Concavity of the $Q\bar{Q}$ potential and AdS/CFT duality

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ABSTRACT: We review our study of generalised concavity conditions for potentials between static sources obtained from Wilson loops coupling both to gauge bosons and a set of scalar fields. It involves the second derivatives with respect to the distance in ordinary space as well as with respect to the relative orientation in internal space. In addition we discuss the use of this field theoretical condition as a nontrivial consistency check of the AdS/CFT duality.

1. Introduction

The AdS/CFT correspondence maps [1, 2, 3, 4], in its most prominent example, $\mathcal{N}=4$, SU(N) super Yang-Mills gauge theory on the conformal boundary of AdS_5 to the type IIB string theory in $AdS_5\times S^5$ background. An essential part of the mapping recipe equates the Wilson loop $W(\mathcal{C})$ in the gauge theory and the string partition function $Z(\mathcal{C})$ under the boundary condition that the string world sheet takes the contour $\mathcal{C}$ as its boundary [5, 6]

$$W(\mathcal{C}) \;=\; Z(\mathcal{C}) \;=\; e^{-A(\mathcal{C})}\;. \qquad (1.1)$$

The last equation in (1.1), with A denoting the area of the minimal surface in AdS sense, is valid in the large N limit and for large t'Hooft coupling $g_{YM}^2 N$.

To be more precise, the Wilson loop under consideration is given by

$$W(\mathcal{C}) = \mathrm{tr}\big(P\exp\int_{\mathcal{C}}\{iA_\mu(x(s))\dot{x}^\mu(s) + \phi_I(x(s))\theta^I(s)|\dot{x}|\}ds\big)\;, \qquad (1.2)$$

with A_μ, ϕ_I denoting the gauge and scalar fields of $\mathcal{N}=4$ SYM, respectively. $x^\mu(s)$ specifies the contour $\mathcal{C}$ in 4-dim. (Euclidean) space-time and $\theta^I(s)$ (with $(\theta^I)^2=1$) fixes the corresponding S^5 part. The Wilson loop with the structure (1.2) is a special case of the generic one and has BPS properties [7].

By standard arguments the Wilson loop can be related to the potential between static sources separated by a distance L in ordinary space and an angle Θ between $\theta_{\bar{Q}}$ and θ_Q on S^5. Then the relevant contour $\mathcal{C}$ is a rectangle of size $L\times T$ in ordinary space-time and the S^5 part stays at the position $\theta_{\bar{Q}}$ or θ_Q along the large T-sides of the rectangle and interpolates linearly on the great circle through $\theta_{\bar{Q}}$ and θ_Q along the L-sides of the rectangle. As a result one finds

$$V(L,\Theta) \;=\; -\lim_{T\to\infty}\frac{1}{T}\log W(\mathcal{C})\;. \qquad (1.3)$$

First principles of quantum field theory like Osterwalder-Schrader reflection positivity have been shown to enforce concavity for the static $Q\bar{Q}$-potential, $\frac{d^2V(L)}{dL^2}\le 0$, in usual non-SUSY Yang-Mills, [8, 9]. Following our paper [10], we now discuss the search for analogous statements for the static potential $V(L,\Theta)$ in $\mathcal{N}=4$ SYM. Before going into the details, let us recall the virtues of such inequalities in the context of AdS/CFT. Since they test the consequences of OS positivity, they are one of the rare occasions where one is able to check the AdS/CFT correspondence without extensive input by the superconformal invariance. In addition, it makes sense to test the arising inequalities already for approximate potentials. If they would be violated in the limit where the approximation is known to be-

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come reliable, this would indicate a breakdown of the AdS/CFT correspondence.

2. Generalised concavity for potentials derived from BPS Wilson loops

We start with the functional ($A\dot{x} = A_\mu \dot{x}^\mu$, $\phi\theta = \phi_j \theta^j$, $\mu = 0,..,3$, $j = 4,..,9$)

$$U_{ab}[x,\theta] = \left(P \exp \int \{iA(x(s))\dot{x}(s) + \phi(x(s))\theta(s)|\dot{x}|\} ds\right)_{ab}. \quad (2.1)$$

The expectation value of its trace for a closed path $x(s)$ yields the Wilson loop under investigation.

