The effects of the Kaluza Klein states on the gauge couplings unification

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ABSTRACT: We consider the full two-loop effects of the Kaluza-Klein states on the running of the canonical gauge couplings in models with large extra spatial dimensions. Part of the contribution of the Kaluza-Klein states is perturbatively exact, while that induced by the wavefunction renormalisation of the low-energy spectrum is valid in two-loop order. From the gauge couplings unification condition, we estimate the corrections induced on $\alpha_3(M_z)$ by towers of Kaluza Klein states associated with some states of the low-energy spectrum assumed to be that of the Minimal Supersymmetric Standard Model (MSSM).

1. Introduction

The possibility of unification of the gauge couplings in supersymmetric (string-inspired) models with *large* extra spatial dimensions has recently been investigated in some detail [1, 2, 3, 4]. This was motivated by the renewed interest [1] in the phenomenology of the models with large additional spatial dimensions [5, 6, 7]. Additional space dimensions to the four-dimensional space-time are common in string theories and their presence has strong theoretical motivations and phenomenological implications for the high energy physics. String models are usually formulated in higher dimensional spaces compactified to the four dimensional space-time. The presence of spatial (compactified) dimensions imposes that $\Phi(x, y)$ and $\Phi(x, y + 2\pi R_o)$ be equal where $\Phi(x, y)$ is a field of our model, x denotes the four dimensional space-time coordinates and y stands for the additional (compactified) spatial dimensions which we assume to be all of equal radius, R_o. The coefficients (operators) of the Fourier expanded field Φ with respect to the new spatial coordinates y represent the Kaluza-Klein (KK) modes which we denote by $\Phi^{(n)}(x)$ for further reference. As we increase the energy scale the new dimensions will open up, which from the point of view of the four-dimensional field theory (adopted throughout this work) corresponds to the appearance in the spectrum of additional heavy states (excited modes). The so-called "zero-modes", $\Phi^{(0)}(x)$, are usually identified with (some) states of the low-energy spectrum which in the present work will be considered to be those of the MSSM. The mass of the excited modes will be of the order of the inverse size of the extra (compact) spatial dimensions considered, $\mu_0 \equiv 1/R_o$,

$$\mu_{\vec{n}}^{o2} = \mu_0^2(n_1^2 + n_2^2 + \cdots n_\delta^2) + m_0^2 \qquad (1.1)$$

where δ is the number of additional dimensions, $\vec{n} \equiv (n_1, n_2, \cdots, n_\delta)$, the integers n_i are Kaluza-Klein excitation numbers with integer values and m_0 denotes the mass of "zero-modes" which will be neglected in this work since m_0 will be much smaller than μ_0.

Additional symmetries imposed on the wavefunctions $\Phi(x, y)$ and the exact details of the compactification procedure can result in very different scenarios for the assignment of the excited modes to the low energy spectrum. We adopt the orbifold compactification procedure outlined in [1] and, although the assignment of excited modes is model dependent, in general these additional states may exist for the gauge sector, for the Higgs fields and even for the fermions [1]. As we will discuss later this leads to different phenomenological predictions, due to the different beta function coefficients such choices bring in.

The purpose of the present work is to present the two-loop Renormalisation Group Evolution (RGE) equations in models with (large) extra dimensions and to give some insights into the phenomenology of the models with Kaluza-Klein momentum states which may lower significantly the unification/string scale through their effects on the RGE. The low-energy spectrum is assumed to be that of the Minimal Supersymmetric Standard Model (MSSM) with an additional set of Kaluza-Klein states whose "zero-level" modes are identified with the corresponding MSSM states. Our results are subject to further (Type I/I') string thresholds at the high scale.

We evaluated[1] the *exact* perturbative contribution of the Kaluza-Klein states to the "running" of the gauge couplings with the scale[2]. Our approach to quantifying these effects is different from that of [1] where this is done in a one-loop calculation where the thresholds induced by the Kaluza-Klein modes as well as the higher order perturbative contributions to the *canonical* gauge coupling[3] are neglected. We find similar results to those of [1] for the power-law running [1, 5, 7] of the couplings, induced by the (large number of) Kaluza-Klein states. We also included in the present analysis the two-loop effects *above* the decompactification scale μ_0, due to one-loop wavefunction renormalisation of the low-energy spectrum assumed to be that of the MSSM. Such effects are present because the anomalous dimensions of the matter fields of the low-energy spectrum "feel" (in one-loop order) the presence of "zero-modes" (identified with the MSSM states, hence giving the usual MSSM two-loop contribution) and that of the excited (heavy) modes described by the interaction Lagrangian between the MSSM states and the (excited) Kaluza-Klein states. The one-loop corrected wavefunctions of the MSSM states induce the two-loop contribution to the running of the gauge couplings. Such two-loop effects (above μ_o scale) were not included in the previous analyses of this topic [1, 2].

