Stable non-BPS D-branes of type I

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ABSTRACT: We review the boundary state description of the non-BPS D-branes in the type I string theory and show that the only stable configurations are the D-particle and the D-instanton. We also compute the gauge and gravitational interactions of the non-BPS D-particles and compare them with the interactions of the dual non-BPS particles of the heterotic string finding complete agreement. In this way we provide further dynamical evidence of the heterotic/type I duality.

KEYWORDS: D-branes, Boundary states.

Dirichlet branes (or D-branes for short) are a key ingredient in our understanding of the duality relations between superstring theories. They are described by a boundary conformal field theory, and admit a two-fold interpretation: on the one hand, the D-branes are objects on which open strings can end, and on the other hand they can emit or absorb closed strings [1]. These two descriptions can be related to each other by world-sheet duality.

Introducing D-branes in a theory of closed strings amounts to extend their conformal field theory by introducing world-sheets with boundaries and imposing appropriate boundary conditions on the closed string coordinates X^μ. In the operator formalism these boundary conditions are implemented through the so called boundary state [1] $|Dp\rangle$, whose bosonic part is defined by the following eigenvalue problem

$$\partial_\tau X^\alpha(\sigma, 0) |Dp\rangle_X = 0 \ ,$$
$$\left(X^i(\sigma, 0) - x^i \right) |Dp\rangle_X = 0 \ , \qquad (1)$$

where the index $\alpha = 0, \ldots, p$ labels the longitudinal directions, the index $i = p + 1, \ldots, 9$ labels the transverse directions and the x^i's denote the position of the brane in the transverse space. *World-sheet* supersymmetry requires that

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[1]For a recent review on the boundary state formalism and its applications, see Ref. [2]

analogous equations must be also imposed on the left and right moving fermionic fields ψ^μ and $\widetilde{\psi}^\mu$. These equations, which define the fermionic part of the boundary state, are

$$\left(\psi^\alpha(\sigma, 0) - i\eta\, \widetilde{\psi}^\alpha(\sigma, 0) \right) |Dp, \eta\rangle_\psi = 0 \ ,$$
$$\left(\psi^i(\sigma, 0) + i\eta\, \widetilde{\psi}^i(\sigma, 0) \right) |Dp, \eta\rangle_\psi = 0 \ , \quad (2)$$

where $\eta = \pm 1$. Notice that there are two consistent implementations of the fermionic boundary conditions corresponding to the sign of η, and consequently there are two different boundary states

$$|Dp, \eta\rangle = |Dp\rangle_X |Dp, \eta\rangle_\psi \qquad (3)$$

both in the NS-NS and in the R-R sectors. The overlap equations (1) and (2) allow to determine the explicit structure of the boundary states (3) up to an overall factor. This normalization can then be uniquely fixed by factorizing amplitudes with closed strings emitted from a disk [3, 4] and turns out to be given by (one half of) the brane tension measured in units of the gravitational coupling constant, *i.e.*

$$T_p = \sqrt{\pi} \left(2\pi \sqrt{\alpha'} \right)^{3-p} \ . \qquad (4)$$

We would like to remark that even if each boundary state $|Dp, \eta\rangle$ is perfectly consistent from the conformal field theory point of view, not all of

them are acceptable in string theory. In fact, to describe a physical D-brane a boundary state has to satisfy three requirements [5]:

i) to be invariant under the closed string GSO projection (and also under orbifold or orientifold projections if needed);

ii) the tree level amplitude due to the exchange of closed strings between two boundary states, after modular transformation in the open string channel, has to make sense as a consistent open string partition function at one-loop;

iii) the open strings introduced through the D-branes must have consistent couplings with the original closed strings [2].

