i\sigma^i}{2}\right\}Q_R\,, \\ Q_L &\to U^\dagger Q_L \end{aligned} \tag{3.4}$$

where $U^\dagger U = I$ and the $\sigma_{i=1,2,3}$ matrices, which are the usual Pauli matrices (with σ_0 the unity matrix), belong to the algebra of $U(1\mid 1)$, where σ_0, σ_3 and σ_1, σ_2 correspond to the bosonic and fermionic generators respectively. The supertrace Str (invariant under $U(1\mid 1)$) is defined as

$$Str\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a - d\,,$$

[4]This is a consequence of the fact that the quenched approximation violates unitarity

where, in general, a, d are complex numbers and b, c complex Grassman numbers. From the transformations in Eq.(3.4) we see that four currents are associated to the $U(1 \mid 1)$ symmetry, which are $J^i_\mu = \bar{Q}^a_R \sigma^i \gamma^\mu Q^a_R$ or in components [10]

$$\begin{aligned} J^0_\mu &= \frac{1}{2}(i\bar{\lambda}^a \gamma_\mu \gamma_5 \lambda^a + \bar{\eta}^a \gamma_\mu \eta^a) \\ J^+_\mu &= \bar{\lambda}^a_R \gamma_\mu \eta^a_R, \quad J^-_\mu = \bar{\eta}^a_R \gamma_\mu \lambda^a_R \\ J^3_\mu &= \frac{1}{2}(i\bar{\lambda}^a \gamma_\mu \gamma_5 \lambda^a - \bar{\eta}^a \gamma_\mu \eta^a) \end{aligned}$$

From the bosonic statistic of the ghost fields η^a it follows that only J^3_μ is anomalous. Indeed for the J^0_μ anomaly the fermionic-statistic loop versus the bosonic one cancels exactly, while for the J^3_μ case these are summed up. As for the trace anomaly it can be shown that the ghost contribution to the trace of the tensor-energy momentum exactly cancel the contribution of the gluino loop.

In order to generalize the VY effective lagrangian we introduce new composite fields which have particular transformation properties under $U(1 \mid 1)$. In terms of the gluino and ghost fields these are given by

$$\begin{aligned} \hat{\phi} &\equiv \sigma^i \hat{\phi}^i, \quad \hat{\phi}^i = c(g)\bar{Q}^a_R \sigma^i Q^a_L, \\ \hat{\chi} &= \frac{ic(g)}{2} \sigma_{\mu\nu} F^{a\mu\nu} Q^a \end{aligned} \tag{3.5}$$

with transformation properties

$$\hat{\phi} \to U \hat{\phi} U, \quad \hat{\chi}_R \to U \hat{\chi}_R \tag{3.6}$$

Then we look for the most general low energy effective lagrangian $\mathcal{L}$ in terms of the fields in Eq.(3.5). This lagrangian can be decomposed as follows [10]

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{int} + \mathcal{L}_{anom}$$

where $\mathcal{L}_{kin}$, $\mathcal{L}_{int}$ are invariant under chiral $U(1 \mid 1)$ and naive scale transformations. The $\mathcal{L}_{anom}$ is not invariant, but it is completely fixed by requiring to reproduce the anomalies of the quenched fundamental theory. Moreover the anomalous $U_{\sigma_3}(1 \mid 1)$ transformations breaks $U(1 \mid 1)$ as

$$U(1 \mid 1) \to Z_{4N_c} \times SU(1 \mid 1)$$

We do not give here the expression for the lagrangian $\mathcal{L}$ that, however, can be found in Ref. [10]. We only point out that the coefficients of the $U(1 \mid 1)$ invariant terms in $\mathcal{L}_{kin}$ and $\mathcal{L}_{int}$ are fixed by requiring that, in the classical unquenched limit ($\eta^a \to 0$), this lagrangian approaches continuosly to the corresponding one of VY, since supersymmetry should be recovered in this limit. The only term which is non-analytic in the unquenched limit and is responsible for the mass splitting is the anomalous term $\mathcal{L}_{anom}$, as explained in Ref. [10].

