Review of $B\Bar{B}$ Mixing Results

Mourad Daoudi  
Stanford Linear Accelerator Center, MS-78, P.O.Box 4349, Stanford, CA 94309, USA  
E-mail: daoudi@SLAC.Stanford.EDU

Abstract: A review of $B\Bar{B}$ mixing results at the end of July 1999 is presented. Emphasis is put on recent measurements of $\Delta m_d$ and $\Delta m_s$. For $\Delta m_d$, the new world average is $\Delta m_d = 0.473 \pm 0.016$ ps$^{-1}$. For $\Delta m_s$, the new world average 95% CL limit is 12.4 ps$^{-1}$, with a sensitivity of 14.2 ps$^{-1}$. Other related results are covered very briefly.

1. Introduction

The main motivation for performing $B\Bar{B}$ mixing measurements lies in the determination of the CKM matrix element $V_{td}$, which represents one of the constraints on the Unitarity Triangle.

$V_{td}$ is accessible experimentally through the box diagrams of figure 1a, by measuring the mass difference $\Delta m_d$ in $B_d$ mixing. $\Delta m_d$ and $V_{td}$ are related by:

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_b m_t^2 \frac{F}{m_W^2} \eta_{QCD} \times B_{B_d} f_{B_d}^2 |V_{td}|^2.$$ 

Similarly $B_s$ mixing provides a measurement of $V_{ts}$.

While $\Delta m_d$ is measured with very good precision, the determination of $V_{td}$ is limited by theoretical uncertainties in the decay constant $f_{B_d}$ and the bag factor $B_{B_d}$. However, in the ratio $\Delta m_s/\Delta m_d$ most hadronic uncertainties cancel:

$$\frac{\Delta m_s}{\Delta m_d} = \frac{\eta_{B_s} M_{B_s} f_{B_s}^2 B_{B_s}}{\eta_{B_d} M_{B_d} f_{B_d}^2 B_{B_d}} \left| \frac{V_{ts}}{V_{td}} \right|^2$$

$$= (1.14 \pm 0.08)^2 \left| \frac{V_{ts}}{V_{td}} \right|^2.$$ 

With $|V_{ts}| \simeq |V_{cb}|$, a precise measurement of both $\Delta m_d$ and $\Delta m_s$ provides a strong constraint on $V_{td}$. This underlines the importance of $B_s$ mixing measurements.

In the Wolfenstein parametrization the CKM matrix is defined by four parameters, two of which are quite well known, $\lambda = \sin \theta_C = 0.2205$ and $\Lambda \simeq |V_{cb}|/\lambda^2 \sim 0.8$, and two that are not well determined, $\rho$ and $\eta$ whose values define the apex of the Unitarity Triangle. In the $\rho\eta$-plane, $V_{td}$ is represented by a circle centered at (1,0). It provides one of three constrains on the Unitarity Triangle, the other two being $V_{ts}$ and $\epsilon_K$ (figure 2). $\Delta m_d$ and $\Delta m_s$ are given by:

$$\Delta m_d \propto A^2 \lambda^6 \left[ (1 - \rho)^2 + \eta^2 \right],$$

$$\Delta m_s \propto A^2 \lambda^4.$$
in terms of $\rho$, $\eta$, $\lambda$, and $A$. Note that, unlike $\Delta m_d$, $\Delta m_s$ has no dependence on $\rho$ and $\eta$. Therefore, a precise measurement of $\Delta m_s$ provides in essence a measure of the product $f_{B_s} \sqrt{B_{B_s}}$.

Another important aspect is the fact that $B_s$ mixing is complementary to the CP violation related measurements that will be performed at $B$ factories in the near future. The measurements of $V_{td}$ and $\sin 2\beta$ being essentially orthogonal, they will provide together an excellent constraint on the apex of the Unitarity Triangle in the $B$ system alone. Whereas, measurements of $V_{ub}$ and $\sin 2\beta$ alone will not be sufficient to provide as good a constraint.

Improvements in lattice calculations continue to be made. For the latest developments in the determination of the decay constant $f_{B_d}$ and the bag factor $B_{B_d}$, see the talk by S. Hashimoto at this conference. Further details can be found in Refs. [1, 2]. It is generally accepted that the product $f_{B_d} \sqrt{B_{B_d}}$ is in the range [160, 240] MeV.