Using these prescriptions, it is rather simple to find the boundary state for the supersymmetric BPS Dp-branes of type II. In particular, the GSO projection of the type II theories forces us to retain only the following linear combinations

$$|Dp\rangle_{\rm NS} = \frac{1}{2}\Big[|Dp,+\rangle_{\rm NS} - |Dp,-\rangle_{\rm NS}\Big]$$
$$|Dp\rangle_{\rm R} = \frac{1}{2}\Big[|Dp,+\rangle_{\rm R} + |Dp,-\rangle_{\rm R}\Big] \qquad (5)$$

in the NS-NS and in the R-R sectors respectively, with $p = 0, 2, 4, 6, 8$ for IIA, $p = -1, 1, 3, 5, 7, 9$ for IIB. The normalization of the boundary states (5) can be deduced by requiring that the spectrum of the open strings living on the Dp-brane (called p-p strings) be supersymmetric. To read the spectrum of these open strings from the boundary state, one has first to evaluate the closed string exchange amplitude

$$\langle Dp|\, P\, |Dp\rangle \quad , \qquad (6)$$

where P is the closed string propagator

$$P = \frac{\alpha'}{2} \int_0^\infty dt \ {\rm e}^{-t(L_0 + \widetilde{L}_0 - 2a)} \qquad (7)$$

(with $a_{\rm NS} = 1/2$ and $a_{\rm R} = 0$), and then perform the modular transformation $t \to 1/s$ to exhibit

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[2]This last condition, which is rather difficult to prove in general, does not give more constraints than the first two in the case of type I and II theories.

the open string channel. Applying this procedure, one finds the following relations

$$_{\rm NS}\langle Dp,\eta|\, P\, |Dp,\eta\rangle_{\rm NS} = \int_0^\infty \frac{ds}{s}\, {\rm Tr}_{\rm NS}\, q^{2L_0 - 1} \quad ,$$

$$_{\rm NS}\langle Dp,\eta|\, P\, |Dp,-\eta\rangle_{\rm NS} = \int_0^\infty \frac{ds}{s}\, {\rm Tr}_{\rm R}\, q^{2L_0} \quad ,$$

$$_{\rm R}\langle Dp,\eta|\, P\, |Dp,\eta\rangle_{\rm R}$$
$$= \int_0^\infty \frac{ds}{s}\, {\rm Tr}_{\rm NS}\, (-1)^F\, q^{2L_0 - 1} \quad , \quad (8)$$

$$_{\rm R}\langle Dp,\eta|\, P\, |Dp,-\eta\rangle_{\rm R}$$
$$= \int_0^\infty \frac{ds}{s}\, {\rm Tr}_{\rm R}\, (-1)^F\, q^{2L_0} = 0 \quad ,$$

where $q = {\rm e}^{-\pi s}$. It is then clear that in order to obtain the supersymmetric (*i.e.* GSO projected) open string amplitude

$$\int_0^\infty \frac{ds}{s}\, \Bigg[{\rm Tr}_{\rm NS}\, \left(\frac{1 + (-1)^F}{2}\right) q^{2L_0 - 1}$$
$$- {\rm Tr}_{\rm R}\, \left(\frac{1 + (-1)^F}{2}\right) q^{2L_0}\Bigg] \qquad (9)$$

one must consider the following boundary state

$$|Dp\rangle = |Dp\rangle_{\rm NS} \pm |Dp\rangle_{\rm R} \qquad (10)$$

where the sign ambiguity is related to the existence of branes and anti-branes. Note that both the NS-NS and the R-R components of the boundary state (10) have the same normalization so that the tension of a Dp-brane essentially equals the density of its charge under the R-R potential: this is the BPS relation which is typical of the supersymmetric and stable branes of type II.

