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Summary of lattice results for decay constants, mixing, etc.

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ABSTRACT: A summary of recent lattice results for the B meson decay constant f_B, bag parameter B_B, and related 'B' parameters is given. The quenched lattice results for f_B have been stable over the past few years, and recent simulations without the quenched approximation show a significant sea quark effects. All the recent unquenched (two light flavours) results are consistent with $f_B = 210(30)$ MeV. Calculation of B_B is less satisfactory, as the results of different formulations of heavy quark are not in good agreement. First lattice studies of $(\Delta\Gamma/\Gamma)_{B_s}$ and b hadron lifetime ratios are also discussed.

1. Introduction

In the determination of the Cabbobo-Kobayashi-Maskawa (CKM) matrix elements, model independent theoretical calculation of hadronic matrix elements, such as B meson decay constant and bag parameter, is essential. Lattice QCD provides a method to calculate them starting from QCD without introducing any model dependence, at least in principle.

For this reason, the lattice calculation of the heavy-light meson decay constants and bag parameters has been studied for more than a decade. The main problem was to construct a reliable method to treat the heavy quark with mass m_Q which is greater then the lattice cut-off. It was achieved by introducing effective theories that work for small spatial momenta on the lattice. The heavy quark effective theory (or static approximation) [1] treats an infinitely heavy quark, and its extension for finite heavy quark mass is also formulated (NRQCD [2]). The relativistic quark actions may also be treated as an effective theory for heavy quark (Fermilab action [3]). A naive use of the relativistic actions for relatively light heavy quark (∼ charm quark) combined with an extrapolation to the bottom is another possibility. In this talk I summarize the recent results for the heavy-light decay constant using these different formulations, each of which has different systematic uncertainties, and show that those are consistent with each other. It means that the systematic errors associated with the heavy quark are now under control.

Among remaining systematic uncertainties, the error of neglecting the effect of light quark loops (quenched approximation) is the most important one. Reliable estimate of the size of the quenching effect is difficult without actually performing simulations including dynamical quarks. Recently, such studies have been started by several groups and a significant effect for the heavy-light decay constant due to quenching is found (Section 2).

The calculation of the B meson bag parameter B_B is not so conclusive, as we find a disagreement between static/NRQCD and relativistic results (Section 3).

I also discuss the recent lattice calculations of other 'B' parameters, that are related to the width difference of B_s mesons (Section 5) and the lifetime ratios of b hadrons (Section 6).

Other applications of lattice QCD covered by separate talks are semileptonic decay form factors [4], quark masses [5], and quarkonia and hybrid spectrum [6]. The most recent review of lattice studies of B physics is found in [7].

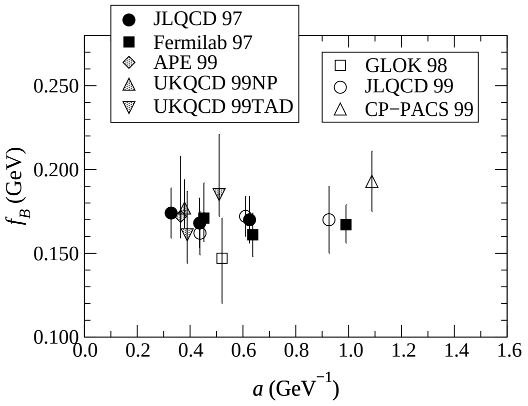

Figure 1: Recent quenched lattice calculations of f_B using $O(a)$-improved actions.

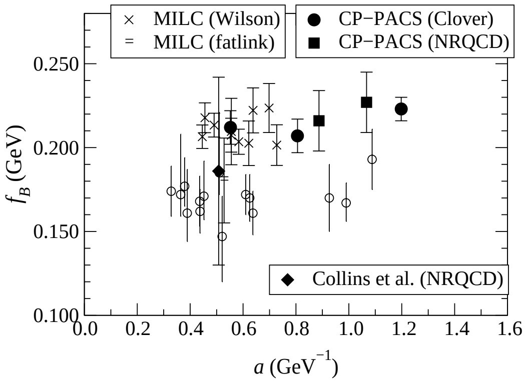

Figure 2: Dynamical lattice calculations of f_B. Results are from MILC [18, 19], Collins *et al.* [20] and CP-PACS [22, 23]. Quenched results as shown in Figure 1 are also plotted with small open symbols.