A reflection operation $\mathcal{R}$ is defined by

$$(\mathcal{R}x)^1(s) = -x^1(s) \; ; \quad (\mathcal{R}x)^\alpha(s) = x^\alpha(s), \quad \alpha \neq 1$$
$$\mathcal{R}U_{ab}[x,\theta] = \overline{U_{ab}[\mathcal{R}x,\theta]} \; . \quad (2.2)$$

In addition, it is useful to define in connection with an isometry $\mathcal{I} \in O(6)$ of S^5 acting on the path $\theta(s)$

$$\mathcal{I}U_{ab}[x,\theta] = U_{ab}[x,\mathcal{I}\theta] \; . \quad (2.3)$$

For linear combinations of U's for different contours we extend $\mathcal{R}$ and $\mathcal{I}$ linearly.

Using the hermiticity of the matrices A, ϕ in the form $\overline{A} = A^t$, $\overline{\phi} = \phi^t$ and introducing

$$(\mathcal{B}x)(s) = x(s_f + s_i - s), \; (\mathcal{B}\theta)(s) = \theta(s_f + s_i - s). \quad (2.4)$$

we get [10]

$$\overline{U_{ab}[x,\theta]} = U_{ba}[\mathcal{B}x, \mathcal{B}\theta] \; . \quad (2.5)$$

This, combined with (2.2),(2.3) yields finally

$$\mathcal{R}\mathcal{I}U_{ab}[x,\theta] = U_{ba}[\mathcal{B}\mathcal{R}x, \mathcal{B}\mathcal{I}\theta] \; . \quad (2.6)$$

It is worth pointing out that for the result (2.5) the presence/absence of the factor i in front of the A and ϕ term in U is crucial. One could consider this as another argument for the choice favoured by the investigations of ref. [7].

We now turn to a derivation of the basic Osterwalder-Schrader positivity condition in a streamlined form within the continuum functional integral formulation. All steps can be made rigorously by a translation into a lattice version with local and nearest neighbour interactions.

Let denote $H_\pm = \{x^\mu | \pm x^1 > 0\}$, $H_0 = \{x^\mu | x^1 = 0\}$. Then we consider for a functional of two paths $x^{(1)}, x^{(2)} \in H_+$, λ real

$$f[x^{(1)},\theta^{(1)}; x^{(2)},\theta^{(2)}] = U_{ab}[x^{(1)},\theta^{(1)}] + \lambda U_{ab}[x^{(2)},\theta^{(2)}], \quad (2.7)$$

$$\begin{aligned}\langle f[x,\theta]\mathcal{R}\mathcal{I}f[x,\theta]\rangle &= \int \mathcal{D}A\mathcal{D}\phi f[x,\theta]\overline{f[\mathcal{R}x,\mathcal{I}\theta]}\; e^{-S} \\ &= \int \mathcal{D}A^{(0)}\mathcal{D}\phi^{(0)}\; e^{-S_0} \\ &\cdot \int_{(b.c.)} \mathcal{D}A^{(+)}\mathcal{D}\phi^{(+)} f[x,\theta]\; e^{-S_+} \\ &\cdot \int_{(b.c.)} \mathcal{D}A^{(-)}\mathcal{D}\phi^{(-)} \overline{f[\mathcal{R}x,\mathcal{I}\theta]}\; e^{-S_-} \quad (2.8)\end{aligned}$$

$\pm$, 0 on the fields as well as on the action indicates that it refers to points in $H_\pm$, H_0. The index for the two paths has been dropped, and the boundary condition ($b.c.$) is

$$A^{(\pm)}|_{\partial H_\pm} = A^{(0)}, \quad \phi^{(\pm)}|_{\partial H_\pm} = \phi^{(0)} \; .$$