Now we look at the spectrum in the exponential representation $\hat{\phi} = \rho \exp(i\theta^i \sigma_i) \equiv \rho\hat{\Sigma}$. Note that, in terms of the original field θ, we have $\theta_3 = \theta - \tilde{\theta}$ and $\theta_0 = \theta + \tilde{\theta}$, where $\tilde{\theta}$ is a pure η ghost condensate and it goes to zero in the unquenched limit.

The mass spectrum in terms of the original VY fields is obtained by using the technique explained in Ref. [12] and we find

$$m_\rho = \frac{\beta'}{\beta} m_\chi, \quad m_\theta = (1+1) m_\chi \tag{3.7}$$

where $\beta'(g)$ is the one-loop β function of the pure Yang-Mills theory. This spectrum should be compared with $m_\chi = m_\sigma = m_\theta$ in the unquenched theory. Note that the splitting in the mass spectrum of Eq.(3.7) provides an estimation of the error induced by the quenched approximation. Now we summarize the main conclusion of this analysis [10]

- The mass splitting of the VY supermultiplet results from the non-analiticity of the anomaly structure induced by the ghost field.
- The numerical result obtained in Ref.[8] for the ratio $(m_\rho/m_\chi)_{lat} = 1.1(3)$ is in fair agreement with our theoretical expectation $(m_\rho/m_\chi)_{th} = 11/9 = 1.22$ for $SU(2)$.

4. Strong coupling limit

The lattice strong coupling expansion is a very powerful analytical probe in order to study the critical behaviours of lattice gauge theories and also to test qualitatively their continuum properties. The strong coupling expansion technique has been extensively used in pure Yang-Mills theory and in QCD, often combined also with the large N_c expansion. We recently investigated the strong coupling limit at large N_c of a N=1 SYM theory and now we present the main results and conclusions of this work [11].

The usual computational frameworks of strong coupling expansion can be summarized as follows

- Effective actions [14]-[16]: The Wilson-Dirac lattice action is considered at large N_c and small $\beta = 1/g_0^2$. The large N_c expansion can be recognized as a saddle-point expansion of the gauge functional integral, previously simplified by the $\beta \to 0$ limit.

- Path resummation [15]-[19]: The fermion matrix $\mathbf{M}$ is inverted by using the standard hopping parameter expansion, which expresses the propagator $\mathbf{M^{-1}}$ as a sum over paths on the lattice.

Our technique is based on the path resummation formulas which are valid irrespective of the representation in which the matter lies and for general r. Moreover we keep r arbitrary since it allows the possibility of searching for multicritical points: indeed more freedom in the parameter space is necessary in order to search for the simultaneous restoration of supersymmetry and chiral symmetry. Now we present the general formalism.

The SUSY Yang-Mills action on the lattice can be formally written as

$$S = \beta S_g + \frac{1}{2}\Psi_i\Psi_j M_{ij} \quad ,$$

where βS_g is the pure gauge part and Ψ_i is a Grassman variable representing the field of a Majorana fermion. The matrix $\tilde{M}$ must be antisymmetric and its form is given by

$$\tilde{M} = CM, \quad M = \mathbf{I} - \sum_{\alpha\in I}\mathbf{\Delta}_\alpha$$
$$(\mathbf{\Delta}_\alpha)_{ij} = k\delta_{m\, n+V(\alpha)}\, U_\alpha^{ab}(n)\,(r\mathbf{I}-\gamma_\alpha)_{AB}$$

where U is the gauge link variable, $\mathbf{I}$ and C are the unity and charge conjugation matrix respectively and κ is the hopping parameter. With the indices i and j we simbolically indicate $i = (n, a, A)$, where n, a and A run on the lattice points, the indices of the $SU(N_c)$ adjoint representation, and the Dirac indices respectively.

