

# Heavy Flavour Production

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ABSTRACT: I discuss a few recent topics in the theory and phenomenology of heavy flavour production.

KEYWORDS: .

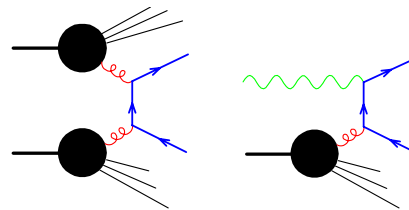
## 1. Introduction

From the point of view of perturbative QCD, the production of a heavy flavour is calculable as long as its mass is sufficiently large. In this limit, low energy hadronization effects should not influence the total cross section and the distributions, and one expects an accurate prediction from the perturbative calculation. Thus, most of the theoretical work in heavy flavour production physics is based upon perturbation theory, even if at times resummation of large contributions are considered.

The production properties of charmed and bottomed hadrons are qualitatively well described by the perturbative calculation. Quantitative comparisons, however, are not always completely satisfactory, although one cannot claim at this moment unjustifiable discrepancies between theory and experiments. A recent review of the phenomenology of heavy quark production is given in ref. [1]. In the present talk I wish to focus upon a few aspects that have received recent attention. In section 2 I will briefly review the theoretical status. In section 3 I will discuss a few aspects of the comparison of theoretical prediction with data, for top, bottom, and charm production.

## 2. Theory

A typical leading order diagram for the process of heavy quark hadroproduction is illustrated in the first diagram of figure 1. The process is computed in the usual improved parton model context, as the convolution of parton densities with



**Figure 1:** Typical diagrams in hadroproduction and photoproduction of heavy flavours.

short distance cross section. At the lowest order, the amplitude is of order  $\alpha_s$ , and the cross section is of order  $\alpha_s^2$ . The heavy quark photoproduction process (the second diagram in figure 1) is of order  $\alpha_{em}\alpha_s$  in the cross section. For both processes, next-to-leading corrections (i.e.,  $\mathcal{O}(\alpha_s^3)$  for hadroproduction, and  $\mathcal{O}(\alpha_{em}\alpha_s^2)$  for photoproduction) have been computed a long time ago [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12]. Going to higher order in perturbation theory seems to be a very difficult task, and no one that I know of has been planning to perform such calculation. Nevertheless, the accuracy of the calculation has been improved in particular regions of phase space, where enhanced contributions arise and have to be summed at all orders.

At very high energy, one expects terms of order Born  $\times [\alpha_s \log S/m_Q^2]^n$  to arise at all orders in perturbation theory. This problem (related to the small-x problem in DIS physics) is relevant, for example, for bottom production at the Tevatron and at the LHC.

When we approach the threshold region for the production of the heavy flavour pair, terms of order Born  $\times [\alpha_s \log^2 \Delta_{thr}]^n$  arise, where  $\Delta_{thr}$

is some measure of the distance from the threshold. These terms are relevant for the production of top at the Tevatron, for the production of  $b$  at HERAb, and for the production of charm at relatively low CM energy.

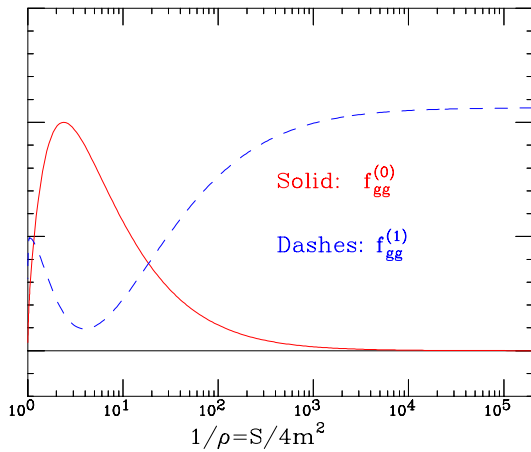
For the production of heavy flavour at high transverse momenta one expects terms of order Born  $\times [\alpha_s \log p_T/m_Q^2]^n$  to arise at all orders in perturbation theory. This problem is relevant for the production of charm and bottom at high transverse momentum at the Tevatron, but also, perhaps, for the production of very high transverse momentum top quarks at the LHC.