2. f_B

Since the mass difference in the B-$\bar{B}$ mixing is experimentally measured quite precisely [8], the determination of $|V_{td}|$ is limited by the systematic errors in the lattice calculation of $f_B^2 B_B$.

In the quenched approximation, for which the effect of sea quark is neglected, the lattice results for f_B are stabilized very well over the past few years. Figure 1 shows the recent quenched results from several groups. The results obtained with the non-relativistic lattice QCD (NRQCD) are given by open symbols (GLOK [9], JLQCD [10], and CP-PACS [11]), while the non-relativistic reinterpretation of the relativistic action [3] is employed for filled symbols (JLQCD [12] and Fermilab [13]). Also, there are results with the conventional approach in which the relativistic action is used for the charm quark mass region and the results are extrapolated to the bottom (APE [14] and UKQCD [15, 16]). The $O(a)$-improvement [17] is applied for these calculations. It is remarkable that the results with different formulations obtained at different lattice spacing a are completely consistent with each other. The MILC collaboration [18] has also done an extensive study with unimproved action, and their result extrapolated to the continuum limit is consistent with the results using the improved action.

The precision achieved in the quenched approximation depends on the method and parameters used, but its typical size is about 10–15% except for the quenching error. Perturbation theory used in the matching between continuum and lattice operators is an important source of the systematic error, which gives $O(\alpha_s^2)$.

The effect of quenching is difficult to estimate without carrying out the simulations with dynamical quarks. The lattice simulation including sea quark effects requires much more computer time than quenched simulation, and had been impractical until recently. Recent simulations by MILC [18, 19], Collins *et al.* [20] and CP-PACS [22, 23], however, are revealing the dynamical quark effect:

The MILC collaboration [18, 19] is performing a bunch of dynamical quark simulations using the staggered quark action for sea quarks (N_F=2). The heavy and light valence quarks are treated as the unimproved Wilson quark action, as in their quenched study. They found a significant raise of f_B with unquenching in the continuum limit, but it is not so clear at finite lattice spacings. A problem in their result is a large a dependence seen in the quenched data due to the use of the unimproved Wilson quark. A linear extrapolation to the continuum limit gives a substantially lower value compared to the data at finite a. On the other hand, their dynamical results do not show a similar a dependence and the continuum limit remains high. For this reason, although their result suggests $f_B^{N_F=2} > f_B^{N_F=0}$, the conclusion is not solid enough. Therefore, they started a new calculation using the fat-link clover action for heavy quark, with which scaling behavior is expected to be improved. A preliminary result favors lower value of f_B, albeit with large statistical error.

Collins *et al.* [20] are also using the staggered sea quarks (N_F=2), while they use the NRQCD action for heavy quark and the $O(a)$-improved action for light valence quark. If we assume the systematic error is completely correlated between quenched and unquenched simulations, their results $f_B^{N_F=2}$=186±5 MeV, where systematic error is omitted, and $f_B^{N_F=0}$=150±10 MeV suggest a large sea quark effect.

At Lattice 99, the CP-PACS collaboration presented two new calculations of f_B on their dynamical lattices (N_F=2) generated with an renormalization group (RG) improved gauge action [21]. They carried out two sets of calculations:

- calculation using the NRQCD action for heavy quark [22]. The $O(a)$-improved action is used for both sea and valence light quarks. Lattice spacing is about a=0.2 fm for quenched and unquenched lattices.

- calculation using the relativistic action for heavy quark [23] combined with the non-relativistic reinterpretation [3]. The $O(a)$-improved action is used for both sea and valence light quarks. Three lattice spacings ranging a=0.09∼0.29 fm are employed to see the systematic error associated with finite a.

Their results for f_B are shown in Figure 2 together with the results from other groups.

It is encouraging that dynamical results are consistent with each other, even though the actions employed for sea quarks and for heavy quark are different. Furthermore, the clear upward shift of f_B with the inclusion of dynamical quarks is remarkable.