Now we will concentrate upon the gauge invariant operators of the form:

$$\mathbf{O}_i(x) = \Psi_{A_1}^{a_1}(x)\ldots\Psi_{A_p}^{a_p}(x)\,(\mathcal{S}_i)_{A_1\ldots A_p}\,\mathcal{C}_i^{a_1\ldots a_p} \tag{4.1}$$

where $\mathcal{C}_i^{a_1\ldots a_p}$ is an invariant color tensor and $(\mathcal{S}_i)_{A_1\ldots A_p}$ a spin tensor. For $p=2$ a basis for S_i is the Clifford algebra basis in d dimension.
We are interested in computing the following quantities at strong coupling

$$\langle\mathbf{O}_i(x)\rangle, \qquad G_{ij}(x-y) \equiv \langle\mathbf{O}_i(x)\mathbf{O}_j(y)\rangle \tag{4.2}$$

where as usual the $\langle\ \rangle$ means the vacuum expectation value.

We will be able to accomplish this goal for $\beta = 0$ and in the large N_c limit, and the corrections to the formulas in powers of β and $\frac{1}{N_c}$ are in principle feasible. When the fermion are integrated out we obtain

$$\prod_i\left(\int d\Psi_i\right)\exp\{-\frac{1}{2}\Psi_i\Psi_j\,\mathbf{M}_{ij} + J_i\Psi_i\} =$$
$$Pf(\mathbf{M})\exp\{-\frac{1}{2}J_iJ_j(\mathcal{M}^{-1}C^{-1})_{ij}\} \tag{4.3}$$

where $Pf(\mathbf{M})$ stands for the Pfaffian of the matrix $\mathbf{M}$. [5] Now the next step is to expand the previous quantities as a sum over all the possible paths γ going from x to y

$$(\mathcal{M}^{-1}(x,y))_{AB}^{ab} = \sum_{\gamma\in\mathcal{S}(x\to y)} W^{ab}(\gamma)\,\Gamma_{AB}(\gamma)$$
$$Pf(\mathbf{M}) = \exp\{\frac{1}{2}\sum_{x\in\mathcal{L}}\sum_{L=1}^{\infty}\sum_{\gamma\in\mathcal{S}_L(x\to x)}$$
$$\frac{1}{L}\,Tr(W(\gamma))\,Tr(\Gamma(\gamma))\}$$

where $W(\gamma)$ is the path ordered product (along the path γ) of the gauge field link variables $U(x)_\alpha^{ab}$ and $\Gamma(\gamma)$ denotes the appropriate product of the spin matrices:

$$\Gamma(\gamma \equiv (x,\vec{\alpha})) = \kappa^L(r-\gamma_{\alpha_1})\cdots(r-\gamma_{\alpha_L})$$

[5] The square of the Pfaffian is the determinant, and up to a sign

$$Pf(\mathbf{M}) = \sqrt{\det(C)\det(\mathcal{M})} = \exp\{\frac{1}{2}Tr(\log(\mathcal{M}))\}$$

We checked that $Pf(\mathbf{M})$ is always positive provided that $|\kappa| < \frac{1}{2d\,(|r|+1)}$

where L is the lengh of the path in lattice space unity and $S_L(x \to y)$ in the sum indicates the sum over the paths of the lenght L which goes from x to y. Now we present the main results obtained by using the results of Ref.[20] for the $SU(N_c)$ group integration for gauge fields in the adjoint representation at large N_c :

- The quenched approximation and the OZI approximations turn out to be exact in the large N_c limit.
- The results obtained at $\beta_{adjoint} = 0$ are exact in the large N_c limit: the corrections of $O(\beta)$ are subleading in the large N_c limit.

Now we give the main formulas (at large N_c) for the condensates $\langle O_i(x)\rangle$ and propagators of the 2-gluino operators $G_{ij}(x)$, defined in Eq.(4.1) (with $p = 2$), obtained after resumming over paths:

$$\begin{aligned} \langle \mathbf{O_i}(x)\rangle &= R_1(\xi) Tr(\hat{S}_i) \\ G_{ij}(x) &= R_2(\xi) \prod_\mu (\int \frac{d\varphi_\mu}{2\pi}) \, e^{\imath \varphi\cdot x} \\ &\times \langle \mathcal{S}_i | \, \widetilde{C}_2^{-1} \, [\Theta_2(\xi)\mathbf{I} - \tilde{\mathbf{A}}_2(\varphi)]^{-1} |\mathcal{S}_j\rangle \\ \tilde{\mathbf{A}}_2(\varphi) &\equiv \kappa^2 \sum_{\alpha\in I} e^{\imath\varphi_\alpha} (r-\gamma_\alpha)\otimes(r-\gamma_\alpha) \end{aligned} \quad (4.4)$$