## 2.1 Total cross section

Radiative corrections to the total cross section are usually parametrized as

$$\sigma_{ij} = \frac{\alpha_s^2(\mu)}{m^2} \left[ f_{ij}^{(0)}(\rho) + 4\pi\alpha_s \left( f_{ij}^{(1)}(\rho) + \bar{f}^{(1)}(\rho) \log \frac{\mu^2}{m^2} \right) \right]. \quad (2.1)$$

In figure 2 I plot the functions  $f$  for the  $gg$  process.



**Figure 2:** Gluon-Gluon contribution to the heavy flavour photoproduction cross section

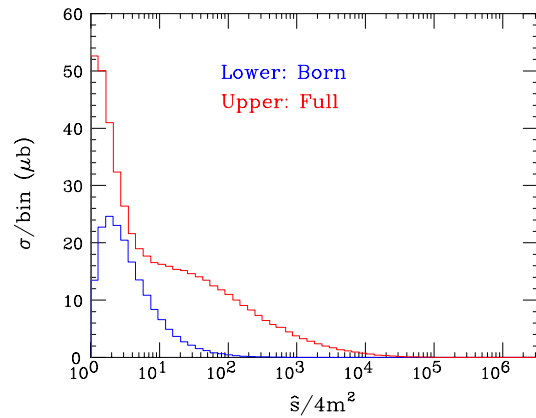
Notice the behavior near threshold. Defining as usual

$$\rho = \frac{4m^2}{\hat{s}}, \quad \beta = \sqrt{1 - \rho} = \text{velocity} \quad (2.2)$$

the most singular terms near threshold have the form

$$\frac{f_{gg}^{(1)}}{f_{gg}^{(0)}} \rightarrow \frac{A}{\beta} + B \log^2 \beta + C \log \beta + \dots \quad (2.3)$$

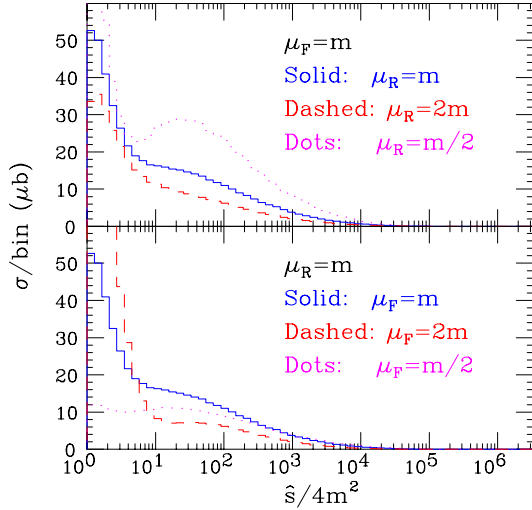
due to Coulomb  $1/\beta$  singularities and to Sudakov double logarithms. Near threshold, these terms may require special treatment (e.g., resummation). Notice also the constant asymptotic behavior of  $f_{gg}^{(1)}$  at large  $S$ . It may cause problems for heavy quark production far above threshold. This problem has been recently investigated in the context of the LHC Workshop at CERN [13]. Plotting the cross section as a function of the partonic  $\hat{s}$  may help to understand the origin of large corrections. For bottom at the LHC (from the LHC workshop), such plots are given in figures 3 and 4. We see that large corrections



**Figure 3:** The contribution to the total cross section as a function of the partonic  $\hat{s}$

come from the region near threshold, and from the region far above threshold. In figures 4 we study the scale dependence of this distribution. We find large scale variation near threshold and large scale variation far from threshold.

The inclusion of resummation effects can reduce the scale variation, provided it is performed at least at the next-to-leading level. Resummation of threshold effects up to the next-to-leading level is well understood [14] [15], while resummation of high energy logarithms has only been studied at the leading logarithmic level [16] [17] [18] [19].