Although it is a difficult task to combine the results from different groups, we can crudely say that all available data is consistent with the estimates listed in Table 1, where I also list the results for f_{B_s} and f_{B_s}/f_B. I do not attempt to extrapolate these results to the physical $N_F = 3$ limit. To do so, it seems necessary to understand the systematic errors coming from the use of different actions and lattice spacings. The sea quark mass dependence should also be clarified.

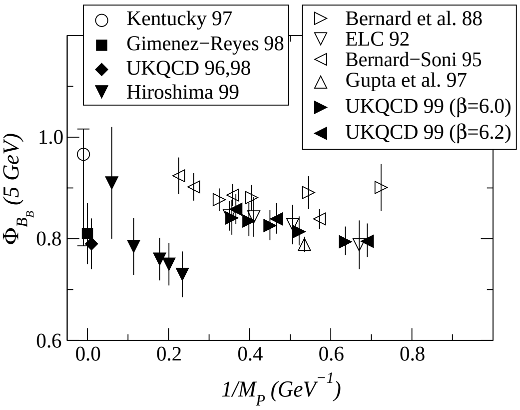

Figure 3: $1/M$ dependence of $\Phi_{B_B}(5GeV)$. The static and NRQCD data are from Kentucky [28], Giménez-Reyes [26] (Reanalysis of [24]), UKQCD [25] (Reanalyzed by [26]), and Hiroshima [29]. The relativistic calculations are Bernard *et al.* [34], ELC [35], Bernard-Soni [36], Gupta *et al.* [37], and UKQCD [31]. Open symbols are obtained with Wilson quark for heavy and/or light quarks, and filled ones are $O(a)$-improved.

	N_F=2	N_F=0
f_B	210 ± 30 MeV	170 ± 20 MeV
f_{B_s}	245 ± 30 MeV	195 ± 20 MeV
f_{B_as}/f_B	1.16 ± 4	1.15 ± 4

Table 1: Summary of the results for decay constants.

3. B_B

In contrast to the achievement for f_B, the lattice calculation of B_B is not satisfactory.

In the static approximation, the results by Giménez and Martinelli [24] and by UKQCD [25], both of which are obtained using the clover ($O(a)$-improved) action for light quark, have been reanalyzed in a recent paper by Giménez and Reyes [26]. They corrected an error in the previous perturbative calculations of the matching of four-quark operators, and applied the tadpole improved lattice perturbation theory [27]. Then they found the two calculations agree and a disagreement, which existed between the static-clover and static-Wilson results[28], is greatly reduced.

The $1/M$ corrections have recently been considered by the Hiroshima group [29, 30]. They performed a calculation using the NRQCD action, and found significant decrease of $B_B(m_b)$ as one includes the $1/M$ corrections. The perturbative matching between continuum and lattice operators, however, was done with the coefficient in the static limit, and thus the large systematic error of $O(\alpha_s/(aM))$ is left unremoved.

In the calculation with relativistic actions, the UKQCD collaboration presented the first calculation with the $O(a)$-improved action at Lattice 98 [31], which has been updated at this conference [16]. To obtain the result at the B meson mass, an extrapolation from charm mass regime is necessary, and they found a clear negative slope in $1/M$.

At Lattice 99, APE group [32] has presented the first result obtained using non-perturbative renormalization of four-quark operators [33]. They found a similar dependence of B_B on $1/M$, but their final numerical results are not yet available at the time I wrote this contribution.

Figure 3 presents a compilation of lattice data for

$$\Phi_{B_B}(\mu_b) \equiv \left(\frac{\alpha_s(M_P)}{\alpha_s(M_B)}\right)^{2/\beta_0} B_B(\mu_b) \qquad (3.1)$$

with $\mu_b = 5$ GeV as a function of $1/M_P$. The renormalization factor is introduced to cancel the $\ln(M/\mu_b)$ dependence appearing in the matching factor [16]. It is encouraging that all relativistic results including the early works [34, 35] show a reasonable agreement with each other, and that the recent UKQCD data show a nice scaling between two lattice spacings (β=6.0 and 6.2). The extrapolation to the static limit ($\sim$ 0.92), however, seems considerably higher than the $O(a)$-improved results in that limit. It suggests that there is unknown sources of systematic error in either or both of static (NRQCD) and relativistic calculations. Higher order perturbative corrections (for both) and $O((aM)^2)$ uncertainty in the relativistic calculations are their potential candidates. For this reason, my summary of the current available data includes a large systematic uncertainty: $B_B(m_b) = 0.80(15)$. On the other hand, many groups agree that the B_B is almost independent of light quark mass and the ratio B_{B_s}/B_B is close to unity: $B_{B_s}/B_B = 1.00(3)$.