1.1 Time-integrated mixing measurements

The probability that a $B^0$ meson mix into a $B^0$ or remain unmixed as a function of proper time is:

$$P_{B^0 \rightarrow B^0}(t) = \frac{\Gamma e^{-\Gamma t}}{2} (1 + \cos(\Delta m t))/2.$$ 

The time-integrated mixed fraction is given by:

$$\chi = \frac{(\Delta m/\Gamma)^2}{2[1 + (\Delta m/\Gamma)^2]}.$$ 

At the $\Upsilon(4S)$ resonance, where $B_d$ mesons are produced essentially at rest, $\Delta m_d$ can extracted from a measurement of $\chi_d$. Because $\Delta m_s$ is large (1/$\lambda^2$ $\approx$ 20 times larger than $\Delta m_d$), it can be obtained only in a time-dependent mixing measurement ($\chi_s$ $\sim$ 0.5), which makes it rather very challenging.

At the $Z^0$ resonance, where all $B$ species are produced, a measurement of $\bar{\chi} = f_s\chi_d + f_s\chi_s$ allows the extraction of $f_s \equiv f(B_s)$, the fraction of $b$ quarks that hadronize into a $B_s$.

1.2 Time-dependent mixing measurements

The requirements for a time-dependent mixing measurement are: 1) a precise reconstruction of the proper time of the $B$ meson, 2) a determination of the flavor $b$ or $\bar{b}$ of the $B$ meson both at production (initial-state tag) and at decay (final-state tag).

A variety of initial-state tags are used, they fall in three categories: 1) vertex and jet charge, high-momentum lepton and kaon, in the opposite hemisphere, 2) fragmentation kaon or pion in the same hemisphere, 3) polarized forward-backward asymmetry. The latter is unique to SLD, it exploits the parity-violating $Zb\bar{b}$ coupling and the presence of a highly polarized electron beam at the SLC.

The most widely used final-state tag is the signal of a high-$P_t$ lepton. Other tags include, for example, the sign of a partially or fully reconstructed $D_s^+$ meson, and the dipole charge (see below).

2. $B$ productions fractions

The LEP B Oscillations Working Group[3] has compiled a new value for $\bar{\chi} = 0.1186 \pm 0.0048$. Furthermore, with the inclusion of recent $\Delta m_d$ measurements by CLEO and CDF, mentioned below, a new world average value for $\chi_d$ was derived: $\chi_d = 0.176 \pm 0.009$.

A measurement of the charged $B$ branching ratio made by DELPHI[6] recently resulted in an increase of the $B_u$ production fraction: $f(B_u) = (40.5 \pm 1.2)\%$. While, the average $b$-baryon production fraction has decreased slightly: $(9.5 \pm 2.0)\%$. As a result, the new $B_s$ production fraction becomes:

$$f(B_s) = (9.6 \pm 1.3)\%.$$ 

This value is in good agreement with a separate determination using the measurement of the branching ratio $B(b \rightarrow B_s) \times B(B_s \rightarrow D_s^- l^+ \nu X)$ which yields: $f(B_s) = (9.7 \pm 2.3)\%$. The new value of $f(B_s)$ is somewhat smaller compared to that evaluated a year ago at the ICHEP’98 conference: $(10.8 \pm 1.3)\%$. This will have the effect of reducing one’s sensitivity to $\Delta m_s$, compared to one’s previous estimate assuming a larger value of $f(B_s)$.

3. $B_d$ mixing measurements

Over the last year there have been two new measurements of $\Delta m_d$: one from CLEO and the other
from CDF.

The CLEO time-integrated measurement is based on a 4.2 Million $B\bar{B}$ event sample. The final-state $b$ flavor is tagged using the sign of a slow pion in the partially-reconstructed decay chain: $B_d^0 \rightarrow D^{*+}\pi^- (\rho^-)$, $D^{*+} \rightarrow D^0\pi^+$. The initial-state is tagged with a high-momentum lepton in the rest of the event. A value of $\chi_d = 0.195 \pm 0.026 \pm 0.032$ is obtained, from which the following result for $\Delta m_d$ is extracted:

$$\Delta m_d = 0.512 \pm 0.053 (\text{stat}) \pm 0.032 (\text{syst}) \text{ ps}^{-1}.$$  

This result is competitive with those from LEP, CDF, and SLD obtained in time-dependent analyses.