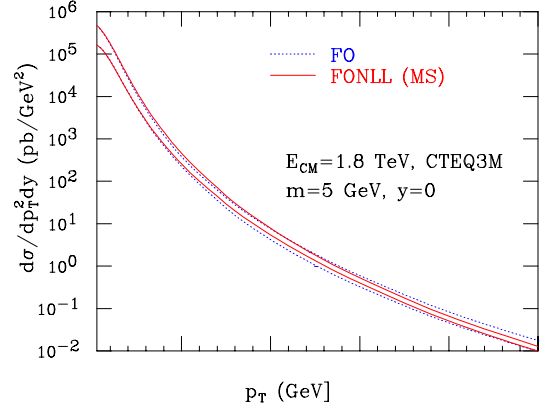


**Figure 4:** Scale dependency of the contribution to the total cross section. In the upper plot  $\mu_F = m$ , in the lower plot  $\mu_R = m$ .

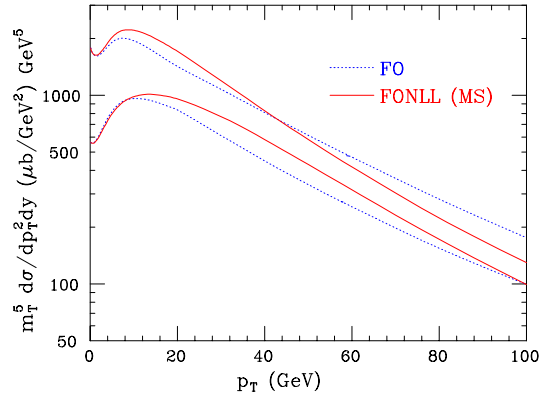
## 2.2 Differential distributions

The formalism for the resummation of the large  $\log p_T/m$  logarithms is made up of many ingredients: the perturbative parton density for heavy quarks, the perturbative fragmentation function and the inclusive cross section for single hadron production in hadronic collisions. Recent approaches to this problem merge the fixed order calculation with the resummed one, maintaining the accuracy of both approaches without overcounting [20] [21]. A summary of these improved results, that merge the NLO fixed order calculation with the next-to-leading-log resummation [20] are illustrated in figures 5, 6 and 7. The reduction in scale dependence at large  $p_T$  is quite evident in the figures.

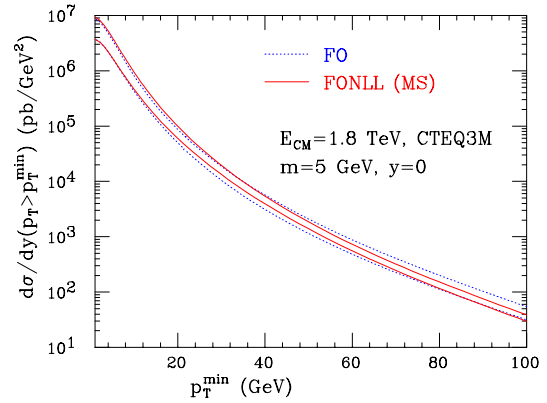
Work is being done to extend this method to photoproduction [22], for fixed target and HERA studies. One preliminary result is the study of mass effects, that have to be added to the resummed calculation in the merged approach. One compares the cross section computed at the NLO level, with a similar calculation where mass suppressed effects have been neglected. This cannot be achieved simply by taking the limit  $m \rightarrow 0$  in the full calculation, because of the presence of logarithms  $\log m/p_T$  in the NLO result. In figure 8 we plot the cross section at fixed  $p_T$  as a function of the mass, in a logarithmic scale,