With the abbreviation

$$h[A^{(0)},\phi^{(0)},x,\theta] = \int_{(b.c.)} \mathcal{D}A^{(+)}\mathcal{D}\phi^{(+)} f[x,\theta] e^{-S_+}, \quad (2.9)$$

the standard reflection properties of the action imply

$$\begin{aligned}\langle f[x,\theta]\mathcal{R}\mathcal{I}f[x,\theta]\rangle &= \int \mathcal{D}A^{(0)}\mathcal{D}\phi^{(0)}\; e^{-S_0} \\ &\cdot h[A^{(0)},\phi^{(0)},x,\theta] \cdot \overline{h[A^{(0)},\phi^{(0)},x,\mathcal{I}\theta]} \; . \quad (2.10)\end{aligned}$$

For $\mathcal{I} = \mathbf{1}$ the integrand of the final integration over the fields in the reflection hyperplane H_0 is non-negative, hence

$$\langle f[x,\theta]\mathcal{R}f[x,\theta]\rangle \geq 0 \; . \quad (2.11)$$

For nontrivial $\mathcal{I}$ the situation is by far more involved. If there would be no boundary condition, the result of the half-space functional integral in (2.9) would be invariant with respect to

$\theta \to \mathcal{I}\theta$. A given boundary configuration in general breaks $O(6)$ invariance on S^5. But due to the $O(6)$ invariance of the action, the functional integration measure and the $\phi\theta$ coupling in f, we have instead

$$h[A^{(0)},\mathcal{I}\phi^{(0)},x,\mathcal{I}\theta] \;=\; h[A^{(0)},\phi^{(0)},x,\theta]\ . \tag{2.12}$$

This implies

$$\langle f[x,\theta]\mathcal{R}\mathcal{I}f[x,\theta]\rangle = \int \mathcal{D}A^{(0)}\mathcal{D}\phi^{(0)}e^{-S_0} \tag{2.13}$$
$$\cdot\ \frac{1}{2}\Big(\ h[A^{(0)},\phi^{(0)},x,\theta]\ \overline{h[A^{(0)},\phi^{(0)},x,\mathcal{I}\theta]}$$
$$+h[A^{(0)},\phi^{(0)},x,\mathcal{I}^{-1}\theta]\ \overline{h[A^{(0)},\phi^{(0)},x,\theta]}\ \Big)\ ,$$

which says us only (R real numbers)

$$\langle f[x,\theta]\mathcal{R}\mathcal{I}f[x,\theta]\rangle \in R \quad \text{for}\ \ \mathcal{I}^2 = \mathbf{1}\ . \tag{2.14}$$

The statements (2.11) and (2.14) are rigorous ones. Beyond them we found no real proof for sharpening (2.14) to an inequality of the type (2.11) for some nontrivial $\mathcal{I}$. For later application to the estimate of rectangular Wilson loops we are in particular interested in nontrivial isometries keeping the, by assumption common, S^5 position of the endpoints of the contours on H_0 fixed. Then $\mathcal{I} = \mathcal{I}_\pi$, denoting a rotation around this fixpoint with angle π, are the only candidates.

At least for boundary fields $\phi^{(0)}$ in (2.9), which as a map $R^3 \to S^5$ have a homogeneous distribution of their image points on S^5, we can expect that for contours of the type discussed in connection with fig.1 below in the limit of large T the orientation of θ relative to $\phi^{(0)}$ becomes unimportant. Therefore, we conjecture for this special situation

$$\langle f[x,\theta]\mathcal{R}\mathcal{I}_\pi f[x,\theta]\rangle\ \geq\ 0\ . \tag{2.15}$$

From (2.11) and (2.15) for any real λ in (2.7) we get via the standard derivation of Schwarz-type inequalities

$$\langle U_{ab}[x^{(1)},\theta^{(1)}]\ \mathcal{R}\mathcal{I}U_{ab}[x^{(2)},\theta^{(2)}]\rangle^2$$
$$\leq \langle U_{ab}[x^{(1)},\theta^{(1)}]\ \mathcal{R}\mathcal{I}U_{ab}[x^{(1)},\theta^{(1)}]\rangle \tag{2.16}$$
$$\cdot\ \langle U_{ab}[x^{(2)},\theta^{(2)}]\ \mathcal{R}\mathcal{I}U_{ab}[x^{(2)},\theta^{(2)}]\rangle\ .$$

This is a rigorous result for $\mathcal{I} = \mathbf{1}$ and a conjecture for $\mathcal{I} = \mathcal{I}_\pi$.

