Conformal String Sector of M-Theory *

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ABSTRACT: We consider a conformal system of a string and a particle defined in $\boldsymbol{D = 10{+}2}$ space-time dimensions. The extra time-like dimension is a gauge artifact and can be eliminated by choosing a gauge in which the $\boldsymbol{SO(10,1)}$ Lorentz symmetry is manifest. The effective theory of string observables is the 11d supergravity. The same theory compactified on $\boldsymbol{T^2}$ provides a non-petrurbative unified picture of the Type IIA, Type IIB and 11d supergravity. This is confirmed by explicit determination of the $\boldsymbol{R^4}$-terms which are finite and manifestly $\boldsymbol{SL(2,Z)}$ invariant as expected by the U-duality conjecture in nine non-compact dimensions with maximal supersymmetry.

1. Introduction

In space-time dimension lower than ten, all supergravity theories with maximal supersymmetry have a universal massless sector, e.g. the gravity supermultiplet [1]. Indeed, all supergravities with N_{max} are identical modulo field redefinitions, which correspond to the vev's of scalars in the supergravity multiplet [1], and after performing "electric-magnetic" duality-like transformations acting on the gauge fields (classical U-duality transformations). These effective theories can be constructed either by compactification on a $T^{(n+1)}$ torus from the $N = 1, d = 11$ supergravity [1] or by string compactification on $T^{(n)}$ of the type IIA or type IIB ten dimensional superstrings. This universality of the maximal supergravities ($N_{max} = 8$ in four dimensions) leads to the conjecture that they are all identical at the non-perturbative level. This suggests the existence of a more fundamental theory, (M-theory?) [2], which is defined necessarily in dimension higher than ten. In particular, the eleven- dimensional supergravity is the effective "low-energy" local field theory of the would be fundamental theory [2]. Furthermore, the universality conjecture for N_{max} seems to be valid even for less supersymmetric theories with 1/2 and 1/4 of N_{max} [2]–[8]. This suggests further that all superstring theories with the same number of supersymmetries in a given dimension are equivalent at the non-perturbative level. Thus, Heterotic ↔ type I ↔ type IIA ↔ type IIB, must be connected in lower dimensions by perturbative and/or non-perturbative U-duality transformations [2]–[8].

What can be the origin of the U-duality connections? Here we will be focus on a possible geometrical origin due to the presence of some extra "hidden" dimensions which are enable us to describe the complete spectrum of all topological non-perturbative BPS states of the maximal supersymmetric theories in nine dimensions. In the past few years, it has been realized that one "hidden" dimension is not enough to describe the spectrum of the topological BPS states. This implies the existence of more than one hidden dimension. The type IIB theory, for instance, suggests a fundamental theory (F-theory) in 10+2 dimensions [8]. Vafa and others suggested a working algorithm which extends in a consistent way the type IIB non-perturbative BPS spectrum and the U-duality properties in lower dimensions. If there are more than one dimension and especially a time-like one how do we interpret them? One of the possibilities is to define a combined system of a String and a Particle living in $D_{crit} = (10 + 2)$ dimensions [9],[10].

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We will shortly review this possibility in the next section and then we will study in more details the R^4 terms of this theory in a special gauge where $SO(10,1)$ Lorentz invariance is manifest with one physical time.

2. String & Particle

Consider a string and a particle [9],[10] described by a world-sheet $X^\mu(\tau,\sigma)$ and a world-line $Y^\mu(\tau)$

$$S_{str}(X^\mu, A_\alpha, g_{\alpha\beta}, Q_\mu) + S_{par}(Y, B, e, P_\mu) + Q\cdot P\,, \tag{2.1}$$

$$S_{str} = \frac{1}{2}\int_0^T d\tau \int d\sigma\, \sqrt{-g}g^{\alpha\beta}\left(\partial_\alpha X^\mu - P^\mu A_\alpha\right) \times \left(\partial_\beta X^\nu - P^\nu A_\beta\right)\eta_{\mu\nu}, \tag{2.2}$$

$$S_{part} = \frac{1}{2}\int_0^T d\tau\, \left[e^{-1}\left(\partial_\tau Y^\mu - Q^\mu B\right)^2 - em^2\right]. \tag{2.3}$$