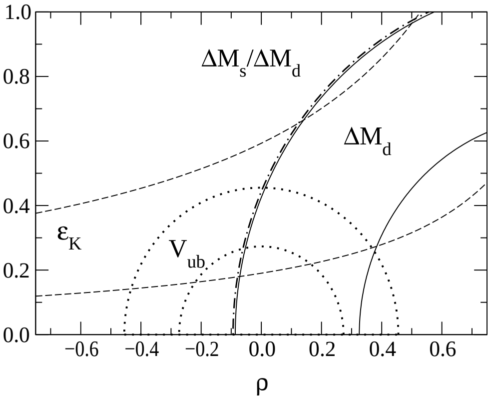

Figure 4: Constraint on the CKM matrix elements.

4. CKM determination

The allowed region on the (ρ, η) plane of the CKM matrix is shown in Figure 4. The two flavour result $f_B = 210 \pm 30$ MeV and the conservative estimate of B_B are used to draw the constraint from ΔM_d. Due to the upward shift of f_B from the previous quenched results, the allowed region favors $\rho > 0$. On the other hand, the ratio f_{B_s}/f_B is not affected by the quenching effect, so that the previous analysis for $|V_{td}/V_{ts}|$ [38] is unchanged.

5. B_s width difference

The width difference in the $B_s - \bar{B}_s$ mixing is given as

$$\Delta\Gamma_s \propto Im \frac{1}{2M_{B_s}} \langle \bar{B}_s | i \int d^4x T \mathcal{H}^{\rm eff}(x) \mathcal{H}^{\rm eff}(0) | B_s \rangle, \qquad (5.1)$$

where $\mathcal{H}^{\rm eff}$ represents the ΔB=1 effective Hamiltonian [39]. Only the final states into which both of B_s and $\bar{B}_s$ can decay contribute. The $1/m_b$ expansion induces two four-quark ΔB=2 operators, whose matrix elements with B_s and $\bar{B}_s$ states are B_B and B_S. B_S is defined through

$$\langle \bar{B}_s | \mathcal{O}_S(\mu) | B_s \rangle = -\frac{5}{3} f_{B_s}^2 M_{B_s}^2 \frac{M_{B_s}^2}{(\overline{m}_b + \overline{m}_s)^2} B_S(\mu), \qquad (5.2)$$

and $\mathcal{O}_S = \bar{b}(1-\gamma_5)s\bar{b}(1-\gamma_5)s$.

At Lattice 99, the Hiroshima group presented a lattice calculation of B_S using the NRQCD action [30]. Their calculation method is the same as that of B_B and they obtain $B_S(m_b)$=1.19(2)(20). It may be compared with the previous lattice study using the unimproved relativistic action $B_S(m_b)$=0.75(±??) obtained from a simulation at charm quark mass [37]. (Their result is converted the above definition by Beneke *et al.* [40].) Error is not specified at the moment, since the extrapolation to the b quark mass is not made. Obviously, more study is necessary to disentangle the mass dependence and eventually to obtain the final result from lattice QCD.

The Hiroshima group obtained the width difference as $(\Delta\Gamma/\Gamma)_{B_s} = 0.16(3)(4)$, using a next-to-leading order formula of Beneke *et al.* [40]. The errors are from f_{B_s} and B_S respectively. The two-flavour result for f_{B_s} discussed in Section 2 is used. The latest experimental bound from DELPHI is $(\Delta\Gamma/\Gamma)_{B_s} < 0.42$ [41].