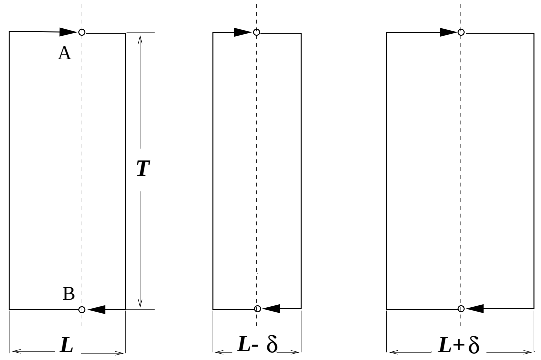

Fig.1 *From left to right the contours* $x^+\circ x^-,\ \ x^+\circ\mathcal{B}\mathcal{R}x^+,\ \ \mathcal{B}\mathcal{R}x^-\circ x^-$.

Let us continue with the discussion of a Wilson loop for a closed contour which crosses the reflection hyperplane twice and which is the result of going first along $x^-\in H_-$ and then along $x^+\in H_+$. In addition we restrict to cases of coinciding S^5 position at the intersection points with H_0 and treat in parallel $\mathcal{I} = \mathbf{1},\mathcal{I}_\pi$ With (2.5),(2.6) we get [10]

$$W[x^+\circ x^-,\theta^+\circ\theta^-]$$
$$\leq \big(W[x^+\circ\mathcal{B}\mathcal{R}x^+,\theta^+\circ\mathcal{B}\mathcal{I}\theta^+]\big)^{\frac{1}{2}}$$
$$\cdot\ \big(W[\mathcal{B}\mathcal{R}x^-\circ x^-,\mathcal{B}\mathcal{I}^{-1}\theta^-\circ\theta^-]\big)^{\frac{1}{2}}\ . \tag{2.17}$$

To evaluate the potential between two static sources ($Q\bar{Q}$) separated by the distance L and located at fixed S^5-positions $\theta_Q,\ \theta_{\bar{Q}}$ we need Wilson loops for rectangular contours of extension $L\times T$ in the large T-limit. We choose the S^5-position on the two L-sides linearly interpolating between θ_Q and $\theta_{\bar{Q}}$ on the corresponding great circle. For this restricted set of contours the Wilson loop becomes a function of $L,\ T$ and the angle between θ_Q and $\theta_{\bar{Q}}$, called Θ.

In addition it is useful to restrict ourselves to contours which are situated in planes orthogonal to the reflection hyperplane and with T-sides running parallel to it in a distance $\frac{L\pm\delta}{2}$, see fig.1. Then $\mathcal{I} = \mathcal{I}_\pi$ reflects $\theta^\pm(s)$, which both lie on the great circle through θ_Q and $\theta_{\bar{Q}}$, with respect to the common S^5-position of the points A and B, see fig.1. As a consequence, (2.17) implies

$$W(L,T,\Theta) \;\leq\; \Big(W(L-\delta,T,\frac{L-\delta}{L}\Theta)\Big)^{\frac{1}{2}} \cdot \Big(W(L+\delta,T,\frac{L+\delta}{L}\Theta)\Big)^{\frac{1}{2}}\;, \tag{2.18}$$

which by standard reasoning yields for the static potential

$$V(L,\Theta) \;\geq\; \frac{1}{2}\Big(V(L-\delta,\frac{L-\delta}{L}\Theta) + V(L+\delta,\frac{L+\delta}{L}\Theta)\Big) \tag{2.19}$$

The last inequality implies the local statement $\frac{d^2}{d\delta^2}V(L+\delta,\frac{L+\delta}{L}\Theta)\leq 0$, i.e.

$$\Big(L^2\frac{\partial^2}{\partial L^2}+2L\Theta\frac{\partial^2}{\partial L\partial\Theta}+\Theta^2\frac{\partial^2}{\partial\Theta^2}\Big)V(L,\Theta)\leq 0\;. \tag{2.20}$$

It means concavity on each straight line across the origin, in the relevant part of the (L,Θ)-plane, $0<L<\infty,\;\; 0<\Theta\leq\pi$.