**Figure 5:** Merged calculation (solid lines) versus the resummed one (dotted line)

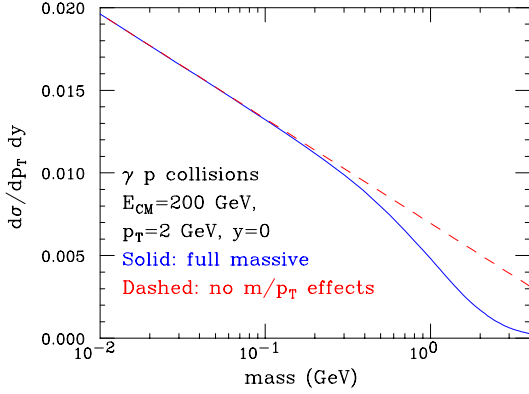


**Figure 6:** Same as in figure 5 with an appropriate weight for an easier comparison.



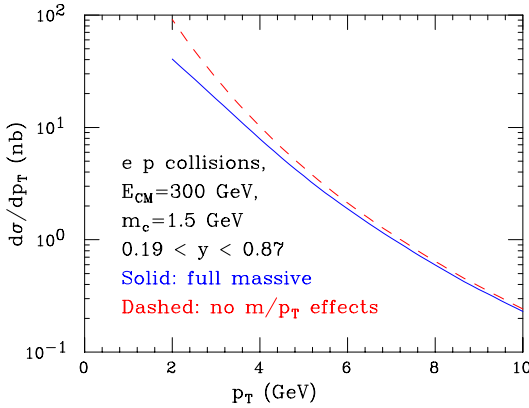
**Figure 7:** Same as in figure 5 for the integrated cross section

comparing the massless limit approach with the full massive one. The linear behavior of the NLO calculation as a function of  $\log m$ , in the



**Figure 8:** Differential cross section at fixed  $p_T$  as a function of the mass for heavy quark photoproduction.

limit of small  $m$ , is quite evident from the figure. The massless limit result is exactly linear. The only mass dependence left there is the logarithmic term, since mass power terms have been dropped. We can see here that mass effects have negative sign, the opposite of what happened in the hadroproduction case (see ref. [20]). In figure 9 we plot the cross section as a function of  $p_T$  and fixed mass, for charm production in electron-hadron collision at HERA. The difference be-



**Figure 9:** As in figure 8, at fixed  $m$  versus  $p_T$ .

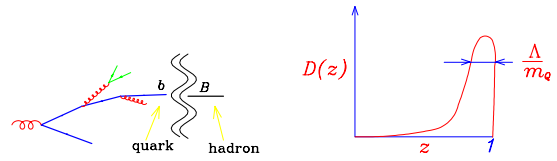
tween the solid and the dashed lines gives the size of mass effects that one can expect there.

### 2.3 Non-perturbative effects

In general, for bottom and charm production, one finds large radiative corrections, and thus one expects large unknown higher order terms.

At the same time, non-perturbative effects (suppressed by powers of  $\Lambda/m_Q$ ) may play a role here, especially for charm production. We have no theory of power suppressed effects in this context. Yet, understanding to what extent we can trust the theoretical machinery of hadroproduction processes (the factorization theorem) is of extreme importance for collider physics. We have to try out models of non-perturbative effects, and compare them with data.

#### 2.3.1 Fragmentation Effects



**Figure 10:** Hadronization is assumed to degrade the quark momenta according to a non-perturbative fragmentation function  $D(z)$ .

One assumes that the hadron momentum is a fraction  $z$  of the quark momentum, distributed according to a fragmentation function (perturbation theory suggests something like this, but the width of the distribution is  $\approx \alpha_s \log p_T/m_Q$ ) For very large quark masses the non-perturbative fragmentation function becomes a delta function, but for  $b$ 's and  $c$ 's this is not quite the case.

#### 2.3.2 Intrinsic Transverse Momentum

One assumes that the incoming partons have an intrinsic transverse momentum  $\langle k_T \rangle \approx \Lambda$  (perturbation theory generates  $p_T \approx \alpha_s \times \text{hard scale}$ ). This affects mostly the transverse momentum of the heavy quark pair, its azimuthal correlation and the transverse momentum distribution of a single quark.

#### 2.3.3 Monte Carlo models

Monte Carlo models of hadronization include many more effects, like color drag from projectile remnants and the like, and can model effects like leading particle enhancements and asymmetries.

### 3. Phenomenology

#### 3.1 Charm production

It is well known that, due to its large sensitivity to the scale choice, to the charm quark mass and to the value of the QCD scale parameter  $\Lambda$ , the total charm cross section has extremely large theoretical uncertainties. It is not a surprise, therefore, that it agrees fairly well with the data. The shape of various distributions is less sensitive to these factors. It turns out that the  $x_f$  and  $p_T$  distributions can be fitted quite well by assuming a hard charm fragmentation function and a 1 GeV primordial transverse momentum of incoming partons, a value which is also favoured by measurements of the azimuthal correlations of the heavy hadron pair. A more extensive review of the data has been given by J. Russ in this proceedings [23]. As an example, I show here two figures from a recent publication of the E791 Collaboration [24]. Figures 11 and 12 show the  $x_f$  and the  $p_T$  distributions for  $D$  mesons, compared to Pythia and to a NLO calculation, performed with  $\epsilon = 0.001$  and  $\langle k_T^2 \rangle = 1 \text{ GeV}^2$ .