The criteria i) - iii) defining physical D-branes do not rely at all on *space-time* supersymmetry, and thus one may wonder whether in type II theories there may exist also non-supersymmetric branes. This problem has been systematically addressed in a series of papers by A. Sen [6] - [9], who constructed explicit examples of non-BPS (and hence non-supersymmetric) branes. In particular in Ref. [8], he considered the superposition of a D-string of type IIB and an anti-D-string (with a Z_2 Wilson line on it) and by suitably condensing the tachyons of the open strings stretching between the brane and the anti-brane, he managed to construct a new configuration of type IIB which behaves like a D-particle, does not couple to any R-R field and is heavier by

a factor of $\sqrt{2}$ than the BPS D-particle of the IIA theory. The boundary state for this non-BPS D-brane has been explicitely constructed in Ref. [10]. This construction can be obviously generalized to the case of a pair formed by two BPS D$(p+1)$-branes with opposite R-R charge (and with a Z_2 Wilson line) which, after tachyon condensation, becomes a non-BPS Dp-brane. Alternatively, this same non-BPS configuration can be described starting from a superposition of two BPS Dp branes with opposite R-R charge and modding out the theory by the operator $(-1)^{F_L}$ whose effect is to change the sign of all states in the R-R and R-NS sectors. In this second scheme, a superposition of a Dp-brane and anti-Dp-brane of type IIA (IIB) becomes in the reduced theory a non-BPS Dp-brane of type IIB (IIA). In either way we therefore find that there exist non-BPS Dp-branes for $p = 0, 2, 4, 6, 8$. For reviews on this subject, see Refs. [11, 12].

These branes are manifestly non-supersymmetric, but nevertheless they satisfy the conditions $i)$ - $iii)$ mentioned above, and thus are perfectly consistent from the closed string point of view. In particular, since they are not charged under any R-R field, the boundary state for these non-BPS D-branes has only the NS-NS component, namely

$$|Dp\rangle = \mu_p |Dp\rangle_{\text{NS}} \qquad (11)$$

where we have introduced a (positive) coefficient μ_p to allow for a different normalization with respect to the standard BPS case. This normalization can be deduced by requiring that the closed string amplitude between two non-BPS branes, after a modular transformation, has the interpretation of the partition function of a non-supersymmetric (*i.e.* without the GSO projection [3]) open string model. Indeed, by requiring that

$$\langle Dp|P|Dp\rangle = \int_0^\infty \frac{ds}{s} \left[\text{Tr}_{\text{NS}}\, q^{2L_0 - 1} - \text{Tr}_{\text{R}}\, q^{2L_0} \right] \qquad (12)$$

we find that $\mu_p = \sqrt{2}$, thus confirming that the non-BPS D-branes are heavier by a factor of $\sqrt{2}$

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[3] Note that the non-supersymmetric GSO projection $(1 - (-1)^F)/2$ cannot correspond to a *single* brane since all NS open string zero-modes are projected out.

than the corresponding BPS ones. Although these non-BPS D-branes of type II may have interesting properties [13], it is clear from (12) that they are not stable, because the absence of the GSO projection on the open strings leaves the NS tachyon on their world-volume. However, these non-BPS branes could become stable in an orbifold of the type II theory, say IIA(B)/$\mathcal{P}$, provided that the tachyon be odd under the projection $\mathcal{P}$. In the orbifold theory, the non-BPS vacuum amplitude of the p-p open-strings is clearly given by

$$\mathcal{Z}_{open} = n \int_0^\infty \frac{ds}{s} \left[\text{Tr}_{\text{NS}} \left(\frac{1 + \mathcal{P}}{2} q^{2L_0 - 1} \right) - \text{Tr}_{\text{R}} \left(\frac{1 + \mathcal{P}}{2} q^{2L_0} \right) \right] (13)$$

where n is a positive integer representing some possible multiplicity. (In our present discussion we take $n = 1$ for the sake of simplicity, but the case $n = 2$ will appear later.) The natural question to ask now is to which boundary state the amplitude (13) could correspond. In the case of a space-time orbifold, the perturbative spectrum of the bulk theory contains only closed strings which can be untwisted (U) or twisted (T) under the orbifold. Therefore, there are four sectors to which the bosonic states belong, namely (NS-NS;U), (R-R;U), (NS-NS;T) or (R-R;T), and there exist different types of boundary states depending on which components in those sectors they have. For example, when the orbifold projection $\mathcal{P}$ acts as the inversion of some space-time coordinates, the boundary state which gives rise to (13) turns out [7, 5, 14] to have only a component in the unstwisted NS-NS sector and another in the twisted R-R sector, *i.e.*

$$|Dp\rangle = \frac{1}{\sqrt{2}} \left(\sqrt{2}|Dp\rangle_{\text{NS;U}} + \sqrt{2}|Dp\rangle_{\text{R;T}} \right) \quad . \qquad (14)$$

In particular, if one computes the exchange amplitude $\langle Dp| P |Dp\rangle$ with this boundary state and performs a modular transformation, one can see that the terms of (13) with $\mathcal{P}$ originate precisely from the twisted part of the boundary state.