Both (2.19) and (2.20) rely on the conjecture (2.15). From the rigorous point of view we are allowed to use (2.17) for $\mathcal{I}=\mathbf{1}$ only. Then the paths generated on the r.h.s. are, with respect to their S^5 properties, no longer of the type with which we started on the l.h.s. On the part of the space time contour orthogonal to H_0 we go e.g. from θ_Q to the common S^5 position of the points A and B and then *back* to θ_Q. Since in the large T-limit, relevant for the extraction of the $Q\bar{Q}$-potential, only the behaviour on the large T-sides matters, we get

$$V(L,\Theta) \;\geq\; \frac{1}{2}\left(V(L-\delta,0)\;+\;V(L+\delta,0)\right)\;. \tag{2.21}$$

This means standard concavity at $\Theta=0$ *and*

$$V(L,\Theta)\;\geq\;V(L,0)\;. \tag{2.22}$$

If the same steps are repeated for rectangles with large T-sides still parallel to H_0, but spanning a plane no longer orthogonal to H_0 one finds

$$V(L,\Theta)\geq\frac{1}{2}\left(V(\alpha(L-\delta),0)+V(\alpha(L+\delta),0)\right),\quad 0\leq\alpha\leq 1\;. \tag{2.23}$$

The only new information gained from (2.23) is that $V(L,0)$ is monotonically non-decreasing in L.

3. Test of the generalised concavity condition for potentials derived via AdS/CFT duality

The simplicity of the calculation recipe for Wilson loops in the classical SUGRA approximation via AdS/CFT duality allows to make statements on universal properties of the arising $Q\bar{Q}$-potential for a large class of SUGRA backgrounds [11, 12]. We now enter a discussion of (2.20) within this framework. The metric of the SUGRA background is assumed in the form

$$G_{MN}dx^Mdx^N \;=\; G_{00}(u)dx^0dx^0 \;+\; G_{||}(u)dx^mdx^m + G_{uu}(u)dudu \;+\; G_{\Omega}(u)d\Omega_5^2\;. \tag{3.1}$$

Then with

$$f(u)\;=\;G_{00}G_{||},\quad g(u)\;=\;G_{00}G_{uu},\quad j(u)=G_{00}G_{\Omega} \tag{3.2}$$

we get along the lines of [6, 11, 12, 13]

$$\begin{aligned}
L^{(\Lambda)} \;&=\; 2\sqrt{f_0}\sqrt{1-l^2}\cdot\int_{u_0}^{\Lambda}\sqrt{\frac{gj}{f}}\frac{du}{\sqrt{j(f-f_0)+(jf_0-j_0f)l^2}}\;,\\
\Theta^{(\Lambda)} \;&=\; 2l\sqrt{j_0}\cdot\int_{u_0}^{\Lambda}\sqrt{\frac{gf}{j}}\frac{du}{\sqrt{j(f-f_0)+(jf_0-j_0f)l^2}}\;,\\
V^{(\Lambda)} \;&=\; \frac{1}{\pi}\cdot\int_{u_0}^{\Lambda}\sqrt{gfj}\;\frac{du}{\sqrt{j(f-f_0)+(jf_0-j_0f)l^2}}\;.
\end{aligned} \tag{3.3}$$