The two-loop RGE equations for the gauge couplings are applied to two models we consider to investigate. The first case will be that of the MSSM with excited modes for the gauge sector and for the two Higgses. The Kaluza Klein state $\mathcal{H}^{(n)}$ for any of the two MSSM Higgs fields will be a massive chiral $\mathcal{N}=1$ multiplet $\mathcal{H}^{(n)}=(H^{(n)},\psi_H^{(n)})$. For the gauge boson sector, a massive gauge boson mode (i.e. a massive $\mathcal{N}=1$ vector supermultiplet) which we call $V^{(n)}$ is represented by an $\mathcal{N}=1$ massless vector supermultiplet $\mathcal{A}^{(n)}=(A^{(n)},\lambda^{(n)})$ (which for $n=0$ will be identified with MSSM gauge bosons) and a $\mathcal{N}=1$ chiral supermultiplet in the adjoint representation, $\tilde{A}^{(n)}=(\phi^{(n)},\psi^{(n)})$. The supermultiplet $\tilde{A}^{(0)}$ is not present as a low-energy state of the MSSM and this is ensured by requiring the $\tilde{A}(x,y)$ field be odd under the Z_2 orbifold compactification procedure and thus has no zero mode component[4] [1]. This completes the description of the assignment of the Kaluza-Klein modes to the low-energy spectrum for this case. For the second case we will consider that only the gauge sector has additional towers of Kaluza-Klein modes while the Higgses and the fermions live on the boundary without any (non-zero) excitations.

The above description of the Kaluza-Klein states for the models considered fixes the contribution of each excited mode to the one-loop

[1] up to a technical approximation in which a discrete sum is replaced by an integral

[2] The detailed derivation of the full two-loop RGE flow for the gauge couplings will be presented elsewhere [8].

[3] Note that for field theory calculations one uses the canonical gauge coupling, as opposed to the *holomorphic* gauge coupling (present in string theory calculations) defined as the coefficient of the gauge kinetic term. For a full discussion and a relationship between the two couplings see [9, 10, 11].

[4] The initial tower of KK states is split into two towers of given parity (odd and even with respect to y) with half the number of states each plus a zero-mode state. For the case when the extra dimensions are compactified on S^1/Z_2 at least, an extra factor $\sqrt{2}$ comes with any coupling of excited modes as compared to the similar coupling of zero-modes, due to the normalisation of the KK basis. In the RGE equations the two effects of reducing the number of states and of increasing α_i by $(\sqrt{2})^2$ will cancel against each other.

beta function coefficients and this will enable us to compute their effects on the evolution of the gauge couplings. Further information is needed for a full two-loop calculation, such as the interaction Lagrangian between zero-modes and excited modes; in the absence of a fully derived MSSM from a higher dimensional string model for the models to be investigated here we will make some generic assumptions about the aforementioned interaction. Moreover, string thresholds at the high scale (not known) may affect our conclusions and an accurate calculation should consider them as well. The two cases we consider may not be the most relevant for phenomenology and there could be difficulties with deriving them from a generic string model, but the method we present is more general and could be used for other cases, too.

2. The evolution equations in models with extra spatial dimensions

In this section we present the effects of the KK states on the running of the gauge couplings below the scale where a more fundamental theory (of strings) will apply. There are good reasons [8] to think that a field theory approach to quantifying the effects of the Kaluza-Klein states using standard RGE techniques is appropriate for those states situated well below the string scale. In our (four-dimensional) field theory approach, we therefore truncate the (*infinite*) tower of KK states to a finite number of states situated below this scale, and under these circumstances the use of the Renormalisation Group Evolution (RGE) techniques is indeed justified. The result (eq. 2.1) is derived using the the Novikov-Shifman-Vainshtein-Zakharov "beta function" [9, 10, 11]. The "running" of the gauge couplings due to Kaluza-Klein states alone is actually perturbatively *exact*, it includes the sum of their threshold effects and their mass renormalisation effects (to all orders in perturbation theory)) due to any additional gauge and/or Yukawa interactions. In fact the two effects "conspire" together to give a dependence of the RGE equations on the bare mass (eq.(2.1) only [8] of the KK states. The origin of this effect can be traced to the fact that the *holomorphic* gauge coupling runs at one-loop level only in $\mathcal{N}=2$ theories. However, the contribution to the RGE due to the MSSM matter wave-function renormalisation is valid in two-loop order only, therefore the overall effect, eq.(2.1), is valid in this approximation only. Following this idea, we showed [8] that the overall two-loop value of the gauge couplings just below the scale μ_o has the following structure