In the case of a world-sheet orbifold, however, this simple picture does not hold. To illustrate this point, we consider the specific case of the type I theory which is the orbifold of the

IIB theory by the world-sheet parity Ω [15]. The distinctive feature of this model is that the perturbative states of the twisted sector of the *bulk* theory now correspond to unoriented *open* strings which should then be appropriately incorporated in the boundary state formalism. Let us briefly summarize how this is done (for more details see [16]). The starting point is the projection of the closed string spectrum onto states which are invariant under Ω. The corresponding closed string partition function is obtained by adding a Klein bottle contribution to the modular invariant (halved) torus contribution. The Klein bottle is a genus one non-orientable self-intersecting surface which may be seen equivalently as a cylinder ending at two crosscaps. A crosscap is a line of non-orientability, a circle with opposite points identified, and thus the associated crosscap state $|C\rangle$ is defined by

$$X^\mu(\sigma+\pi,0)\,|C\rangle \;=\; X^\mu(\sigma,0)\,|C\rangle \;\;,\qquad(15)$$
$$\partial_\tau X^\mu(\sigma+\pi,0)\,|C\rangle \;=\; -\partial_\tau X^\mu(\sigma,0)\,|C\rangle \;\;,$$

and by the analogous relations appropriate for world-sheet fermions. As is clear from these equations, the crosscap state does not have any space-time interpretation but nevertheless it is related to the boundary state of the BPS space-time filling D9 brane through

$$|C\rangle \;\propto\; \mathrm{i}^{L_0+\tilde{L}_0}\,|D9\rangle \;\;.\qquad(16)$$

The normalization of $|C\rangle$, which may be fixed up to an overall sign using the action of Ω on the massless closed string modes and the world-sheet duality, turns out to be 32 times the normalization of the boundary state for the D9-brane. Consequently, the (negative) charge for the unphysical 10-form R-R potential created by the crosscap must be compensated by the introduction of 32 D9 branes. In this way we then introduce unoriented open strings which start and end on these 32 D9 branes, whose vacuum amplitude is given by

$$\mathcal{Z}_{open} = \frac{1}{2}\Big(2^{10}\langle D9|P|D9\rangle \qquad(17)$$
$$+\, 2^5\langle D9|P|C\rangle + 2^5\langle C|P|D9\rangle\Big) \;\;,$$

where the first line represents the contribution of the annulus and the second line the contribution of the Möbius strip. By adding to (17) the contribution of the Klein bottle we obtain a modular invariant expression, in which the tadpoles for the massless unphysical states cancel if and only if we choose the still unfixed overall sign in front of the crosscap state to be $+$. A moment thought shows that this corresponds to choose the open string gauge group to be $SO(32)$ (the other sign instead leads to the gauge group $Sp(32)$). Thus, we can say that the type I theory possesses a "background" boundary state given by

$$\frac{1}{\sqrt{2}}\Big(|C\rangle + 32|D9\rangle\Big) \;\;.\qquad(18)$$

where the factor of $1/\sqrt{2}$ has been introduced to obtain the right normalization of the various spectra. Performing a modular transformation, we can rewrite the amplitude $\mathcal{Z}_{open}$ of eq. (17) in the open string channel as follows

$$\int_0^\infty \frac{ds}{s}\left[\mathrm{Tr}_{\mathrm{NS}}\left(\frac{1+(-1)^F}{2}\frac{1+\Omega}{2}\,q^{2L_0-1}\right)\right.$$
$$\left.-\,\mathrm{Tr}_{\mathrm{R}}\left(\frac{1+(-1)^F}{2}\frac{1+\Omega}{2}\,q^{2L_0}\right)\right] \;\;,$$

where the part depending on Ω comes from the Möbius contribution. Thus, we see that in the type I theory the crosscap state plays the same role that the twisted part of the boundary state had in the space-time orbifolds. In the following, we shall use this remark in order to classify the stable non-BPS branes of type I theory.