6. Lifetime ratios

The ratios of lifetime of b hadrons, such as $\frac{\tau(B^-)}{\tau(B^0)}$ and $\frac{\tau(\Lambda_b)}{\tau(B^0)}$, provide an important test of the theoretical method to calculate the inclusive hadronic decay rates [42]. In the $1/m_b$ expansion, the leading contribution to the decay rate comes from a diagram in which the b-quark decay proceeds without touching the spectator quark, so that it does not contribute to the lifetime ratios. The $O(1/m_b^2)$ correction to the ratios is also small for the same reason, and the first correction involving the spectator quark effect is of $O(1/m_b^3)$, which is parametrized by the 'B parameters' of ΔB=0 four-quark operators. The UKQCD collaboration computed these matrix elements for the first time and obtained $\frac{\tau(B^-)}{\tau(B^0)} = 1.03(2)(3)$ [43], which is consistent with the recent experimental result 1.07(2) [44].

It is a known problem that the lifetime of Λ_b is surprisingly shorter than that of B mesons $\tau(\Lambda_b)/\tau(B^0) = 0.79(5)$ [44]. It is, therefore, interesting to see whether it is explained with the theoretical calculation, in which the similar matrix elements of four quark operators for Λ_b are required. The UKQCD group has studied these matrix elements [45], and found that the spectator effect is large $\sim -6\%$. Although their result $\tau(\Lambda_b)/\tau(B^0) = 0.91(1)\sim 0.93(1)$, depending on the light quark mass, is much higher than the experimental value, higher statistics calculations at smaller lattice spacings seem necessary to draw a definite conclusion.

7. Summary

Recent lattice calculations of the B meson decay constants and B parameters are summarized. For the decay constant f_B, quenched results are stable over the past few years, and our interest is now in the sea quark effects. Several recent unquenched simulations with two flavors of dynamical quarks suggest larger f_B and f_{B_s}, while their ratio remains unchanged.

The results for B_B are less satisfactory, even in the quenched approximation. The source of the disagreement between the static/NRQCD and relativistic calculations have to be understood. First lattice results for the B_s width difference and the B hadron lifetime ratios are also discussed.

As we found a non-negligible effect of quenching in f_B, it is important to study the other quantities without the quenched approximation. Simulations with two dynamical flavours (u and d quarks) are being carried out by several groups including European, US and Japanese collaborations, and we expect more and more results for heavy quark physics in the near future. Realistic three flavour simulations including the strange quark have to be done to eventually obtain the predictions from the first principles (QCD).

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References

[1] E. Eichten, *Nucl. Phys. Nucl. Phys.* **4** *(Proc. Suppl.)* (1988) 170.

[2] B.A. Thacker, G.P. Lepage, *Phys. Rev.* **D 43** (1991) 196; G.P. Lepage *et al.*, *Phys. Rev.* **D 46** (1992) 4052.

[3] A.X. El-Khadra, A.S. Kronfeld and P.B. Mackenzie, *Phys. Rev.* **D 55** (1997) 3933.

[4] L. Lellouch, "Exclusive semileptonic B decays: dispersive bounds and lattice results these", to appear in these proceedings.

[5] G. Martinelli, "Quark masses", to appear in these proceedings.

[6] C. Michanel, "Quarkonia and hybrids from the lattice", to appear in these proceedings.

[7] S. Hashimoto, talk presented at Lattice 99, the XVII International Symposium on Lattice Field Theory, Pisa, Italy, June 29-July 3, 1999, `hep-lat/9909136`.

[8] M. Daoudi, "B-$\bar{B}$ mixing results", to appear in these proceedings.

[9] A. Ali Khan *et al.*, *Phys. Lett.* **B 427** (1998) 132.

[10] JLQCD Collaboration (K-I. Ishikawa *et al.*), `hep-lat/9905036`.

[11] CP-PACS collaboration (A. Ali Khan *et al.*), talk presented at Lattice 99.

[12] JLQCD collaboration (S. Aoki *et al.*), *Phys. Rev. Lett.* **80** (1998) 5711.

[13] A.X. El-Khadra *et al.*, *Phys. Rev.* **D 58** (1998) 014506.