$$
\begin{aligned}
\alpha_i^{-1}(\mu_0) &= \alpha_i^{-1}(\Lambda) \\
&+ \frac{\tilde{b}_i}{2\pi}\left\{\frac{\pi^{\delta/2}}{\delta\,\Gamma(1+\delta/2)}\left[\left(\frac{\Lambda}{\mu_0}\right)^{\delta}-1\right]-\ln\frac{\Lambda}{\mu_0}\right\} \\
&+ \frac{b_i}{2\pi}\ln\frac{\Lambda}{\mu_0}+\frac{T^i(G)}{2\pi}\ln\frac{\alpha_\Lambda}{\alpha_i(\mu_0)} \\
&+ \frac{1}{4\pi}\sum_{k=1}^{3}\left(b_{ik}-2\,b_k\,T_k(G)\delta_{ik}\right)\int_{\mu_0}^{\Lambda}\frac{d\ln\mu}{2\pi}\,\alpha_k(\mu) \\
&+ \frac{1}{4\pi}\sum_{k=1}^{3}\left(b_{ik}-2\,b_k\,T_k(G)\delta_{ik}-b_{ik}^H\right) \\
&\times \int_{\mu_0}^{\Lambda}\frac{d\ln\mu}{2\pi}\,\alpha_k(\mu)\,f(\mu,\mu_0;\delta) \\
&+ \frac{1}{4\pi}\sum_{k=1}^{3}b_{ik}^H\int_{\mu_0}^{\Lambda}\frac{d\ln\mu}{2\pi}\,\alpha_k(\mu)\,g(\mu,\mu_0;\delta) \qquad (2.1)
\end{aligned}
$$

We used the following notations: Λ represents the high scale of the model where all couplings meet, $\alpha_i(\Lambda)=\alpha_\Lambda$, $\tilde{b}_i$ is the sum over the Dynkin indices for the distinct representations (of the gauge group) that have a tower of Kaluza-Klein states, $\mu_0\equiv 1/R_o$, b_i is the one-loop MSSM beta function coefficient, ($b_i=33/5,1,-3$). The one-loop gauge wavefunction renormalisation effects[5] are accounted for by the last term in the third line of eq.(2.1). This term is perturbatively exact [9], and hence includes all higher (than one-loop) orders as well. Further, b_{ik} is the two loop MSSM-only beta function, so the term with the coefficient $(b_{ik}-2b_kT_k\delta_{ik})$ accounts for the "standard" two-loop MSSM effects to α_i^{-1} due to the wavefunction renormalisation (induced by the MSSM gauge bosons) of the three MSSM generations and Higgs fields; b_{ik}^H is the two loop contribution due to the two Higgs doublets via their one-loop wavefunction renormalisation induced by *excited* KK modes (of the gauge sector) only. The term with the coefficient $(b_{ik}-$

[5] this is a two-loop order contribution to α_i^{-1}

$2b_k\ T_k(G)\delta_{ik} - b_{ik}^H)$ is the similar contribution coming from the one-loop wavefunction renormalisation of the three MSSM families (due to *excited* KK gauge modes).

Finally, the functions f, g are model dependent form factors which encode some information regarding the contribution $\Delta\gamma_\phi$ of the excited Kaluza Klein states to the anomalous dimensions γ_ϕ of the MSSM fields, i.e. $\gamma_\phi = \gamma_\phi^o + \Delta\gamma_\phi$ where γ_ϕ^o is just the MSSM anomalous dimension of the MSSM field ϕ induced by the MSSM states only. These form factors could be computed in a Lagrangian formulation of the theory in higher dimensions compactified to the four dimensional space-time. In the absence of a fully derived MSSM from such a theory, we assumed that

$$\Delta\gamma_\phi(m) = -f_\phi(m,\mu_0;\delta)\sum_{j=1}^{3} 2\alpha_k(m)C_k(\phi) \tag{2.2}$$

for energy scales $\mu_0 \leq m \leq \Lambda$ and where ϕ stands for the MSSM fermions. $C_k(\phi)$ is the Casimir operator for the representation ϕ. For the two Higgses the similar form factor is denoted by g. Below the scale μ_0 the form factors f, g vanish while above this scale can be taken equal. In the limit $\delta \to 0$ we recover the usual MSSM running ($f(m,\mu_0;0) = g(m,\mu_0;0) = 0$). Below the scale μ_0 the usual MSSM spectrum and RGE equations apply.