We have seen before that the type IIB theory contains unstable non-BPS Dp-branes with $p = 0, 2, 4, 6, 8$ which are described by the boundary state (11). Now, we address the question whether these D-branes become stable in the type I theory, *i.e.* we examine whether the tachyons of the p-p open strings are removed by Ω. As explained in [10], the world-sheet parity can be used to project the spectrum of the p-p strings only if $p = 0, 4, 8$. Thus, the non-BPS D2 and D6 branes will not be further considered. However, in order to be exhaustive, we must take into account also another kind of configuration, namely the superposition of a Dp-brane and an anti-Dp-brane of type IIB. This pair clearly does not carry any R-R charge, is represented by a boundary state of the form (11) and is unstable due to the

presence of tachyons in the open strings stretching between the brane and the anti-brane. In the type I theory, however, these tachyons might be projected out. A systematic analysis [17, 10] shows that in this case Ω can be used as a projection only if $p = -1, 3, 7$.

In conclusion, we have to analyze the stability of the non-BPS Dp-branes of type I with $p = -1, 0, 3, 4, 7, 8$ whose corresponding boundary states $|Dp\rangle$ are given by eq. (11) with suitable values of μ_p. To address this problem, we need to consider the spectrum of the unoriented strings living on the brane world-volume (the p-p sector), and also the spectrum of the open strings stretched between the Dp-brane and each one of the 32 D9-branes of the background (the p-9$\oplus$9-p sector), in which tachyonic modes could develop.

Let us first analyze the p-p sector, whose total vacuum amplitude is given by

$$\mathcal{A}_{\text{tot}} = \frac{1}{2}\Big(\mathcal{A} + \mathcal{M} + \mathcal{M}^*\Big) \tag{19}$$

where $\mathcal{A}$ and $\mathcal{M}$ are respectively the annulus and the Möbius strip contributions

$$\mathcal{A} = \langle Dp|P|Dp\rangle \quad \text{and} \quad \mathcal{M} = \langle Dp|P|C\rangle \ . \tag{20}$$

After a modular transformation, in the open string channel these amplitudes read respectively

$$\mathcal{A} = \mu_p^2\, V_{p+1}\, (8\pi^2\alpha')^{-\frac{p+1}{2}} \int_0^\infty \frac{ds}{2s}\, s^{-\frac{p+1}{2}}$$
$$\times \left[\frac{f_3^8(q) - f_2^8(q)}{f_1^8(q)}\right] \ , \tag{21}$$

and

$$\mathcal{M} = 2^{\frac{7-p}{2}}\, \mu_p\, V_{p+1}\, (8\pi^2\alpha')^{-\frac{p+1}{2}} \int_0^\infty \frac{ds}{2s}\, s^{-\frac{p+1}{2}}$$
$$\left[\mathrm{e}^{\mathrm{i}(p-9)\pi/4} \frac{f_4^{p-1}(\mathrm{i}\,q)\, f_3^{9-p}(\mathrm{i}\,q)}{f_1^{p-1}(\mathrm{i}\,q)\, f_2^{9-p}(\mathrm{i}\,q)}\right.$$
$$\left. -\mathrm{e}^{\mathrm{i}(9-p)\pi/4} \frac{f_3^{p-1}(\mathrm{i}\,q)\, f_4^{9-p}(\mathrm{i}\,q)}{f_1^{p-1}(\mathrm{i}\,q)\, f_2^{9-p}(\mathrm{i}\,q)}\right] \ , \tag{22}$$