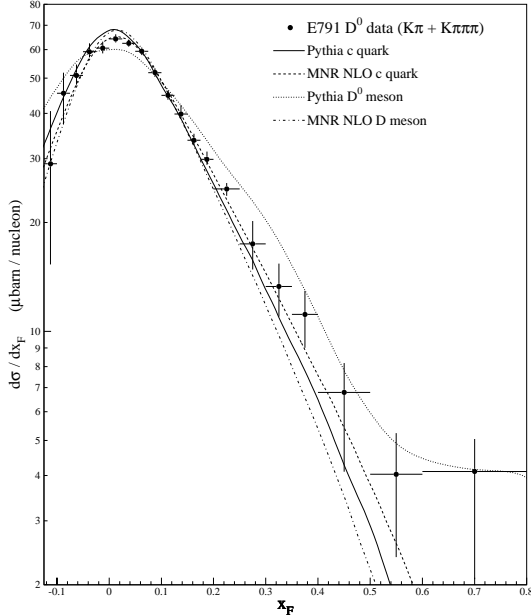


Figure 11: Charm  $x_f$  distribution

Charm production is a good place where to study non-perturbative effects and fragmentation properties [23]. These studies may help in predicting

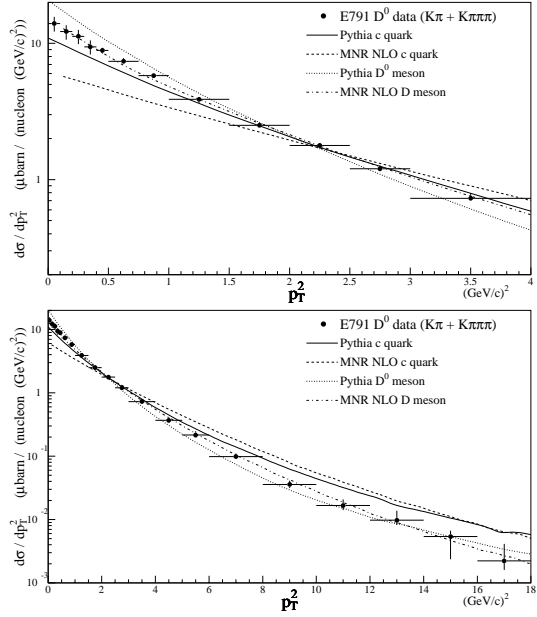


Figure 12: Charm  $p_T$  distribution

the impact of non-perturbative effects in bottom production. At the CERN LHC workshop, the problem of predicting charge asymmetries in  $B$  production was considered, since these asymmetries may fake a CP violation signal. In order to do this, one uses models of hadronization that generate asymmetries, tuned to the measured  $D$  production asymmetries [25].

#### 3.2 Understanding Tevatron data

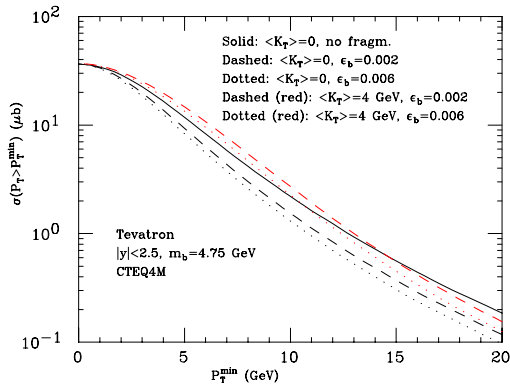
As shown by K. Sumorok [26], Tevatron data for the transverse momentum spectrum in  $b$  production are higher than QCD predictions. This problem has been around for a long time, although it has become less severe with time. The prediction band for the bottom spectrum is rather wide. This signals the presence of large uncertainties. In my opinion, it is not unlikely that we may have to live with this discrepancy, which is certainly disturbing, but not strong enough to question the validity of perturbative QCD calculations. To put it in simple words, the QCD  $\mathcal{O}(\alpha_s^3)$  corrections for this process are above 100% of the Born term, and thus it is not impossible that higher order terms may give contributions of the same order. Nevertheless it is useful to look for higher order perturbative effects and

non-perturbative effects that can rise the cross section.