We defined $f_0=f(u_0)$ etc. Λ is a cutoff at large values of u. In the following our discussion will be restricted to values of L and Θ for which all expressions appearing under square roots above are positive and where the inversion $u_0=u_0(L,\Theta)$, $l=l(L,\Theta)$ is well defined. (3.3) implies

$$V^{(\Lambda)}=\frac{1}{\pi}\int_{u_0}^{\Lambda}\sqrt{\frac{g}{fj}}\sqrt{j(f-f_0)+(jf_0-j_0f)l^2} +\frac{1}{2\pi}\sqrt{f_0}\sqrt{1-l^2}\;L^{(\Lambda)}+\frac{1}{2\pi}\sqrt{j_0}\;l\;\Theta^{(\Lambda)}. \tag{3.4}$$

Now we differentiate with respect to u_0 and l. After this Λ can be sent to ∞ ending up with a relation for the renormalised potential V:

$$\frac{\partial V}{\partial u_0}=\frac{1}{2\pi}\sqrt{f_0}\sqrt{1-l^2}\frac{\partial L}{\partial u_0}+\frac{1}{2\pi}\sqrt{j_0}l\frac{\partial\Theta}{\partial u_0},$$
$$\frac{\partial V}{\partial l}=\frac{1}{2\pi}\sqrt{f_0}\sqrt{1-l^2}\ \frac{\partial L}{\partial l}+\frac{1}{2\pi}\sqrt{j_0}l\frac{\partial\Theta}{\partial l}. \quad (3.5)$$

For V defined by (3.3) implicitly as a function of L and Θ this means

$$\frac{\partial V}{\partial L}\ =\ \frac{1}{2\pi}\sqrt{f_0}\sqrt{1-l^2},\quad \frac{\partial V}{\partial \Theta}\ =\ \frac{1}{2\pi}\sqrt{j_0}\ l\ , \quad (3.6)$$

i.e. V is monotonically nondecreasing both in L and Θ. The monotony in Θ is in agreement with our rigorous result (2.22).

Calculating now second derivatives one arrives at ($f_0'=\frac{df(u_0)}{du_0}$ etc.)

$$\Big(L^2\frac{\partial^2}{\partial L^2}+2L\Theta\frac{\partial^2}{\partial L\partial\Theta}+\Theta^2\ \frac{\partial^2}{\partial\Theta^2}\Big)V(L,\Theta)$$
$$=\frac{1}{4\pi\sqrt{f_0j_0}}\ (Lf_0'\sqrt{j_0(1-l^2)}+\Theta j_0'\sqrt{f_0}\ l)\cdot(L\frac{\partial u_0}{\partial L}+\Theta\frac{\partial u_0}{\partial\Theta})$$
$$+\frac{1}{2\pi\sqrt{1-l^2}}\ (\Theta\sqrt{j_0(1-l^2)}-L\sqrt{f_0}\ l)\cdot(L\frac{\partial l}{\partial L}+\Theta\frac{\partial l}{\partial\Theta})\ . \quad (3.7)$$

Neglecting for a moment the issue of internal space dependence by restricting oneselves to the case $\Theta=l=0$, one finds usual concavity in L from (3.7) if $f_0'\ \frac{\partial u_0}{\partial L}\leq 0$. The last inequality is for $f'>0$ guaranteed by theorem 1 of ref.[12]. [1]

Therefore, for $\Theta=0$ standard concavity of $Q\bar{Q}$-potentials with respect to the distance in usual space is guaranteed for the wide class of SUGRA backgrounds covered by theorem 1 of ref.[12].

However, due to the more complicated structure of the l.h.s. of (3.7) for $\Theta\neq 0$ we did not found a similar general statement in the generic case. We can only start checking (2.20) case by case.