where $\widetilde{C}_2 \equiv C^{-1}\otimes C^{-1}$ with C the charge conjugation matrix, ξ is a function of r, k given in Ref.[11], and S_i are matrices of the Clifford algebra basis in d-dimension. The expressions for the functions $R_2(x)$ and $\Theta_2(x)$ can be found in Ref.[11]. We have analogous expressions for the p-gluino propagators provided that the function R_2, the vectors $|S_i\rangle$ and the matrix $\tilde{\mathbf{A}}_2(\varphi)$, appearing in Eq.(4.4), are substituted with the corresponding ones for the p-gluino operators.

The main difficulty in order to calculate the propagators is given by the calculation of the inverse matrix $\Theta_p(\xi)\mathbf{I} - \tilde{\mathbf{A}}_{\rm p}(\varphi)$ for general p-gluino operators. This goal has been achieved for the 2-gluino operators in any dimension by means of the gamma-fermions techniques developed in Ref.[11]. In general for the p-gluino sector (with $p > 2$) we have been able to invert this matrix only in the particular limit where the spectrum of the p-gluino propagators becomes degenerate.

In general the procedure to obtain the masses can be summarized as follows: extract the eigenvalues of the matrix $\Theta_p(\xi)\mathbf{I} - \tilde{\mathbf{A}}_{\rm p}(\vec{\varphi} = \vec{0})$, which are functions of the temporal momentum φ_0. Then determine φ_0^{pole} which is the (complex) value of φ_0 for which the eigenvalues vanishes. Finally the lattice masses are given by

$$M = -log(|e^{\imath\varphi_0^{pole}}|). \quad (4.5)$$

Note that the lattice masses are dimensionless quantities and depend only on k and r. The physical masses are proportional to M/a and so the states whose lattice mass vanish at the critical line, are the states that survive this continuum limit.

Now we present below the main results for the spectrum of the 2-gluino and 3-gluino operators in d=4.

- Chiral symmetry is spontaneusly broken.
- The pseudoscalar is the lightest states and the critical lines where the scalar or the lightest fermion become massless are outside of the physical region in the (k, r) plane.
- All the meson masses become degenerate only for $r \to \infty$ and $\kappa \to 0$ with the product $\kappa r = fixed$. In particular all the mesons become massless in the limit where $\kappa r = \frac{1}{2\sqrt{2d-1}}$.
- In this limit : the lightest fermion mass can not be degenerate with the lightest meson sector and for $p > q$ any mass in the p-gluino sector (the fermions have p odd) is higher than any other in the q-gluino sector in this limit.

From these results we argue that there are no points in the (k, r) plane giving a possible candidate for a supersymmetric continuum limit.

5. Conclusions

In order to estimate the error induced by the quenched simulations on the spectrum, we implemented the quenching in the fundamental theory by introducing a ghost field. Although SUSY is lost upon quenching, it turns out that a new

$U(1 \mid 1)$ symmetry arises, explicitly broken by the chiral anomaly to $Z_{4N_c} \times SU(1 \mid 1)$. Then we carried out this new symmetry and the corresponding anomalies in a low energy lagrangian scheme. The result is that the anomaly structure entails a controlable splitting of the VY multiplet by giving the scalar mass 20% heavier then the fermionic one. These results are in fair agreement with the numerical quenched ones within the statistical errors and provides a first estimate of the systematic error associated to the quenching in lattice SUSY computations.

From the side of the strong coupling limit at the large N_c : we used the hopping parameter (k) expansion in terms of random walks resummed for any value of the Wilson parameter r and close to the origin in k. We found exact analytical results for the condensates, propagators and spectrum in the large N_c limit, for arbitrary dimensions and general r. By analysing these results our main conclusion is that the quenched and the usual OZI approximations are exact at $\beta_{adjoint} = 0$ and in the large N_c limit. Moreover we proved that in the strong coupling regime there are no critical lines or points in the (k, r) plane giving a possible candidate for a supersymmetric continuum limit, at least in the validity region of the hopping parameter expansion close to the origin.

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