where f_1, f_2 f_3 and f_4 are the standard one-loop functions defined for example in Ref. [1]. The spectrum of the p-p open strings can be analyzed by expanding the total amplitude $\mathcal{A}_{\text{tot}}$ in powers of q. The leading term in this expansion is

$$\mathcal{A}_{\text{tot}} \sim \int_0^\infty \frac{ds}{2s}\, s^{-\frac{p+1}{2}}\, q^{-1}$$
$$\times \left[\mu_p^2 - 2\,\mu_p \sin\big(\frac{\pi}{4}(9-p)\big)\right] \ . \tag{23}$$

The q^{-1} behavior of the integrand signals the presence of tachyons in the spectrum; therefore, in order not to have them, we must require that

$$\mu_p = 2\,\sin\big(\frac{\pi}{4}(9-p)\big) \ . \tag{24}$$

Since μ_p has to be positive, the only possible solutions are

p	-1	0	7	8
μ_p	2	$\sqrt{2}$	2	$\sqrt{2}$

(25)

From this table we see that in the type I theory there exist two even non-BPS but stable Dp-branes: the D-particle and the D8-brane. Both of them have a tension that is a factor of $\sqrt{2}$ bigger than the corresponding BPS branes of the type IIA theory. Moreover, there exist two odd non-BPS but stable Dp-branes of type I: the D-instanton and the D7-brane. Their tension is twice the one of the corresponding type IIB branes, in accordance with the fact that, as mentioned above, they can be simply interpreted as the superposition of a brane with an anti brane, so that the R-R part of the boundary state cancels while the NS-NS part doubles.

This classification of the stable non-BPS D-branes of type I based on the table (25) is in complete agreement with the results of Refs. [17, 18] derived from the K-theory of space-time.

Let us now analyze the p-9 $\oplus$ 9-p sector. The relevant quantity to consider is the "mixed" cylinder amplitude

$$\mathcal{A}_{\text{mix}} = \frac{32}{2}\Big(\langle Dp|P|D9\rangle + \langle D9|P|Dp\rangle\Big) \ , \tag{26}$$

which, after a modular transformation in the open string channel, reads

$$\mathcal{A}_{\text{mix}} = 2^5\, \mu_p\, V_{p+1}\, (8\pi^2\alpha')^{-\frac{p+1}{2}} \int_0^\infty \frac{ds}{2s}\, s^{-\frac{p+1}{2}}$$
$$\times \left[\frac{f_3^{p-1}(q)\, f_2^{9-p}(q)}{f_1^{p-1}(q)\, f_4^{9-p}(q)} - \frac{f_2^{p-1}(q)\, f_3^{9-p}(q)}{f_1^{p-1}(q)\, f_4^{9-p}(q)}\right] \ ,$$

where the first and second term in the square brackets account respectively for the NS and R sector. This expression needs some comments. First, for $p = -1, 0$ we see that there are no tachyons in the spectrum; moreover, the values of μ_p for the D-instanton and D-particle are crucial in order to obtain a sensible partition function for

open strings stretching between the non-BPS objects and the 32 D9-branes. Indeed, they are the smallest ones that make integer the coefficients in the partition functions. Secondly, for $p = 7, 8$ we directly see the existence of a NS tachyon, so that the corresponding branes are actually unstable [10]. Hence, only the D-instanton and the D-particle are fully stable configuration of type I string theory [10, 12]. Nevertheless, the strict relation connecting the D0-brane and the D(-1)-brane to the D8-brane and the D7-brane respectively suggests however that also the latter may have some non trivial meaning. Finally, we observe that the zero-modes of the Ramond sector of these p-9 $\oplus$ 9-p strings are responsible for the degeneracy of the non-BPS Dp-branes under the gauge group SO(32): in particular the D-particle has the degeneracy of the spinor representation of SO(32), as discussed in [8, 9]. Thus the D-particle accounts for the existence in type I of the non-perturbative non-BPS states required by the heterotic/type I duality.