As our first example we consider the original calculation of Maldacena [6] for the $AdS_5\times S^5$ background. The result was ($R^2=\sqrt{2g_{YM}^2N}$)

$$V(L,\Theta)\ =\ -\ \frac{2R^2}{\pi}\frac{F(\Theta)}{L}\ , \quad (3.8)$$

with

$$F(\Theta)=(1-l^2)^{\frac{3}{2}}\cdot\left(\int_1^\infty\frac{dy}{y^2\sqrt{(y^2-1)(y^2+1-l^2)}}\right)^2,$$
$$\Theta=2l\int_1^\infty\frac{dy}{\sqrt{(y^2-1)(y^2+1-l^2)}}\ . \quad (3.9)$$

Due to this special structure ($L\frac{\partial V}{\partial L}=-V$, $L^2\frac{\partial^2}{\partial L^2}V=2V$, $\frac{\partial\Theta}{\partial u_0}=0$), (2.20) is equivalent to

$$\Theta^3\ \frac{d^2}{d\Theta^2}\left(\frac{F}{\Theta}\right)\ \geq 0\ . \quad (3.10)$$

A numerical calculation of $\frac{F}{\Theta}$ confirms (3.10) clearly, see fig.2.

Next we discuss the large L confining potential including internal space dependence and α' corrections of the background derived in [13]. It has the form ($\gamma=\frac{1}{8}\zeta(3)R^{-6}$, $\hat{T}$ temperature parameter)

$$V(L,\Theta)\ =\ \frac{\pi R^2\hat{T}^2}{2}(1-\frac{265}{8}\gamma)\cdot L+\ \frac{R^2}{4\pi}(1+\frac{15}{8}\gamma)\ \frac{\Theta^2}{L}\ +\ O(1/L^3)\ . \quad (3.11)$$

Although this potential for $\Theta\neq 0$ violates naive concavity $\frac{\partial^2 V}{\partial L^2}\leq 0$, there is *no* conflict with the correctly generalised concavity (2.20). Applied to (3.11) the differential operator just produces zero.

4. Concluding remarks

The $Q\bar{Q}$-potential derived [6] from the classical SUGRA approximation for the type IIB string in $AdS_5\times S^5$ fulfils our generalised concavity condition at $\Theta\geq 0$. This adds another consistency check of this most studied case within the AdS/CFT duality.

Potentials have been almost completely studied only for $\Theta=0$ in other backgrounds. At least partly, this might be due to the wisdom to approach in some way QCD, where after all there is no place for a parameter like this angle between different orientations in S^5. However, one has to keep in mind that this goal, in the approaches discussed so far, requires some additional limiting procedure. Before the limit the

[1] Our f and g are called f^2 and g^2 in that paper.

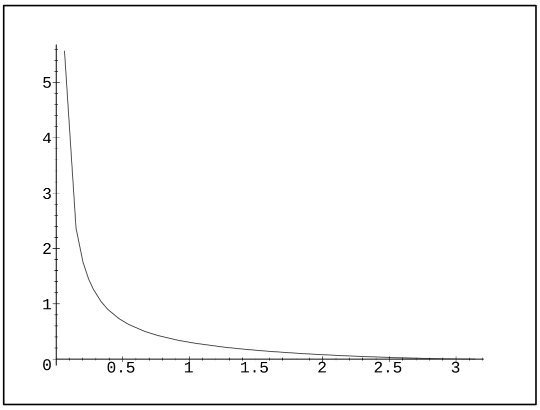

Fig.2 $\frac{F}{\Theta}$ *as a function of* Θ. *Use has been made of the representation in terms of elliptic integrals given in [6].*

full 10-dimensionality inherited by the string is still present. Fluctuation determinants in all 10 directions have to be taken into account for quantum corrections [14, 15, 16] and the Θ-dependence of the potentials is of course not switched off.

Although we proved in classical SUGRA approximation monotony in L and Θ as well as concavity at $\Theta = 0$ for a whole class of backgrounds, we were not able to get a similar general result on concavity for $\Theta > 0$. Further work is needed to decide, whether at all general statements for $\Theta > 0$ are possible. Alternatively one should perform case by case studies for backgrounds derived e.g. from rotating branes [17], type zero strings [18] or nonsupersymmetric solutions of type IIB string theory [19].

On the field theory side further work is necessary to really prove the conjectured inequality (2.15), otherwise the available set of rigorous constraints on the L *and* Θ dependent potential, beyond the standard concavity at $\Theta = 0$, would contain only the very mild condition (2.22).

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