Inclusion of resummation of  $p_T$  logarithms [20] brings about a moderate increase in the cross section in the intermediate transverse momentum region (figures 5-7). This increase is difficult to quantify (see ref. [20]), but it is certainly positive and goes in the right direction.

In analogy to the case of charm production, the agreement between theory and data improves if one does not include any fragmentation effects. It is then natural to ask whether the fragmentation functions commonly used in these calculations are appropriate. Following the LEP measurements, fragmentation functions have appeared to be harder than previously thought. It will be interesting to see whether SLD new data [27] will help in clarifying this issue.

One can ask whether also for  $b$  production an intrinsic transverse momentum for the incoming partons may explain the discrepancy. Such study has been performed in [13]. In figure 13 we show the effect of the intrinsic  $k_T$ , and we also show the sensitivity to the  $\epsilon$  parameter in the fragmentation function. We see that for



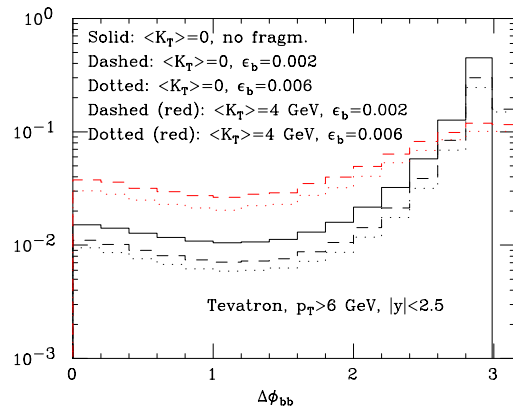
**Figure 13:** The  $b$  cross section at the Tevatron: the effect of a large intrinsic transverse momentum, and the sensitivity to the fragmentation parameter  $\epsilon$ .

$p_T^{min} < 20$  GeV, the  $k_T$  effect is sizeable, even in presence of fragmentation, provided we allow for unphysically large intrinsic  $k_T$ . Furthermore, uncertainty on the fragmentation function ( $\epsilon$  parameter) are relatively small. At this point it is fair to ask whether such large values of  $\langle k_T \rangle$  are compatible with other observables (such large values have also been invoked to explain direct

photon production data [28]). There are observables, like the distribution of the azimuthal distance  $\Delta\phi$  of the  $b$  pair, that are particularly sensitive to the intrinsic transverse momentum. The  $\Delta\phi$  distribution is trivial at leading order:  $b$  and  $\bar{b}$  are emitted back-to-back, so

$$\frac{d\sigma}{d\Delta\phi} \propto \delta(\phi - \pi). \quad (3.1)$$

An intrinsic  $k_T$  smears out the  $\delta$  function. For  $\langle k_T \rangle = 4$  GeV the effect is quite dramatic, as can be seen in figure 14 [13]. However, CDF and



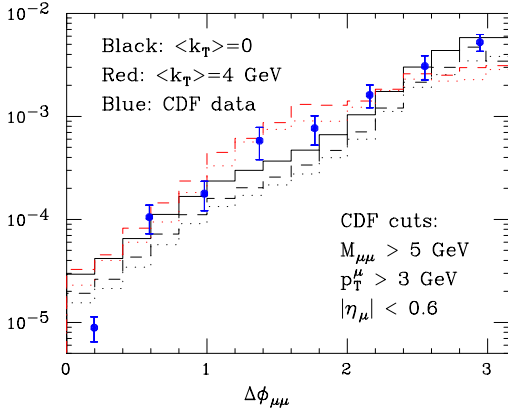
**Figure 14:** The  $b\bar{b}$  azimuthal correlation at the Tevatron: the effect of a large intrinsic transverse momentum, and the sensitivity to the fragmentation parameter  $\epsilon$ .