The two actions S_{str} and S_{part} are invariant under independent reparametrizations on the world-sheet and on the world-line. Then one can choose the usual conformal gauge for the string, $\sqrt{-g}g^{\alpha\beta} =\eta^{\alpha\beta}$, and $e=1$ for the particle. The equations of motion for $X^\mu(\tau,\sigma)\,, Y^\mu(\tau)\,,$ in the gauges we have chosen, are:

$$\partial_+\left(D_-X^\mu\right) + \partial_-\left(D_+X^\mu\right) = 0,\ \partial_\tau q^\mu = 0, \tag{2.4}$$

where

$$\begin{aligned} D_\pm X^\mu &= \left(\partial_\pm X^\mu - P^\mu A_\pm\right), \\ q^\mu &= \left(\partial_\tau Y^\mu - Q^\mu B\right). \end{aligned} \tag{2.5}$$

There are additional gauge invariances, which may be understood in the spirit of gauged WZW models

$$\begin{aligned} \delta_1 X^\mu &= P^\mu \Lambda_1(\tau,\sigma)\,, \quad \delta_1 A_\alpha = \partial_\alpha \Lambda_1(\tau,\sigma)\,, \\ \delta_2 Y^\mu &= Q^\mu \Lambda_2(\tau)\,, \qquad \delta_2 B = \partial_\tau \Lambda_2(\tau)\,. \end{aligned} \tag{2.6}$$

We can solve the constraints [9],[10] and the equations of motion (for $m\neq 0$)

$$Q^\mu \sim p^\mu, \quad P^\mu \sim q^\mu, \quad p\cdot q = 0. \tag{2.7}$$

In the light-like case $P^2=0=q^2$ ($m=0$), the action S_{str} has no A_+A_- term, and acquires an additional gauge symmetry:

$$\delta_3\ X^\mu(\tau,\sigma) = 0, \quad \delta_3\ A_\pm = \pm\partial_\pm \Lambda_3(\tau,\sigma)\,. \tag{2.8}$$

In the background of the *massless particle*, two string components, rather than only one, are eliminated by the gauge invariances and thus, $\partial_\pm X^\mu - c_1 q^\mu$, has no components along the light-like q^μ and $q\cdot\partial_\pm X = 0$. For a *massive particle*, these two conditions correspond to one and the same component [9],[10].

The equation of motion for the string is easily solved, since it has the free string form $\partial_+\partial_- X^\mu = 0$. The general solution is given in terms of left- and right- movers

$$X_\mu = X_\mu^{(+)}\left(\sigma^+\right) + X_\mu^{(-)}\left(\sigma^-\right) + c_1(\sigma^+ + \sigma^-)q_\mu,$$

$$X_\mu^{(\pm)}\left(\sigma^\pm\right) = \frac{1}{2}\left(x\mu + \frac{\sigma^\pm}{2\pi}p_\mu\right) - i\sum_{n\neq 0}\frac{1}{n}\alpha_{n\mu}^{(\pm)}\, e^{in\sigma^\pm}. \tag{2.9}$$

The solution of the particle equation,

$$Y^\mu(\tau) = y^\mu + \left(q^\mu + c_2 p^\mu\right)\tau, \tag{2.10}$$

shows that *the particle moves freely, except for the orthogonality constraint* $p\cdot q = 0$. Due to this constraint, at the quantum level there are anomalies in arbitrary space-time dimensions [9], [10]; the String & Particle System is anomaly free only in special dimensions depending on the parameter. Furthermore, $m=0$ when super-reparametrization invariance is present on the world-sheet of the string and on the world-line of the particle, $m=0$.

- When $m=0$ $D_{cr}=28$, for the bosonic string with $SO(26,2)$ Lorenzt invariance.
- For superstring $m=0$, $D_{cr}=12$, with $SO(10,2)$ Lorentz symmetry.

All quantization approaches, BRST quantization, Light-cone quantization or gauge WZW approach, give the same critical dimensions [9],[10].