These same methods may be used in order to study the stability of a non-BPS Dp-brane in presence of another Dq-brane. Indeed the spectrum of open strings stretching between two such (distant) objects at rest has a vacuum amplitude given by

$$\frac{\mu_p \mu_q}{2} \Big({}_{\text{NS}}\langle Dp|P|Dq\rangle_{\text{NS}} + {}_{\text{NS}}\langle Dq|P|Dp\rangle_{\text{NS}} \Big) \ . \tag{27}$$

The overall factor of one-half indicates that, respectively to the IIB case, only the Ω symmetric combinations are retained. By explicitly computing this amplitude, one can see that for $|p-q| \leq 3$ and for sufficiently small values of the distance between the branes, a NS tachyon develops in the open string spectrum, thus signalling the unstability of the configuration. As a first consequence of this, we can conclude that the superposition of two non-BPS D-particles with trivial quantum numbers, decays into the vacuum [8]. As a matter of fact, a stable non-BPS Dp-brane is its own anti-brane, as may be also inferred from the K-theory analysis which shows that the conserved D-brane charge in that case is Z_2 valued [17]. This analysis shows that there is no hope to form a stable superposition of N non-BPS Dp-branes of type I as they always exert an attractive force

on each other, as may be seen from (27). This is to be contrasted with the case of a space-time orbifold [14] where, for some particular values of the compactification radii, a compensation occurs between the attractive force due to exchange of untwisted NS-NS states and the repulsive force due to exchange of twisted R-R states. As a second consequence, and for analogous reasons, the superposition of a D-particle and of a D1-string is unstable and decays in the vacuum. Note that in that case, the 0-1 open string is T-dual of the 8-9 string responsible for the unstability of the D8-brane.

Up to now we have investigated the flat ten dimensional case. However, this analysis can be easily extended also to the case in which some directions are compactified. Contrarily to the BPS D-branes, the non-BPS one are not stable in all moduli space. As an example, let us consider the non-BPS D-particle and compactify one space direction along a circle of radius R. Then, one can observe that a tachyon develops in the 0-0 open string sector if $R < R_c = \sqrt{\alpha'/2}$, so that below the critical radius R_c the configuration is unstable. The corresponding stable non-BPS configuration which carries the same quantum numbers in this range of moduli is a superposition of wrapped D1 and anti-D1 strings with a Z_2 Wilson line. Notice that when the time direction is compactified, the D-particle is stable for any value of the radius.

We now present the basic ideas and results about the gravitational and gauge interactions of two stable non-BPS D-particles of type I string theory (the detailed calculations and analysis of these interactions can be found in [19]).

In the type I theory, D-branes interact via exchanges of both closed and open bulk strings. Since the dominant diagram for open strings has the topology of a disk, it gives a subleading (in the string coupling constant) contribution to the diffusion amplitude of two branes which is thus dominated by the cylinder diagram, *i.e.* by the exchange of closed strings. In the long distance limit, this accounts for the gravitational interactions. Let us now use this observation to calculate the dominant part of the scattering amplitude between two D-particles of type I moving

with a relative velocity v. This process can be simply analyzed using the boundary state formalism as explained in [20]. What we need to compute is the cylinder amplitude between the boundary state of a static D-particle $|D0\rangle$ and the boundary state of a moving D-particle $|D0, v\rangle$. The latter is simply obtained by acting with a Lorentz boost on $|D0\rangle$, *i.e.*

$$|D0, v\rangle = e^{i\pi\theta J_{01}}|D0\rangle \ , \qquad (28)$$

where θ is the rapidity along direction of motion (which we have taken to be x^1) defined through $v = \tanh\pi\theta$, and J_{01} is the corresponding Lorentz generator. Thus, the amplitude we are looking for is

$$\mathcal{A} = \langle D0|P|D0, v\rangle + \langle D0, v|P|D0\rangle \ , \qquad (29)$$

which indeed reduces to (27) for $v \to 0$. From this expression, we can extract the long range gravitational potential energy, which, in the non relativistic limit, reads [19]