D0 analysis of the azimuthal correlation of muon pairs coming from  $b$ 's [26] do not seem to favour such a large intrinsic transverse momentum, as can be seen in figures 15 and 16 [13]. We thus conclude that it is unlikely that large  $k_T$  effects are present in  $b$  hadroproduction.

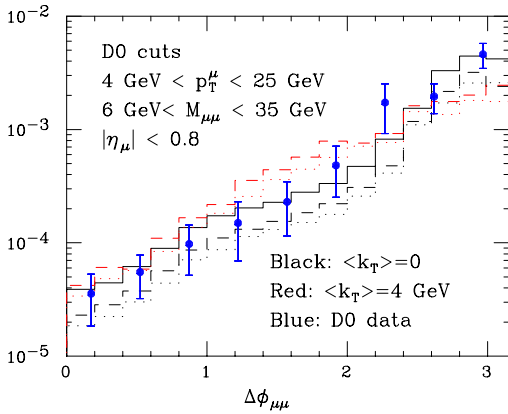
### 3.3 Top production

A remarkable success of the theory of heavy flavour production has been the fairly accurate prediction of the top cross section at the Tevatron. In fig. 17 the recent CDF and D0 data are shown [29], compared to the calculation of ref. [14]. The narrow theoretical prediction band is due to several factors:

- the heaviness of the top quark makes the perturbative prediction quite reliable;
- the process is dominated by the valence quark parton densities;



**Figure 15:** CDF results on azimuthal correlations compared with the perturbative calculation, with and without intrinsic  $k_T$



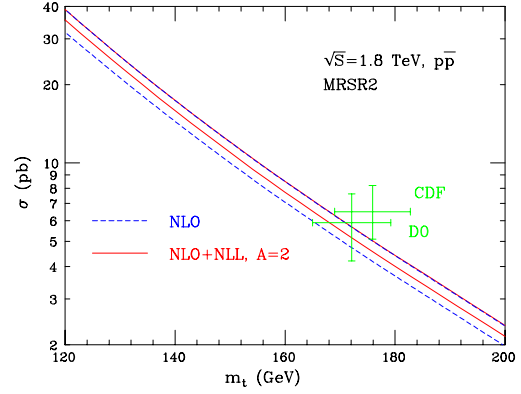
**Figure 16:** D0 results on azimuthal correlations compared with the perturbative calculation, with and without intrinsic  $k_T$

- uncertainties in  $\alpha_s$  are compensated by evolution effects;
- resummation of threshold effects reduces the uncertainties.

It is reassuring to see that the theory works better where we expect it to work better, e.g. for very heavy quarks.

### 3.4 Bottom at HERA

HERA has results on  $b$  production cross section, with very high central values. These have been discussed by Sefkow [30]. Roughly speaking, Zeus and H1 cross sections have central values a factor of 5 higher than QCD predictions. The theoretical error on this cross section is quite small, and



**Figure 17:** Resummed cross section for top production at the Tevatron, compared with recent experimental results.

it is very difficult (if not impossible) to justify these large values. It is also very difficult to explain why should charm work so well, if bottom is so far out. I believe that, in this case, it is wise to wait for the experimental errors to narrow down.

## 4. Summary and conclusions

As we have seen, there are many indications that the perturbative mechanism of heavy flavour production is at work in all known cases. The prediction of the top cross section has been a remarkable success; charm distributions are qualitatively well described by perturbation theory, and so are the bottom distributions. There are areas, however, where the agreement is less than perfect. Tevatron data is quite high, although large theoretical uncertainties are present there. HERA data on  $b$  production also shows problems, although in this case we should perhaps wait for more precise experimental studies.

From the theoretical point of view, new tools have been introduced to deal with special kinematic regions, like the production near threshold, and the production at very high transverse momentum. In this area there are many improvements that may come in the future. Threshold resummation for the  $p_T$  spectrum is one of them. Perhaps the most important missing resummation calculation is a full next-to-leading log resummation of the high energy logarithms, which is of great interest for LHC and Tevatron physics.

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