At CDF, a new time-dependent measurement was performed. It consists of the reconstruction of a $B_d$ decaying to either a $D^+$ or a $D^{*+}$:

- $B_d^0 \rightarrow D^+ X$, $D^+ \rightarrow K^-\pi^+\pi^+$,
- $B_d^0 \rightarrow D^{*+} X$, $D^{*+} \rightarrow D^0\pi^+, D^0 \rightarrow K^-\pi^+$.

The sign of the $D^{(*)}$ tags the $b$ flavor of the $B_d$ at decay while its flavor at production is given by a high momentum lepton in the opposite hemisphere. A nice feature of this analysis is the fact that $\Delta m_d$ is extracted from a maximum likelihood fit in which the tagging dilution factor ($D_0$) is also fitted. Effectively, the mistag rate is determined directly from the data. figure contains a compilation of about 25 measurements of $\Delta m_d$ from the four LEP experiments, CLEO, CDF, and SLD, all very consistent with one another. The new world average value is:

$$\Delta m_d = 0.473 \pm 0.016 \text{ ps}^{-1}.$$  

Further improvements in $B_d$ mixing are expected from CLEO and SLD which have relatively large portions of data not yet analyzed.

### 4. $B_s$ mixing measurements

Over the last few years, a lot of efforts have been spent on $B_s$ mixing. Before reviewing some of the recent analyses, let us mention the first experimental limits on the ratio $\Delta \Gamma_s/\Gamma_s$ which provide indirect upper limits on $\Delta m_s$. Unlike $B_d$, recent calculations indicate that for $B_s$ the ratio $\Delta \Gamma/\Gamma$ is sizeable. Experimentally, the limits are derived by fitting in a $B_s$-enriched sample the proper time distribution to a sum of two exponentials. The best limit comes from DELPHI:

$$\Delta \Gamma_s/\Gamma_s < 0.42 \text{ at } 95\% \text{ CL.}$$

Limits on $\Delta \Gamma_s/\Gamma_s$ were also set by L3 and CDF. Though the above limit does not provide a strong constraint on $\Delta m_s (< 110 \text{ ps}^{-1})$, future improvements are expected as additional and perhaps improved measurements are made.

Three ingredients are required for a $\Delta m_s$ measurement, besides a large data sample. They are a high $B_s$ enrichment ($f_s$), a small mistag rate ($\eta$), and a good proper time resolution ($\sigma_t$). This is illustrated by the following expression which gives the statistical significance to a $\Delta m_s$ signal:

$$S = \sqrt{N/2} f_s (1 - 2\eta) \exp \left[-(\Delta m_s \sigma_t)^2/2 \right].$$

$N$ is the total number of $B$ decays in the sample. The proper time resolution can be expressed as...
| Experiment          | Preliminary Average | Uncertainty | Updated Data
|---------------------|---------------------|-------------|-----------------
| Preliminary Average | \(0.35\)             | \(0.4\)     | \(0.45\)       |
| SLD                 | \(0.473\)           | \(\pm 0.016\) | \(\pm 0.017\) |
| OPAL                | \(0.567\)           | \(\pm 0.029\) | \(\pm 0.028\) |
| L3                  | \(0.472\)           | \(\pm 0.049\) | \(\pm 0.042\) |

Figure 4: Measurements of \(\Delta m_d\).

\[ \sigma = \sigma_l^2 + \sigma_B^2 \]

where \(\sigma_l\) is the decay length resolution and \(\sigma_B\) is the boost resolution. Clearly, for a large value of \(\Delta m_s\), all the sensitivity is at very short proper time, and the decay length resolution is the dominant term there.

A widely used approach to reconstruct a \(B_s\) meson and tag its final-state \(b\) flavor is to intersect a high-\(P_t\) lepton with a downstream vertex which may be a fully- or partially-reconstructed \(D_s^\pm\) meson. Sometimes the \(B_s\) fraction in the sample is enhanced by requiring that a kaon or a \(\phi\) meson be present in this vertex.