$$V^{\text{grav}}(r) = (2\kappa_{10})^2 \frac{M_0^2}{7\,\Omega_8\,r^7}$$
$$\times \left(1 + \frac{1}{2}\,v^2 + o(v^2)\right) \ , \qquad (30)$$

where r is the radial coordinate, Ω_8 is the area of the unit 8-dimensional sphere, $M_0 = T_0/\kappa_{10}$ is the D-particle mass and κ_{10} is the gravitational coupling constant in ten dimensions. Hence the boundary state calculation correctly reproduces the gravitational potential we expect for a pair of D-particles in relative motion.

Although they are subdominant in the string coupling constant, the interactions of the D-particle with the open strings of the bulk are nevertheless interesting because they account for the gauge interactions. Since the non-BPS D-particles of type I are spinors of $SO(32)$, their gauge coupling is fixed by the spinorial representation they carry (except possibly by the overall strength). The stringy description of such a coupling has been provided in [19] where we have shown that it is represented by an open string diagram with the topology of a disk with two boundary components, one lying on the D9-branes from which the gauge boson is emitted, and the other lying on the D-particle (see Figure 1).
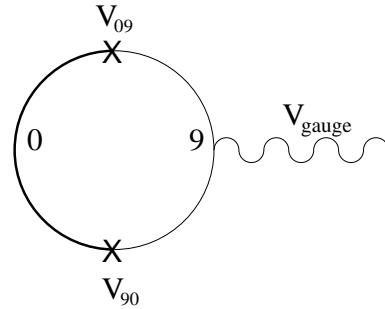

Figure 1: The disk diagram describing the gauge coupling of a type I D-particle.

At the points where the two boundary components join, we thus have to insert a vertex operator V_{90} (or V_{09}) that induces the transition from Neumann to Dirichlet (or from Dirichlet to Neumann) boundary conditions in the nine space directions. As we have mentioned before, the $SO(32)$ degeneracy of the D-particle is due to the fermionic massless modes of the open strings stretching between the D-particle and each of the 32 D9-branes; therefore it is natural to think that the boundary changing operators V_{90} and V_{09} are given by the vertex operators for these massless fermionic modes [19]. By construction, these operators carry Chan Paton factors in the fundamental representation of SO(32), while the vertex operator V_{gauge} for the gauge boson carries a Chan-Paton factor in the adjoint. As a consequence, the diagram represented in Figure 1 must be considered as the one point function of the gauge boson in the background formed by a D-particle seen as an object in the bi-fundamental representation of $SO(32)$. Hence, we do not see the entire gauge degeneracy of the D-particle because the degrees of freedom we use to describe it are not accurate enough. This is reminiscent from the fact that, in the boundary state formalism, also the Lorentz degeneracy of a D-brane is hidden. Using this result, we can easily compute the Coulomb potential energy $V^{\text{gauge}}(r)$ for two D-particles placed at a distance r. Indeed, this is simply obtained by gluing two diagrams like that of Figure 1 with a gauge boson propagator, and its explicit expression turns out to be [19]

$$V^{\text{gauge}}(r) = -\frac{g_{\text{YM}}^2}{2}\,\frac{1}{7\,\Omega_8\,r^7}$$

$$\times \left(\delta^{AB} \delta^{CD} - \delta^{AC} \delta^{DB} \right) \quad , \qquad (31)$$

where $g_{\rm YM}$ is the gauge coupling constant in ten dimensional type I string theory, and A, B, C and D are indices in the fundamental representation of $SO(32)$.

We conclude by recalling that the non-BPS D-particles of type I are dual to perturbative non-BPS states of the $SO(32)$ heterotic string which also have gravitational and gauge interactions among themselves. These can be computed using standard perturbative methods and if one takes into account the known duality relations and renormalization effects, one can explicitly check that they agree with the expressions (30) and (31). This agreement provides further dynamical evidence of the heterotic/type I duality beyond the BPS level.

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