Although the form factors f and g are model dependent, some general information about their form is available. As emphasized in [6, 12], for orbifold compactifications, the couplings $\alpha_{j,\vec{n}}$ between two massless twisted modes (i.e. a low-energy state without KK excitations) situated at the *same* fixed point of the orbifold and one untwisted (i.e. one with KK modes, as a KK boson) of level $\vec{n}$ is changed from the ordinary gauge coupling α_j, and hence $f(m,\mu_0,\delta)$ is expected to be proportional to

$$f(m,\mu_0,\delta) \propto N_{KK}(m,\mu_0,\delta)\rho^{-\mu_{\vec{n}}^{o2}\alpha_H} \tag{2.3}$$

where α_H is the four dimensional heterotic string coupling, $\alpha_H = 1/M_H^2$ and ρ is a number which depends on the orbifold twist [12], being of order 10 or larger. Also $N_{KK}(m,\mu_0,\delta)$ is an average number of Kaluza-Klein states coupling to the two massless *twisted* modes and accounts for summing up all the effects due to Kaluza Klein modes which renormalise the wave function of the low-energy (twisted) states. If $\mu_{\vec{n}}^{o2} \equiv \vec{n}^2\mu_0^2 \ll \Lambda^2$ and since Λ is identified with the string scale M_H, then

$$f(m,\mu_0,\delta) \propto N_{KK}(m,\mu_0,\delta) \tag{2.4}$$

In this case of "large" compactification radius situated well below the string scale, the two loop effects due to Kaluza-Klein will have an enhancement role due to N_{KK}.

If $\mu_{\vec{n}}^{o2} \equiv \vec{n}^2\mu_0^2 \lesssim \Lambda^2$ then the couplings $\alpha_{j,\vec{n}}$ are strongly suppressed and two loop effects induced by excited Kaluza-Klein modes become less significant and therefore the predictions from the RGE equations are less dependent to the exact details (the interaction Lagrangian between the light states and the Kaluza-Klein states) of the theory, encoded in f and g. If so we are mainly left with the usual MSSM two-loop terms, but the validity of (2.1) is rather limited in this case.

We also note that the power-law behaviour of the RGE for the gauge couplings should be restricted, for consistency with the orbifold models to values of $\delta = 1, 2$ as the higher powers would introduce a scale dependence not present in the large radius limit of the string calculations. More explicitly, the one-loop string corrected relation between the gauge couplings, at the string scale is

$$\alpha_i^{-1}(M_{string}) = \alpha_{string}^{-1} + \frac{1}{4\pi}\Delta_i \tag{2.5}$$

where the values of Δ_i have a large radius behaviour

$$\Delta_i \propto (Radius)^2 \tag{2.6}$$

for Calabi-Yau [13] compactifications.

The origin of this apparent discrepancy[6] between the string thresholds scale dependence restricted to R_o^2 in the large radius limit and that we obtained (R_o^δ) is in the exact structure of the spectrum of the model. In our calculation we considered that all towers of Kaluza-Klein states are $\mathcal{N} = 2$ supersymmetric for all $\delta = 1, 2, ..6$.

[6]Note that Δ_i includes both momentum and winding modes

In string models this is true only for $\delta = 1, 2$, the rest of the spectrum having $\mathcal{N} = 4$ supersymmetry and hence does not induce any RGE running of the holomorphic coupling. The overall running will therefore be due to the $\mathcal{N} = 2$ sector only. This remains also true when we evaluate the canonical gauge coupling with the sum over the Kaluza-Klein states restricted to $\mathcal{N} = 2$ sector, thus explaining the R_o^2 dependence[7].

3. Two models and their predictions

In this section we consider two models and estimate the size of the corrections on $\alpha_3(M_z)$ induced by the KK states, assuming the gauge couplings unification.