The extraction of a \(\Delta m_s\) limit relies on the so-called Amplitude Fit method, where the \((1 - \cos \Delta m_s t)\) term in the above expression for the time-dependent mixed probability is replaced by \((1 - A \cos \Delta m_s t)\). Values of the amplitude parameter \(A\) are scanned for each trial value of \(\Delta m_s\) in a maximum likelihood fit. At the true mixing frequency, the amplitude \(A = 1\), and \(A = 0\) for all other values of \(\Delta m_s\). The 95% CL limit corresponds to the smallest value of \(\Delta m_s\) for which the measured amplitude and its error satisfy the relation: \(A + 1.645 \sigma_A = 1\). Similarly, the sensitivity of a given measurement corresponds to the smallest value of \(\Delta m_s\) for which \(1.645 \sigma_A = 1\) holds.

A total of eleven analyses have been performed by ALEPH, CDF, DELPHI, OPAL, and SLD. This summer, OPAL and DELPHI have produced updates on their \(B_s\) mixing results. These are described below. For completeness, important results from other experiments are also reviewed.

4.1 ALEPH inclusive lepton analysis

This is the most sensitive \(B_s\) mixing measurement to date. \(B_s\) mesons are reconstructed as the intersection of a high-\(P_t\) lepton and an inclusive \(D\) vertex. The final-state flavor is given
by the lepton sign, while the initial-state tag relies on vertex charge or a high-momentum lepton in the opposite hemisphere, or a fragmentation kaon in the decay hemisphere. A total of 33,023 decays are selected. The decay length resolution is modeled by a double-Gaussian distribution with a core (82%) $\sigma_l = 280 \, \mu m$ and a tail (18%) $\sigma_l = 1060 \, \mu m$. The relative boost resolution is estimated by $\gamma = 7.1\%$ in the core ($72\%$) and $\gamma = 21\%$ in the tails ($28\%$).

Figure 5 shows the amplitude vs. $\Delta m_s$ distribution obtained in this analysis. The derived limit is:

$$\Delta m_s > 9.5 \, ps^{-1} \text{ at 95\% CL},$$

with a sensitivity of $9.6 \, ps^{-1}$. Combined with two other analyses ($D_s^0$ and $D_s^h$), ALEPH's sensitivity becomes $10.6 \, ps^{-1}$ and the overall limit is:

$$\Delta m_s > 9.6 \, ps^{-1} \text{ at 95\% CL}.$$

4.2 OPAL inclusive lepton update

OPAL uses a rather sophisticated technique to reconstruct in 3-D a $B_s$ vertex that contains a high-$P_t$ lepton [15]. The final-state is tagged with the lepton sign, while the initial-state is given by either jet charge or a high-$P_t$ lepton in the opposite hemisphere. In this update, OPAL included their 1995 data and made substantial improvements to their jet charge technique. The final analysis is based on 47,109 single-lepton and 6,031 di-lepton events. The amplitude vs. $\Delta m_s$ distribution is shown in Figure 6. The sensitivity is $7.0 \, ps^{-1}$ and the limit is:

$$\Delta m_s > 5.2 \, ps^{-1} \text{ at 95\% CL}.$$

4.3 DELPHI exclusive $B_s$ and $D_s^+ h^+$ update

DELPHI has made the first attempt at an exclusive $B_s$ reconstruction [11]. This approach has the advantage of an excellent proper time resolution, with the $B_s$ boost being determined exactly. The following decay channels are utilized:

$$B_s \to D_s^- \pi^+ \, (a_1^+),$$
$$B_s \to D^0 K^- \pi^+ \, (a_1^+),$$

where the $D_s^-$ and $D^0$ are reconstructed in 6 and 2 decay modes, respectively. From the full data sample corresponding to 3.6 Million hadronic $Z$’s, 11 fully- and 33 partially-reconstructed $B_s$ decays are selected with a corresponding $B_s$ purity of $70\%$ and $45\%$, respectively. With such a small efficiency this analysis by itself has no sensitivity to $\Delta m_s$. However, because of its excellent proper time resolution it provides a non-negligible contribution at large $\Delta m_s$ values.