3. String & Particle Partition function and R^4-terms

There is a gauge choice which solves the orthogonality condition $q_\mu\partial_\pm X^\mu = 0$ with $SO(1,10)$ covariance for all string observables in the particle background with momentum q_μ:

$$q_\mu = (\ \epsilon, 0 \mid q_{10}, q_9,\ 0,0,0,0,0,0,0,0\ )$$

$$\partial_\pm X^\mu = (0, V^0 \mid V_\pm^{10}, V_\pm^9,\ V^I\ ),\quad I = 1, ..., 8. \tag{3.1}$$

In this gauge, one of the two time-like string coordinates is eliminated and one is left with the one physical time coordinate as in usual string theories. Notice that two orthogonal time-like vectors *cannot exist in a space with a single time-like dimension.* The initial $SO(2,10)$ covariant constraints $q_\mu \partial_\pm X^\mu = 0$ become in this gauge:

$$q_i \partial_\pm X^i = 0\ ,\quad i = 9, 10. \tag{3.2}$$

These constraints eliminate the quantum excitations of the string oscillators parallel to the vector q_i like in the gauge WZW-models. There is however a remnant of the zero-mode topological sector defined by the $9th$ and $10th$ left- and right-moving compactified coordinates of the string. The reduced (2,2) Lorentzian lattice $\Gamma(2,2)_{q_i}(T,U)$ depends on both T and U moduli $T \sim iR_9R_{10}$, $U \sim iR_9/R_{10}$.

$$\Gamma(2,2)_{q_i} = e^{-\pi \,\mathrm{Im}\,\tau (\, p_i G^{ij} p_j + n^i G_{ij} n^j\,) + 2i\pi \mathrm{Re}\tau\, p_i n^i}, \tag{3.3}$$

$$p_i = m_i + B_{ij} n^j \text{ with } p_i q^i = n^i G_{ij} q^j = 0. \tag{3.4}$$

The orthogonality constraint projects the $\Gamma(2,2)$ lattice on to a sum of $\Gamma(1,1)$ sectors according to the co-prime integers (p,q) defined as $q^i = \hat{M} \hat{q}^i = \hat{M}(p,q)$. Then,

$$\Gamma(2,2)_{q^i} = e^{-\pi \,\mathrm{Im}\,\tau \left(M^2 \hat{q}_i G^{ij} \hat{q}_j + \frac{N^2}{\hat{q}_i G^{ij} \hat{q}_j} \right) + 2i\pi \mathrm{Re}\tau M N}. \tag{3.5}$$

In terms of T and U moduli:

$$\hat{q}_i G^{ij} \hat{q}_j = \frac{1}{\mathrm{Im}\, T} \frac{|p + qU|^2}{\mathrm{Im}\, U}. \tag{3.6}$$

The sum over (p,q)-sectors yields to $SL(2,Z)_U$ invariant results.

In the zero winding sector $N = 0$ the sum over M and (p,q) can be reorganized to *two* Kaluza-Klein momenta,

$$m_i = M \epsilon_{ij} \hat{q}^j$$

$$\Gamma(2,2)_{q^i}|_{(N=0)} = e^{-\pi \,\mathrm{Im}\,\tau (\, m_i G^{ij} m_j\,)}. \tag{3.7}$$

After Poisson re-summation on m_i:

$$\Gamma(2,2)_{q^i}|_{(N=0)} = \frac{\mathrm{Im}\, T}{\mathrm{Im}\, \tau} e^{\frac{-\pi (\, \tilde{m}^i G_{ij} \tilde{m}^j\,)}{\mathrm{Im}\, \tau}}. \tag{3.8}$$

This result has an obvious eleven dimensional interpretation. Thus the sum over all (p,q) string sectors give rise to two instead of one Kaluza-Klein momenta, with an obvious eleven-dimensional interpretation.

We can go even further and calculate the R^4-gravitational corrections. This can be easily done using the techniques developed in refs[11],[12] where the flat space-time is replaced by a non trivial gravitational background deformed by the left- and right- helicity operators [11],[12] $vQ_{left},\ \bar{v}Q_{right}$ with:

$$R \neq 0 \to v, \bar{v} \neq 0.$$

From the deformed partition function [11],[12]:

$$Z(\tau, \bar{\tau}|v, \bar{v}) = \frac{(\mathrm{Im}\,\tau)^{-1}}{\eta^2(v)\, \bar{\eta}^2(\bar{v})} \frac{(\mathrm{Im}\,\tau)^{-5/2}}{\eta^5\ \bar{\eta}^5} \frac{\Gamma_{2,2}(p,q)}{\eta\ \bar{\eta}} \times \frac{1}{2} \sum_{a,b} (-)^{a+b+ab} \frac{\vartheta[^a_b](v)\ \vartheta[^a_b]^3}{\eta^4} \times \frac{1}{2} \sum_{\bar{a},\bar{b}} (-)^{\bar{a}+\bar{b}+\bar{a}\bar{b}} \frac{\bar{\vartheta}[^{\bar{a}}_{\bar{b}}](\bar{v})\ \bar{\vartheta}[^{\bar{a}}_{\bar{b}}]^3}{\bar{\eta}^4}. \tag{3.9}$$

one obtains the R^4-term from the term which is proportional to $v^4\, \bar{v}^4$ -term. Details of these technics appear in refs[11],[12]. Here we display and give some comments on our results.

$$F_{R^4} = \int_{\mathcal{F}} \frac{\partial\tau \partial\bar{\tau}}{t^2}\ t^{1/2}\ \Gamma_{2,2}(p,q), \tag{3.10}$$

$$F_{R^4} = \mathrm{Im}\, T \left(\int_{\mathcal{F}} \frac{\partial\tau \partial\bar{\tau}}{t^2} + \int_0^\infty \frac{dt}{t^{5/2}}\ e^{-\pi \frac{\tilde{m}^i G_{ij} \tilde{m}^j}{t}} \right). \tag{3.11}$$

In the second term $\tilde{m}^i \neq 0$ ($t = \operatorname{Im}\tau$).

$$F_{R^4} = \operatorname{Im}T \left(\frac{\pi}{3} + \frac{\operatorname{Im}T^{-3/2}}{2\pi} \sum_{\tilde{m}^2} \frac{\operatorname{Im}U^{3/2}}{|\tilde{m}^1 + \tilde{m}^2 U|^3} \right)$$

$$= \operatorname{Im}T\ \frac{\pi}{3} + \operatorname{Im}T^{-1/2} \frac{\zeta(3)}{2\pi}\ E_{3/2}(U), \quad (3.12)$$

where $E_{3/2}$ is the Eisenstein series with weight w=3/2:

$$E_{3/2}(U) = \sum_{p,q} \frac{\operatorname{Im}U^{3/2}}{|p+qU|^3}, \quad (p,q) \text{ coprimes.} \quad (3.13)$$

The same results was obtained previously by M.B.Green, P.Vanhove, and M.Gutperle in refs[13] calculating the one loop graviton scattering in the eleven dimensional supergravity compactified on T^2 after a regularization of the UV divergent term proportional to $\operatorname{Im}T$ and assuming the validity of Type IIA $\leftrightarrow$ Type IIB string duality.

In our case the calculations are done without any assumption at all. The result is UV-finite giving rise to an $SL(2,Z)$ invariant answer as expected from the type IIB string theory and Type IIA $\leftrightarrow$ Type IIB duality.

4. Conclusions

We presented a suitable conformal system of a String and a Particle in $D = (10+2)$ dimensions. We show by using the extra symmetries of this system that one of the two time-like coordinates can be eliminated from all string observables remaining and leads to an effective string dimensionality $D_{string} = (10+1)$. The 11th dimension arise after the resuming all topological non-trivial (p,q) string sectors. These sectors appear as solution of the orthogonality constrain between string and particle compactified momenta.

It is interesting that all perturbative and non-perturbative BPS states in nine dimensions are appearing with masses parametrised by the geometrical toroidal moduli of T^2 $\Gamma(2,2)$ lattice. In the R^4 calculations only the perturbative and non- perturbative BPS states contributed giving the finite result conjectured in refs[13] terms of the Eisenstein $E_{3/2}(U)$ function:

$$\left(\operatorname{Im}T\frac{\pi}{3} \ + \ \operatorname{Im}T^{-1/2}\ \frac{\zeta(3)}{2\pi}\ E_{3/2}(U) \right)\ R^4$$

In String & Particle Theory all R^n terms with $n > 4$ are determined unambiguously as in any conventional string theory replacing the $\Gamma_{1,1}$ Lattice of strings by the $\Gamma_{2,2}(p,q)$ restricted lattice of the String & Particle theory. In the R^n with $n > 4$ terms, non BPS-states (perturbative and non-perturbative) give non-zero contribution (contrary to R^4-terms). The String & Particle theory give the correct masses for all these states and the multiplicities are provided by the string oscillators.

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