Model A considers excited KK modes for the MSSM gauge bosons and for the two Higgses[8] while model B includes only Kaluza Klein gauge bosons in addition to the MSSM spectrum. The excited modes are assumed to be $\mathcal{N} = 2$ massless vector multiplets (gauge modes) and $\mathcal{N} = 2$ hypermultiplets (Higgs modes). To ensure that the "zero-level" modes are indeed only $\mathcal{N} = 1$ supersymmetric multiplets, additional symmetry conditions must be imposed. For orbifold compactifications, the chiral adjoint component of $\mathcal{N} = 2$ vector multiplet is considered odd under the discrete group of the orbifold so that it does not have zero modes[9], the three generations of the MSSM are assumed to lie all at the (same) fixed points of the orbifold considered (this avoids the presence of KK states for them) and that after compactification the spectrum of the MSSM is indeed reproduced entirely. These are strong assumptions and must be recovered while deriving such models within a Lagrangian formalism in a $4+\delta$ dimensional space. Assuming that all these conditions are fulfilled, the coefficient $\tilde{b}_i$ of (2.1) should be replaced by the combination

$$\begin{aligned}\tilde{b}_i(\xi) &= \xi[T(R^i_{H_u}) + T(R^i_{H_d})] + T^i(G) - 3T^i(G) \\ &= 2\,\xi\; T_i(H_u) - 2T_i(G) \\ &= \left\{\frac{3}{5}\,\xi;\; \xi - 4;\; -6\right\}_i \end{aligned} \tag{3.1}$$

where $T^i(H_u) = T^i(H_d) = (3/10, 2, 0)_i$. For model A $\xi = 1$ and for model B $\xi = 0$. The last and second-last term in the first line of equation (3.1) represent the contribution of a Kaluza-Klein state of the gauge sector, associated with the $\mathcal{N} = 1$ massive vector supermultiplet which consists of $\mathcal{N} = 1$ massless vector supermultiplet and a chiral supermultiplet in the adjoint representation, hence the terms $-3T_i(G)$ and the term $T_i(G)$ respectively, to account for the excited mode of the gauge boson [10].

We assume that above the scale μ_0 the values of the form factors f and g due to KK gauge effects are fixed to the following value

$$f_{\phi_j}(m, \mu_0; \delta) = N(m, \mu_0; \delta) - 1, \;\; m > \mu_0 \tag{3.2}$$

where N is the total number of KK states below the string scale [1]. This is because above the scale μ_0 only *excited* $\mathcal{N} = 1$ massless vector KK states contribute, and their number is $N(m, \mu_0; \delta) - 1$. Also

$$g_{H_{u,d}}(m, \mu_0; \delta) = 0, \;\; m > \mu_0 \tag{3.3}$$

because above μ_0 scale there is no wave function renormalisation for the MSSM Higgs, due to $\mathcal{N} = 2$ multiplets running inside the loop, corresponding to excited KK for Higgses and vector bosons. For $N(m, \mu_0; \delta) = 1$ we recover the usual result, due to the MSSM gauge bosons only (identified with the "zero-modes" gauge bosons).

From the scale μ_0 down to M_z only the usual MSSM spectrum and RGE running is supposed to apply. Our numerical results are then obtained from eqs.(2.1), and from three additional RGE equations for the MSSM-like running of the couplings below the scale μ_0.

[7] For the numerical analysis of Section 3 we will allow $\delta \geq 2$, choice motivated by the suggestion that suitable orbifold choices can allow [1] for all KK states to fall into $\mathcal{N} = 2$ multiplets.

[8] A detailed description of model A is presented in [1].

[9] This ensures that the chiral adjoint field is not present in the low energy spectrum.

[10] The presence of KK states associated with the Higgses introduces two-loop changes (in the RGE for model A) of Yukawa type but these effects are ignored in our analysis; therefore, the only difference between models A and B is that induced by the coefficient $\tilde{b}_i(\xi)$ in front of the power-law term.