DELPHI choose to combine it with the more inclusive $D_s^+ h^+$ ($D_s \to \phi \pi, K^* K$) analysis [11].
where 2953 candidates are selected with a $B_s$ purity of 40%. The overall sensitivity so obtained is 3.2 ps$^{-1}$ and the 95% CL limit is 4.0 ps$^{-1}$.

4.4 DELPHI $D_s^+ l^\tau$ update

This is overall the second most-sensitive $B_s$ mixing analysis\footnote{[11]}. A total of 436 decays with a fully-reconstructed $D_s$ are selected in the decay modes $D_s^+ \rightarrow \phi \pi^+$, $K^* K^+$, $\phi l^+ \nu$. Another 441 candidates with a partially-reconstructed (missing $\gamma/\pi^0$) $D_s^+ \rightarrow \phi l^+ X$ are also selected. The $B_s$ content in the two sub-samples is estimated to be $230 \pm 18$ and $41 \pm 12$ $B_s$’s, respectively. Clearly, the $D_s$ sign provides the final-state tag, while a complex initial-state package including 9 discriminant variables gives the initial-state $b$ flavor with a 78% purity. The decay length resolution in this analysis is parametrized with a core (86%) Gaussian with $\sigma_1 = 220 \mu$m and a tail (14%) Gaussian with $\sigma_1 = 870 \mu$m. Whereas, the boost resolution is characterized by a 5.4% relative residual in the core (78%) and 17.2% in the tails (22%). The 95% CL limit extracted from this analysis is $\Delta m_s > 5.8$ ps$^{-1}$ at 95% CL, and a sensitivity of 5.1 ps$^{-1}$, as shown by the amplitude distribution of figure 8.

4.5 CDF $\ell l / l$ analysis

Using their di-lepton trigger data, CDF\footnote{[16]} perform a partial reconstruction of the $B_s$ in the decay chain $B_s \rightarrow D_s l^\nu X \rightarrow \phi l^\nu X$. A sample of 1068 candidates is selected with a $B_s$ purity of 61%. A second high-momentum lepton in the opposite hemisphere tags the initial-state flavor. This analysis results in a limit of:

$$\Delta m_s > 5.8 \text{ ps}^{-1} \text{ at } 95\% \text{ CL},$$

and a sensitivity of 5.1 ps$^{-1}$, as shown by the amplitude distribution of figure 8.

4.6 SLD $B_s$ mixing results

Using a data sample equivalent to 350k hadronic $Z^0$’s, SLD has performed three $B_s$ mixing analyses, referred to as Lepton+$D$, Lepton+$Tracks$, and Dipole\footnote{[17]}. The number of selected candidates in the three analyses is 2352, 8864, and 8211, respectively. In the first two, the final-state tag is provided by the high-$P_t$ lepton, while in the last one it is given by the dipole charge, defined as the charge difference between the tertiary and secondary vertices multiplied by the distance separating the two vertices. The initial-state tag is given primarily by the polarized forward backward asymmetry, and by jet charge, vertex charge, a high-momentum lepton or kaon in the opposite hemisphere. An excellent decay length resolution

Figure 7: Amplitude distribution for DELPHI’s $D_s l$ analysis.

Figure 8: Amplitude distribution for CDF’s $\ell l/l$ analysis.
is achieved at SLD, characterized by a core (60%) Gaussian with $\sigma_l = 105-130 \, \mu m$ and a tail (40%) Gaussian with $\sigma_l = 330 - 550 \, \mu m$. Good boost resolution is also realized: $\sigma_{p\gamma}/\beta\gamma = 7 - 9 \%$ for the core (60%) and $\sigma_{p\gamma}/\beta\gamma = 22 - 30 \%$ for the tails (40%). The combined amplitude distribution for the three analyses is shown in figure 9. An overall sensitivity of 6.6 ps$^{-1}$ is obtained, and two separate intervals in $\Delta m_s$ are excluded at the 95% CL:

$$\Delta m_s < 5.3 \quad \text{and} \quad 6.0 < \Delta m_s < 11.5 \, \text{ps}^{-1}.$$  

4.7 Summary of $\Delta m_s$ results

In the following, a summary as of the end of July 1999 of all $B_s$ mixing results compiled by the LEP B Oscillations Working Group[5] is given. figure 11 gives the measured amplitude and its error at a $\Delta m_s$ value of 15 ps$^{-1}$, as well as the sensitivity of each analysis. The world average amplitude at $\Delta m_s = 15 \, \text{ps}^{-1}$ is $2.2 \, \sigma$ away from zero ($1.53 \pm 0.69$).