[14] D. Becirevic, P. Boucaud, J.P. Leroy, V. Lubicz, G. Martinelli, F. Mescia, F. Rapuano, *Phys. Rev.* **D 60** (1999) 074501; A. Abada, D. Becirevic, P. Boucaud, J.P. Leroy, V. Lubicz, G. Martinelli, F. Mescia, talk presented at Lattice 99, `hep-lat/9910021`.

[15] UKQCD collaboration (C.M. Maynard *et al.*), talk at Lattice 99, `hep-lat/9909100`.

[16] UKQCD collaboration (L. Lellouch and C.J.D. Lin), these proceedings.

[17] B. Sheikholeslami and R. Wohlert, *Nucl. Phys.* **B 259** (1985) 572.

[18] C. Bernard *et al.*, *Phys. Rev. Lett.* **81** (1998) 4812.

[19] MILC collaboration (S. Gottlieb *et al.*), poster at Lattice 99, `hep-lat/9909121`.

[20] S. Collins *et al.*, *Phys. Rev.* **D 60** (1999) 074504.

[21] R. Burkhalter (CP-PACS collaboration), *Nucl. Phys.* **73** *(Proc. Suppl.)* (1999) 3.

[22] CP-PACS collaboration (A. Ali Khan *et al.*), talk at Lattice 99.

[23] CP-PACS collaboration (H. Shanahan *et al.*), talk at Lattice 99, `hep-lat/9909052`.

[24] V. Giménez and G. Martinelli, *Phys. Lett.* **B 398** (1997) 135.

[25] UKQCD collaboration, (A.K. Ewing *et al.*), *Phys. Rev.* **D 54** (1996) 3526.

[26] V. Giménez and J. Reyes, *Nucl. Phys.* **B 545** (1999) 576.

[27] G.P. Lepage and P.B. Mackenzie, *Phys. Rev.* **D 48** (1993) 2250.

[28] J. Christensen, T. Draper and C. McNeile, *Phys. Rev.* **D 56** (1997) 6993.

[29] S. Hashimoto, K-I. Ishikawa, H. Matsufuru, T. Onogi, S. Tominaga, N. Yamada, *Phys. Rev.* **D 60** (1999) 094503.

[30] N. Yamada *et al.*, talk at Lattice 99, `hep-lat/9910006`.

[31] UKQCD collaboration (L. Lellouch and C.-J.D. Lin), *Nucl. Phys.* **73** *(Proc. Suppl.)* (1999) 357.

[32] A. Donini, V. Giménez, L. Giusti and G. Martinelli, talk at Lattice 99, `hep-lat/9909041`.

[33] G. Martinelli, C. Pittori, C.T. Sachrajda, M. Testa, A. Vladikas, *Nucl. Phys.* **B 445** (1995) 81.

[34] C. Bernard, T. Draper, G. Hockney and A. Soni, *Phys. Rev.* **D 38** (1988) 3540.

[35] A. Abada *et al.*, *Nucl. Phys.* **B 376** (1992) 172.

[36] A. Soni, *Nucl. Phys.* **47** *(Proc. Suppl.)* (1996) 43.

[37] R. Gupta, T. Bhattacharya and S. Sharpe, *Phys. Rev.* **D 55** (1997) 4036.

[38] See for example, S. Plaszczynski, "Overall determination of the CKM matrix",to appear in these proceedings.

[39] M. Beneke, G. Buchalla, I. Dunietz, *Phys. Rev.* **D 54** (1996) 4419.

[40] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, *Phys. Lett.* **B 459** (1999) 631.

[41] DELPHI collaboration (A. Borgland *et al.*), DELPHI 99-109 CONF 296, reported at EPS-HEP99.

[42] M. Neubert and C.T. Sachrajda, *Nucl. Phys.* **B 483** (1997) 399.

[43] UKQCD collaboration (M. Di Pierro and C.T. Sachrajda), *Nucl. Phys.* **B 534** (1998) 373.

[44] LEP B Lifetime Working Group, see their web page for the latest summary, http://home.cern.ch/~claires/lepblife.html.

[45] UKQCD collaboration (M. Di Pierro, C.T. Sachrajda and C. Michael), `hep-lat/9906031`.