3.1 Numerical results

For the case the Higgs fields have towers of excited Kaluza-Klein modes (model A, $\xi = 1$) we cannot obtain phenomenologically viable value[11] for $\alpha_3(M_z)$ [2]. In this case, the prediction for the strong coupling $\alpha_3(M_z)$ from unification condition lies above the MSSM value [2], in disagreement with the experiment. Additional two-loop effects above decompactification scale which we included here (and were neglected in [2]) further increase $\alpha_3(M_z)$ by an amount of about $3-4\%$ from the results of [2]. The values of the unification scale Λ and of the decoupling scale μ_0 are also systematically increased from the predictions of [2] by an amount of about 10%. This gives an idea of the size of the two-loop corrections for the aforementioned quantities, relative to their predictions which ignored two-loop terms above μ_0 [2]. Two-loop corrections will increase even further from these values if one considers additional towers of KK states for the fermions which would bring an increase of the unified coupling [2] (due to their positive Δb_i) and hence the contributions of the integrals in (2.1) become more significant.

When the MSSM Higgs fields do not have towers of excited states (model B, $\xi = 0$) the results are presented in Table 1. They show that for an acceptable value for $\alpha_3(M_z)$, the unification of the gauge couplings takes place at a scale of about 10^{14} GeV while the scale of new spatial dimensions is only slightly smaller, with values also in the region of 10^{14} GeV and the ratio of the two less than 10. The number of KK states is not very large and it is usually restricted by the values of the one-loop beta coefficients $\tilde{b}_i(\xi)$ which in turn depend on the way we assign Kaluza-Klein states to the low energy spectrum. The validity of perturbation theory also requires that $N\alpha \leq 4\pi$, condition which is respected in our case. We see from Tables 1 and 2 that the unification of the gauge couplings leads to good predictions for the strong coupling $\alpha_3(M_z)$, in agreement with the experimental value[12].

[11] This statement is subject to further threshold corrections at the high scale which may change this value of $\alpha_3(M_z)$

[12] The average value for $\alpha_3(M_z) = 0.119 \pm 0.002$, [15].

The different behaviour of $\alpha_3(M_z)$ for the two models (A and B) considered can be explained from eqs.(2.1) giving

$$\alpha_3^{-1}(M_z) - \alpha_3^{o\,-1}(M_z = \frac{3}{14\pi}(2-3\xi) \times \left\{ \frac{\pi^{\delta/2}}{\delta\,\Gamma(1+\delta/2)}\left[\left(\frac{\Lambda}{\mu_0}\right)^{\delta} - 1\right] - \ln\frac{\Lambda}{\mu_0}\right\} + two - loop\ term \tag{3.4}$$

where $\alpha_3^o(M_z)$ is the MSSM value for the strong coupling. For $\xi = 0$ the Higgs fields do not have Kaluza-Klein towers of excited modes and since the second bracket in (3.4) is positive, $\alpha_3(M_z) \leq \alpha_3^o(M_z) \approx 0.126$ which leads to good phenomenological results, as shown in Table 1. If $\xi = 1$ (model A) the strong coupling is increased above the MSSM prediction and above the experimental upper limit, $\alpha_3(M_z) \geq \alpha_3^o(M_z) \approx 0.126$; two-loop terms in (3.4) cannot change this result, for the perturbation theory to be valid.

One could consider other models of assigning Kaluza-Klein states to the low-energy spectrum. One can include Kaluza-Klein towers for the fermions [1, 4] which will enhance the two-loop KK contribution above the scale μ_0 as mentioned. Other constructions motivated by specific string models [1, 16] are possible, but we will not consider these cases here. It must be mentioned that a good accuracy for the prediction for $\alpha_3(M_Z)$ is achieved if the number of KK states below the string scale is significant so that eq.(2.1) is accurate. Finally, lowering the string scale in the "TeV" region as a result of the power-law running of the gauge couplings is affected by fine tuning effects as emphasized in [2].

4. Conclusions

We estimated in a full two loop analysis the correction on $\alpha_3(M_z)$ following from the condition of gauge couplings unification in two models with Kaluza-Klein states. Summing up the *logarithmic* individual contributions of these states to the RGE equations gives the power-law running of the couplings valid to all orders in perturbation theory and this correctly includes the threshold effects due to log's of ratios of (physical) masses of Kaluza-Klein states of different levels.

Since the contribution of the Kaluza-Klein states to the wave-function renormalisation of the low-energy (MSSM) spectrum is evaluated in one-loop only, our analysis is valid in two-loop order. The two-loop effects above the decompactification scale are more significant when one considers KK states associated with the fermionic sector, since they increase the gauge couplings and thus the two loop contribution. Otherwise, their contribution is in the region of 4% for $\alpha_3(M_z)$ and 10% for the mass scales of the model. Lowering the unification scale in the "TeV" region as an effect of the power-law running is not possible without fine-tuning effects. The results are subject to additional string threshold effects at the high scale, not included in this work.

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ξ	δ	$\rho = \ln(\Lambda/\mu_0)$	Λ (GeV)	μ_0 (GeV)	$\alpha_3(M_Z)$	α_Λ
0	1	0.	3×10^{16}	3×10^{16}	0.1260	0.0433
0	1	0.2	2.512×10^{16}	2.057×10^{16}	0.1254	0.0429
0	1	0.4	1.959×10^{16}	1.313×10^{16}	0.1246	0.0424
0	1	0.6	1.400×10^{16}	7.687×10^{15}	0.1235	0.0418
0	1	0.8	9.001×10^{15}	4.044×10^{15}	0.1221	0.0410
0	1	1	5.079×10^{15}	1.868×10^{15}	0.1204	0.0400
0	1	1.2	2.445×10^{15}	7.365×10^{14}	0.1183	0.0388
0	1	1.4	9.700×10^{14}	2.392×10^{14}	0.1157	0.0374
0	1	1.6	3.037×10^{14}	6.132×10^{13}	0.1125	0.0357
0	1	1.8	7.126×10^{13}	1.178×10^{13}	0.1089	0.0339
0	1	2	1.175×10^{13}	1.590×10^{12}	0.1047	0.0318
0	1	2.2	1.261×10^{12}	1.397×10^{11}	0.0999	0.0296
0	1	2.4	8.005×10^{10}	7.262×10^{9}	0.0946	0.0272
0	1	2.6	2.677×10^{9}	1.988×10^{8}	0.0888	0.0248
0	2	0	3×10^{16}	3×10^{16}	0.1260	0.0433
0	2	0.2	1.976×10^{16}	1.618×10^{16}	0.1246	0.0424
0	2	0.4	9.873×10^{15}	6.618×10^{15}	0.1224	0.0412
0	2	0.6	3.265×10^{15}	1.792×10^{15}	0.1191	0.0393
0	2	0.8	5.841×10^{14}	2.624×10^{14}	0.1143	0.0366
0	2	1	4.179×10^{13}	1.537×10^{13}	0.1076	0.0332
0	2	1.2	7.637×10^{11}	2.300×10^{11}	0.0989	0.0291
0	2	1.4	1.825×10^{9}	4.502×10^{8}	0.0882	0.0245
0	3	0	3×10^{16}	3×10^{16}	0.1260	0.0433
0	3	0.2	1.503×10^{16}	1.231×10^{16}	0.1237	0.0419
0	3	0.4	3.792×10^{15}	2.542×10^{15}	0.1195	0.0395
0	3	0.6	2.740×10^{14}	1.504×10^{14}	0.1123	0.0356
0	3	0.8	2.037×10^{12}	9.153×10^{11}	0.1009	0.0300
0	3	1	2.419×10^{8}	8.900×10^{7}	0.0851	0.0233
0	4	0	3×10^{16}	3×10^{16}	0.1260	0.0433
0	4	0.2	1.153×10^{16}	9.444×10^{15}	0.1229	0.0414
0	4	0.4	1.152×10^{15}	7.723×10^{14}	0.1162	0.0376
0	4	0.6	5.756×10^{12}	3.159×10^{12}	0.1032	0.0311
0	4	0.8	3.718×10^{7}	1.670×10^{7}	0.0824	0.0223
0	5	0	3×10^{16}	3×10^{16}	0.1260	0.0433
0	5	0.2	9.292×10^{15}	7.607×10^{15}	0.1222	0.0411
0	5	0.4	3.001×10^{14}	2.012×10^{14}	0.1126	0.0357
0	5	0.6	2.107×10^{10}	1.156×10^{10}	0.0923	0.0262
0	6	0	3×10^{16}	3×10^{16}	0.1260	0.0433
0	6	0.2	8.095×10^{15}	6.628×10^{15}	0.1218	0.0408
0	6	0.4	7.504×10^{13}	5.030×10^{13}	0.1091	0.0339
0	6	0.6	9.886×10^{6}	5.425×10^{6}	0.0806	0.0216

Table 1: Model B: The values of the unification scale Λ, decoupling scale μ_0, the strong coupling at the electroweak scale and the bare coupling α_Λ in terms of the ratio Λ/μ_0 which is related to the number of KK states [1]. Our two-loop results are based on (2.1). The results correspond to model B, when the Higgs fields do not have Kaluza-Klein states.