The world average amplitude vs. $\Delta m_s$ distribution is shown in figure 12. The overall sensitivity is 14.2 ps$^{-1}$ and the combined limit:

$$\Delta m_s > 12.4 \, \text{ps}^{-1} \quad \text{at 95\% CL.}$$

Note that this limit does not take into account the new smaller value of the $B_s$ production fraction, as discussed above. Compared to last year’s results at the ICHEP’98 conference[18], the $\Delta m_s$ limit is unchanged, while the overall sensitivity has improved slightly (up from 13.8 ps$^{-1}$). The contribution of the limit on $\Delta m_s$ to the determination of the CKM matrix is summarized in Ref.[19].

The bump seen in the amplitude distribution at around 15 ps$^{-1}$ has been the subject of a lot interest. In a recent contribution, D. Abbaneo and G. Boix[20] proposed a procedure to estimate the probability that the observed structure is due to a pure statistical fluctuation. They ran 2000 fast Monte Carlo experiments with a large value of $\Delta m_s$ (150 ps$^{-1}$), each experiment being equivalent to the world sensitivity as of last winter (Moriond’99)[21]. They convert the measured amplitudes into likelihood profiles as a function of $\Delta m_s$. From these profiles which are shown in figure 10, they estimate the probability that the bump in the data (represented by the triangle in the figure) to originate from a fluctuation in a no-signal sample to be $\sim 3 - 5 \%$.  

\newpage 

Figure 9: Combined amplitude distribution for the Lepton+D, Lepton+Tracks, and Dipole analyses at SLD.

Figure 10: Likelihood profiles for 2000 toy Monte Carlo samples with $\Delta m_s = 150 \, \text{ps}^{-1}$. The triangle represents the experimental data as of winter’99.
Figure 11: Measured amplitude at $\Delta m_s = 15 \text{ ps}^{-1}$ for each analysis.

Figure 12: World average amplitude vs. $\Delta m_s$ distribution.
4.8 SLD status and future prospects

Final $B_s$ mixing results from SLD are expected in the near future. Since Moriond’99, substantial improvements have been achieved, they come from the following three sources:

- Improved tracking resolution.
- Improved charge dipole reconstruction.
- Addition of two new analyses.

The design resolution with the pixel vertex detector VXD3 has been realized. It is characterized by a track impact parameter resolution of $7.8 \mu m$ in the $r\phi$-plane and $9.7 \mu m$ in the $rz$-plane. Furthermore, the precise location of the micron-size SLC beam spot is determined with an uncertainty of $4 \mu m$.

![Figure 13: SLD’s dipole charge distribution for data (points) and Monte Carlo (histogram). The dotted and dashed histograms represent the $b$- and $\bar{b}$ components, respectively.](image)

Improvements in the SLD’s topological vertexing algorithm have resulted in a significantly better dipole tag purity (80% compared to 73% previously). The dipole tag, which is unique to SLD, exploits the charge flow in the cascade decay $b \rightarrow c$, and is made possible due to the ability to reconstruct (topologically) well-separated $B$ and $D$ vertices. The dipole charge is reconstructed as the charge difference between the $B$ and $D$ vertices multiplied by the distance separating them. It is shown in figure 13, where the $b$-quark and $\bar{b}$-quark components are represented by the dotted and the dashed histograms, respectively.

One of the two new analyses at SLD, called the $D_s + Tracks$ analysis, relies on the exclusive reconstruction of a $D_s^+$ in the $\phi\pi^+$ and $K^{*0}K^+$ decay modes. It provides both a high $B_s$ purity of 33%, and an excellent decay length resolution $\sigma_l = 46 \mu m$ in the core (60%) and $\sigma_l = 158 \mu m$ in the tails (40%).

In the second new SLD analysis, called the $Lepton + Kaon$ analysis, final-states containing an opposite sign lepton – kaon pair are selected. It is aimed primarily at isolating the semileptonic decays: $B_s \rightarrow D_s^{**+}l^+\nu$, $D_s^{**-} \rightarrow K^-\bar{D}(s)^{(*)0}$. Decays of $B^\pm$ and $B_d$ mesons produce lepton – kaon pairs with the same sign and are therefore suppressed. Thus, resulting in an enhancement of the $B_s$ purity. This analysis is also sensitive to the decays $B_s \rightarrow D_s^-l^-\nu$, $D_s^- \rightarrow \phi X$, $\phi \rightarrow K^-K^+$, in particular with the additional requirement of a second kaon in the $B_s$ vertex. The $B_s$ purity that is achieved is 26%, while a very good decay length resolution is also obtained with $\sigma_l = 71 \mu m$ in the core (60%) and $\sigma_l = 330 \mu m$ in the tails (40%). Both these new analyses have the disadvantage of a small overall efficiency. They have a small sensitivity to $\Delta m_s$, but provide a substantial contribution at high values of $\Delta m_s$ when combined with the other SLD analyses.

![Figure 14: Estimated improvement in SLD’s sensitivity and its impact on the world average.](image)

With these improvements, SLD’s overall $\Delta m_s$
sensitivity is estimated to increase to \( \sim 13 \text{ ps}^{-1} \), and to dominate the world average for \( \Delta m_s > 13 \text{ ps}^{-1} \). The combined LEP+CDF+SLD sensitivity becomes \( \sim 16 \text{ ps}^{-1} \). This is illustrated in figure 14.

Several new experiments\(^{23,24,25}\) specifically geared for \( B \) physics, \( CP \) violation in particular, are planned for the next decade. Their expected reach for \( B_s \) mixing is shown in figure 15. It is anticipated that CDF will be the first experiment to measure \( \Delta m_s \), with a sensitivity of \( 40 \text{ ps}^{-1} \). Note that SLD’s reach could have been extended to \( \Delta m_s = 20 \text{ ps}^{-1} \). However, the run extension that was requested in order to achieve that was not approved due to lack of funding.

\[ f(B_s) = (9.6 \pm 1.3\%) \]  

A relatively significant (\( \sim 2\sigma \)) bump persists in the amplitude spectrum at around \( \Delta m_s = 15 \text{ ps}^{-1} \). An attempt at estimating the probability that this may be due to a statistical fluctuation was performed by G. Abbaneo and G. Boix (using winter’99 data). Their result is a probability of \( \sim 3 - 5\% \).

We have also seen the first attempts at constraining \( \Delta m_s \) from above, by setting a lower limit on the ratio \( \Delta\Gamma_s/\Gamma_s \). The best limit comes from DELPHI:

\[ \Delta\Gamma_s/\Gamma_s < 0.42 \text{ at } 95\% \text{ CL}. \]

In the near future, final results from SLD are expected, with substantial improvements coming from a better tracking resolution, an improved dipole charge reconstruction, and the inclusion of two new analyses. As of this writing, a partial update from SLD\(^{26}\) has already been presented at the Lepton-Photon’99 conference. In fact the world average \( \Delta m_s \) limit and sensitivity compiled there\(^{27,5}\) supersedes the above quoted values.

In the longer term, many \( B \) physics experiments are planned well into the next decade. With a \( \Delta m_s \) reach extending over a very wide range, a precise measurement of \( \Delta m_s \) will be performed by these experiments.

5. Conclusions

We have presented a review of \( B \) mixing results as of the end of July 1999. For \( B_d \) mixing, with a new measurement from CDF, the new world average for \( \Delta m_d \) becomes:

\[ \Delta m_d = 0.473 \pm 0.016 \text{ ps}^{-1}. \]

In \( B_s \) mixing, updates from OPAL and DELPHI were produced recently. The new world average limit is:

\[ \Delta m_s > 12.4 \text{ ps}^{-1} \text{ at } 95\% \text{ CL}, \]

and the new world average sensitivity is \( 14.2 \text{ ps}^{-1} \).

This, without taking into account the smaller value of the \( B_s \) production fraction, which is evaluated by the LEP B Oscillations Working Group to be:

\[ f(B_s) = (9.6 \pm 1.3\%) \]

Acknowledgments

I would like to thank my colleagues David Jackson, John Jaros, and Stéphane Willocq for very fruitful discussions.
# References


See also:


