Black Holes and U-duality in Diverse Dimensions

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ABSTRACT: In this paper we review some properties of BPS black holes of supergravities with $n = 32, 16$ supersymmetries. The BPS condition, a condition on the eigenvalues of the central charge matrix, can be shown to be U-duality invariant. We explicitly work out $D = 4$, $N = 8$ and $D = 5$, $N = 4$ supergravities.

1. Preliminaries

The purpose of this work is to investigate some properties of black hole solutions of different supergravity theories in terms of the corresponding U-duality group.

Black holes are solutions of supergravity theories with point-like sources. They are rotationally invariant and are, in general, charged with respect to the vector fields of the theory. We will consider extremal black hole solutions, that is, solutions on which some of the supersymmetry charges Q are null. In this case, the solutions are parametrized by the set of charges q^Λ and the values of the asymptotic values of the scalar fields ϕ^i of the theory at ∞ (moduli).

We consider the N-extended super Poincaré algebra in a $D = 4, \ldots 9$ dimensional space-time (we follow the notation of Ref.[1]). For $D = 2n + 1$ the irreducible representation of the Clifford algebra $\mathcal{C}(1, D - 1)$, of dimension 2^n, is also irreducible with respect to so$(1, D - 1)$. This is the only spinor representation and consequently it is self-conjugated. For $D = 9$ it is real, and the anticommutator of two supercharges is given by

$$\{Q_{\alpha i}, Q_{\beta j}\} = \left(\Gamma_\mu C p^\mu\right)_{\alpha\beta}\delta_{ij} + C_{\alpha\beta}Z_{ij} \quad (1.1)$$

where $i, j = 1, \ldots N$, $\alpha, \beta = 1, \ldots 2^4$. C is the charge conjugation matrix that in this case is symmetric, so the central charge extension of the super Poincaré algebra, Z_{ij}, is also symmetric.

For $D = 5, 7$ the spinor representation is pseudoreal, so the anticommutator is instead

$$\{Q_{\alpha i}, Q_{\beta j}\} = \left(\Gamma_\mu C p^\mu\right)_{\alpha\beta}\Omega_{ij} + C_{\alpha\beta}Z_{ij}, \quad (1.2)$$

where Ω is the symplectic bilinear form. For $D = 5$, $C = -C^T$, so Z is antisymmetric and for $D = 7$ $C = C^T$ so Z is symmetric.

For $D = 2n$ the irreducible representation of the Clifford algebra $\mathcal{C}(1, D - 1)$ splits into two irreducible pieces under so$(1, D - 1)$, $(\mathbf{2}^{n-1})_\pm$. The projectors on each piece are

$$\mathcal{P}_\pm = \frac{1 \pm \Gamma_{D+1}}{2}, \qquad Q_\pm = \mathcal{P}_\pm Q.$$

For $D = 4, 8$ these representations are complex and pairwise conjugate. The anticommutator is

$$\{Q_{\alpha i+}, Q_{j-}^{\dot\beta}\} = \left(\mathcal{P}_+\Gamma_\mu C p^\mu\right)_\alpha^{\dot\beta}\delta_{ij}$$
$$\{Q_{\alpha i+}, Q_{\beta j+}\} = (\mathcal{P}_+ C)_{\alpha\beta}Z_{ij}. \quad (1.3)$$

When $D = 4$, $C = -C^T$, so Z is antisymmetric and when $D = 8$, $C = C^T$ and Z is symmetric.

Finally, for $D = 6$ the two spinor representations are pseudoreal and independent. This means that one can choose independently the number of supersymmetry charges with chirality $+$ or $-$ (N_+ and N_-).

$$\{Q_{\alpha i+}, Q_{\beta j+}\} = \left(\mathcal{P}_+\Gamma_\mu C p^\mu\right)_{\alpha\beta}\Omega_{ij}$$
$$\{Q_{\alpha i+}, Q_{\beta j-}\} = (\mathcal{P}_+ C Z_{ij})_{\alpha\beta}$$
$$\{Q_{\alpha i-}, Q_{\beta j-}\} = \left(\mathcal{P}_-\Gamma_\mu C p^\mu\right)_{\alpha\beta}\Omega_{ij}. \quad (1.4)$$

There are some rotations of the charge vectors $Q_{\alpha i}$ that leave invariant the momentum term in the anticommutator of supercharges. These transformations are automorphisms of the super Poincaré algebra, and they form the R-symmetry group of the algebra. It is a compact group that we will denote by H. The nature of this group obviously depends on the reality properties the spinors. For $D = 4, \ldots 9$ these are

R-symmetry group H

$D = 9$ SO(N)

$D = 8$ SU(N) $\times$ U(1)

$D = 7$ USp(N)

$D = 6$ USp(N_+) $\times$ USp(N_-)

$D = 5$ USp(N)

$D = 4$ SU(N) $\times$ U(1)

We consider theories with a maximal number of supesymmetries, $n = 32$. From the symmetry properties of Z stated above, it follows that the central charge transforms in the following representation of the R-symmetry group,

Central charge representation of the R-symmetry

$D = 9$ **3** of SO(2), (real symmetric tensor).

$D = 8$ **3(+)** of SU(2)$\times$U(1), (complex triplet).

$D = 7$ **10** of USp(4), (real symmetric tensor).

$D = 6$ **16** of USp(4)$\times$USp(4),

      (bispinor (4,4) of O(5)$\times$O(5)).

$D = 5$ **27** of USp(8), (Ω-traceless

      symplectic antisymmetric tensor).

$D = 4$ **28** of SU(8), (complex antisymmetric

      tensor).   (1.5)

We will see that it is always possible to put the central charge matrix Z_{ij} in normal form (diagonal or skew diagonal) using a transformation of the R-symmetry group (we denote the eigenvalues by Z_A). Then, using the theory of induced representations, one can go to the rest frame $(p = (m, 0, \ldots, 0))$ and the supercharge anticommutator becomes

$$\{S_A, S_B^\dagger\} \propto (m + Z_A)\delta_{AB},$$
$$\{\hat{S}_A, \hat{S}_B^\dagger\} \propto (m - Z_A)\delta_{AB},$$

so we obtain a Clifford algebra. It is clear that if $m = |Z_A|$ for some A (in general one has $m \geq |Z_A|$), there is one pair of oscillators in the Clifford algebra that decouple. Representations with such value of the central charge are representations in which certain supersymmetry charges become trivial . They are called BPS states. If all eigenvalues are equal, half of the oscillators decouple and we say that we have a representation with $\frac{1}{2}$BPS. For less restrictive conditions one obtains representations with $\frac{1}{4}$BPS or $\frac{1}{8}$BPS.

The corresponding supergravity theories can be obtained by compactifying $D = 11$, $N = 1$ supergravity on a torus T^{d+1} or compactifying $D = 10$, type IIA and IIB supergravity on a torus T^d. The scalar fields of the theory parametrize a coset manifold of the form G/H, where G is a non compact group whose maximally compact subgroup is H. For any d, $G = E_{d+1(d+1)}$ [2] (the factor U(1) for $D = 4$ must be supressed to have H as a subgroup of G). G is called the U-duality group. G acts on the non linear manifold of the scalars, and the vector fields also transform in a representation of G. This representation naturally extends (1.5) and is given in the following table for each case [2],

Central charge representation of the U-duality group

$D = 9$ **2+1** of E_2=SL(2)$\times$ O(1,1)

$D = 8$ **(3,2)** of E_3=Sl(3)$\times$Sl(2)

$D = 7$ **10** of E_4=Sl(5)

$D = 6$ **16** of E_5=O(5,5)

$D = 5$ **27** of $E_{6(6)}$

$D = 6$ **56** of $E_{7(7)}$ (1.6)

Let Φ^i be the scalar fields which parametrize G/H and consider a local section on the principal bundle G over the base G/H, $L(\phi) \in G$ (coset representative). The action of G can be expressed as

$$L(\phi_g) = gL(\phi)h(\phi_g) \qquad (1.7)$$

where $h(\phi_g) \in H$ is a transformation in the fiber over ϕ_g. Choosing a representation of G the coset

representative becomes a matrix $L_a^\Lambda(\Phi)$. The indices a, Λ run over the same represenation of G (and of H), the different names used to remind the transformation rule (1.7).

The charges of a black hole solution can be computed by integrating the Hodge dual of the field strengths of the vector fields present in the theory on a $D-2$ spacial surface enclosing the source. We denote these charges by q^Λ. Then, the central charges of the black hole solutions are given by

$$Z_a(q, \Phi) = q_\Lambda^T L_a^\Lambda(\Phi) \qquad (1.8)$$

Notice that the vectors fields are in a certain representation of G, which necesarily coincides with the representation in (1).

When the central charges correspond to BPS states, some supersymmetry generators are null when acting on the black hole solutions, so the solutions are "supersymmetric". The condition to be a BPS black hole is a condition on Z, that is, on q and Φ. Due to the form of (1.8), it is clear that any condition $E_\alpha(Z) = 0$ (where α runs over some representation T of G) that is covariant under G, that is

$$E_\alpha(Zg) = E_\beta(Z)T(g)_\alpha^\beta, \qquad (1.9)$$

will become simply a condition on q, $E_\alpha(q) = 0$. It was shown in [3] that this condition is actually moduli independent or U-duality invariant in all theories in dimensions $D = 4, \ldots, 9$ with $n = 32, 16$ supersymmetries. Here we will review these results, in particular $D = 4$, $N = 8$ and $D = 5$, $N = 4$.

2. Maximal supergravity in dimensions $D = 4, \ldots, 9$

We study the diagonalization of the matrix of central charges in the different cases (Table 1). A matrix is quaternionic (or symplectic) if

$$Z^* = -\Omega Z \Omega.$$

A matrix satisfying this conditions can be understood as a matrix whose entries are quaternions, and these are in the representation

$$\mathrm{Id}_{2\times2}, \quad i\sigma_1, \quad i\sigma_2, \quad i\sigma_3.$$

One has the following results:

1. Any matrix can be brought to a diagonal form by making a transformation

$$Z_D = U_1 Z U_2^\dagger$$

where U_i are orthogonal matrices if Z is real, unitary if Z is complex and unitary symplectic if Z is quaternionic (In the representation of $n \times n$ quaternionic matrices by complex, $2n \times 2n$ matrices, it is ΩZ which is brought to diagonal form).
2. In the case that the original matrices are symmetric, hermitian or symplectic hermitian respectively, $U_1 = U_2$ and the eigenvalues are real.
3. An antisymmetric matrix can be brought to skew diagonal form by a transformation

$$Z_{SD} = U Z U^T$$

where U belongs to the appropriate group as before and the eigenvalues are real.

For $D = 9$ we can trivially diagonalize Z with an R-symmetry transformation by using the result 2. For $D = 8$, an R-transformation U brings the matrix Z to UZU^T. It is easy to see that to diagonalize the 2×2 matrix it is enough to use the result 1 with $U_2^\dagger = U_1^T$, being the eigenvalues real. For $D = 7$, Z is four dimensional and since the spinors are pseudoreal it is quaternionic. Since it is also symmetric, we can take advantage of the isomorphism $\mathrm{Sp}(4) \approx \mathrm{O}(5)$ and decompose Z as

$$Z_{ij} = Z_{IJ}(\gamma^{IJ})_{ab}, IJ = 1, \ldots 5$$

where Z^{IJ} is real and antisymmetric and

$$\gamma^{IJ} = \frac{1}{2}[\gamma^I, \gamma^J],$$

and γ^I are the gamma matrices of O(5). We can skew diagonalize Z^{IJ} using the result 3, with only two independent eigenvalues. For $D = 6$ one can use the result 1 and diagonalize the matrix with an element $(U_1, U_2) \in \mathrm{USp}(4) \times \mathrm{USp}(4)$. The quaternionic property is preserved by a transformation $Z_D = U_1 Z U_2^\dagger$, but since the eigenvalues are not real there are four independent real quantities. Finally, for $D = 5, 4$ one can use the result 3.

For $D = 7, 8, 9$ there are only two independent eigenvalues. In the generic situation, when

D	N	H	Z	eigenvalues
9	2	SO(2)	real symmetric.	2
8	2	SU(2)×U(1)	complex symmetric	2
7	4	USp(4)≈O(5)	quaternionic symmetric	2
6	4,4	USp(4)×USp(4)	quaternionic	4
5	8	USp(8)	quaternionic antisymmetric	4
4	8	SU(8)	complex antisymmetric	4

Table 1: Central charge matrix

the two eigenvalues are different, we have that the bound $m = |Z_A|$ can be reached only by the highest eigenvalue. Then, $\frac{1}{4}$ of the oscillators of the supersymmetry alegra are null on the solution. We say that we have a $\frac{1}{4}$BPS state. If all the eigenvalues are equal, the half of the oscillators decouple and we have a $\frac{1}{2}$BPS state. For $D = 4, 5, 6$ we have four eignevalues and the generic case, when all of them are different preserves $\frac{1}{8}$ of the supersymmetry. If the eigenvalues are equal by pairs we have $\frac{1}{4}$BPS states and if they are all equal we have $\frac{1}{2}$BPS states. Exotic cases like having only one pair of equal eigenvalues and the rest different or having three equal eigenvalues and a different fourth one are forbidden on physical grounds as we will see in the next example.

2.1 BPS states in $D = 4$, $N = 8$ Supergravity.

As we have seen in this case Z is a 8×8 complex antisymmetric matrix, so it can be skew diagonalized

$$Z_{SD} = UZU^T$$

with 4 independent eigenvalues. Since we can use only transformations $U \in$ SU(8), the eigenvalues are not real. Instead there is an overall phase that cannot be removed [4]. In fact, this phase is an extra parameter of the solution which usually is set to zero and all the eigenvalues real. Their absolute value, can be computed as the square root of the eigenvalues of the matrix $ZZ^\dagger$.

$\frac{1}{2}$**BPS** Since all the eigenvalues are real, the matrix $ZZ^\dagger$ is a multiple of the identity,

$$ZZ^\dagger = \text{Tr}(ZZ^\dagger)\frac{1}{8}\text{Id}. \tag{2.1}$$

This is an SU(8) covariant constraint. The idea is to find an $E_{7,7}$ covariant constraint that neces-

sarily implies (2.1). Then we will be in the situation (1.9), where the $\frac{1}{2}$BPS condition is moduli independent. Nevertheless, the eigenvalues will depend on the moduli.

We consider the irreducible representation of $E_{7,7}$ **56** (the number indicates the dimensionality of the representation). Under SU(8) it decomposes as $\mathbf{56} = \mathbf{28} + \bar{\mathbf{28}}$. **28** is the twofold antisymmetric representation of SU(8) in which Z sits. We will write a vector of **56** symbolically as $\tilde{Z} = (Z, \bar{Z})$. We consider now the quartic invariant of this representation of $E_{7,7}$ [6, 8, 9],

$$I = 4\text{Tr}(Z\bar{Z})^2 - (\text{Tr}Z\bar{Z})^2 + 2^4(PfZ + Pf\bar{Z}), \tag{2.2}$$

where

$$PfZ = \frac{1}{2^4 4!}\epsilon^{ABCDRPGH}Z_{AB}Z_{CD}Z_{RP}Z_{GH}$$

is the Pfaffian of Z. Let us take the second derivative of this invariant

$$\frac{\partial^2 I}{\partial \tilde{Z} \partial \tilde{Z}}.$$

It is a quadratic polynomial which is a symmetric tensor, so it sits in the $(\mathbf{56} \times \mathbf{56})_S = \mathbf{1596}$ representation of E_7 which is reducible and decomposes as $\mathbf{1463} + \mathbf{133}$. **133** is the adjoint of E_7, so we can take the projection

$$\frac{\partial^2 I}{\partial \tilde{Z} \partial \tilde{Z}}|_{Adj_{E_7}}. \tag{2.3}$$

Since **133** decomposes as $\mathbf{63} + \mathbf{70}$ under SU(8) (**63** is the adjoint representation of SU(8)), the expression (2.3) splits into two SU(8) covariant polynomials

$$V_A^C = \frac{\partial^2 I}{\partial Z_{AB} \bar{\partial} Z^{CB}}|_{Adj_{SU(8)}} \approx$$

$$(Z_{AB}\bar{Z}^{CB} - \frac{1}{8}\delta_A^C Z_{PQ}\bar{Z}^{PQ}), \tag{2.4}$$

$$V^+_{[ABCD]} = \frac{\partial^2 I}{\partial Z_{AB}\bar{\partial}\bar{Z}^{CB}}|_{\mathbf{70}} \approx \frac{\partial^2 I}{\partial Z_{[AB}\partial Z_{CD]}} -$$

$$\frac{1}{4!}\epsilon^{ABCDPQRS}\frac{\partial^2 I}{\partial \bar{Z}^{[AB}\partial \bar{Z}^{CD]}}. \qquad (2.5)$$

The **63** piece, (2.4) is just (2.1), which is the constraint we want. But the **70** part $V^+_{[ABCD]}$ is not, in principle, zero. Here is where the G/H structure of the theory enters. Consider $L(\Phi)$: $G/H \mapsto G$, the local section or coset representative of G/H. Using this map we can make a pull-back of the Maurer-Cartan left invariant forms on G to an open set of G/H,

$$\Omega(\phi) = L^{-1}(\phi)dL(\phi) = \omega_i T_i + P_\alpha T_\alpha. \quad (2.6)$$

$T_i \in \mathcal{H}$ are the generators in the Lie algebra of H and $T_\alpha \in \mathcal{K}$, where $\mathcal{G} = \mathcal{H} + \mathcal{K}$ is a Cartan decomposition of $\mathcal{G}$, the Lie algebra of G. ω_i are the components of the spin connection of the bundle $G \mapsto G/H$ and P_α is the vielbein of the invariant metric on G/H.

In the case of $E_{7,7}/SU(8)$, the equation (2.6) takes the form

$$\nabla_{SU(8)}Z_{AB} = \frac{1}{2}\epsilon_{ABCD}\bar{Z}^{CD}.$$

Taking the covariant derivative $\nabla_{SU(8)}$ of the equation $V^C_A = 0$ and using the Maurer-Cartan equations above, one obtains

$$V^{ABCD}_+ = 0,$$

as we wanted to show.

Notice that the reality of the eigenvalues follows in fact from (2.5), so it follows from the $E_{7,7}$ invariance.

$\frac{1}{4}$**BPS.** In this case we have that the eigenvalues are equal by pairs. It can be shown [3] that the $E_{7,7}$ covariant condition for this to happen is

$$\frac{\partial I}{\partial Z_{AB}} = 0 \quad (\Rightarrow \frac{\partial I}{\partial \bar{Z}^{AB}} = 0),$$

where I is again the quartic invariant. The reality also follows from the $E_{7,7}$ invariance.

$\frac{1}{8}$BPS is the generic case when all the eigenvalues are different. One can always make the highest eigenvalue (which is the mass of the BPS state) real, so no equation is needed to assure reality. Having three or two eigenvalues equal

and the rest different makes the quartic invariant negative [5]. Since this invariant has an interpretation as the entropy of the black hole squared, $I \propto S^2$ it cannot be negative, so these cases are excluded.

For all the other theories in Table 1 a similar analysis can be carried out [3, 4, 6, 7]

3. Supergravities with 16 supersymmetries. BPS states of $D = 5$, $N = 4$ supergravity

Theories with sixteen supersymmetries are obtained as compactifications of the heterotic string on tori T^d ($1 \leq d \leq 6$). They can also be obtained by compactifying $D = 11$ supergravity or $D = 10$ Type IIA and IIB supergravity on manifolds preserving less supersymmetries, as K_3. In these theories there are matter fields and the U-duality group G depends on the matter content as well as on the dimension of spacetime. The maximal compact subgroup is a direct product $H_R \times H_M$ where H_R is the R-symmetry of the corresponding supersymmetry algebra and H_M is the group acting on the matter multiplets. If n is the number of matter multiplets this group is $H_M = O(n)$. G is of the form O(10-D,n)×O(1,1) for $5 \leq D \leq 9$ while for $D = 4$ it is SL(2)×O(6,n). The R-symmetry groups are O(10-D) for $5 \leq D \leq 9$ and O(6)×O(2)≈SU(4)×U(1) for D=4.

The G and H_R representations of the central charges are given in the following tables.

Central charge representation of H_R.

$D = 9$	**1**	O(1) = Id
$D = 8$	**1$^{\mathbf{c}}$** complex	U(1) ≈ O(2)
$D = 7$	**3** real	SU(2) ≈ USp(2)
$D = 6$	**4** real	O(4) ≈ USp(2) × USp(2)
$D = 5$	**1 + 5** real	O(5) ≈ USp(4)
$D = 4$	**6$^{\mathbf{c}}$** complex	O(6) × O(2) ≈
		SU(4) × U(1)

From the above table, and according to our previous analysis, it follows that the matrix of central charges, Z, has only one independent real eigenvalue for $D = 6, \dots 9$ and two independent

eigenvalues for $d = 5, 6$. Therefore, for $D = 6, \ldots 9$ only $1/2$ BPS states can occur while for $D = 4, 5$ both, $1/2$ and $1/4$ BPS states can occur.

Central charge representation of G

$$D = 6, \ldots 9 \quad \mathbf{d} + \mathbf{n} \text{ real vector}$$
$$\mathrm{O}(d, n) \times \mathrm{O}(1, 1)$$
$$D = 5 \quad \mathbf{1} + (\mathbf{5} + \mathbf{n}) \text{ (singlet+vector)} r$$
$$\mathrm{O}(5, n) \times \mathrm{O}(1, 1)$$
$$D = 4 \quad (\mathbf{2}, \mathbf{6} + \mathbf{n})$$
$$\mathrm{Sl}(2) \times \mathrm{SO}(6, n)$$

We will briefly outline here the case of $D = 5$. It corresponds to heterotic string on T^5 or $D = 11$ supergravity on $K^3 \times T^2$. The number of matter multiplets is $n = 21$, although our analysis is independent of such number.

The central charge $\hat{Z}$ is an antisymmetric quaternionic matrix. This implies that the matrix $Z = \hat{Z}\Omega$ is hermitian and quatenionic. The 4×4 matrix depends on 6 real parameters. We want to exploit the isomorphism $\mathrm{O}(5) \approx \mathrm{USp}(4)$, so we decompose Z as

$$Z = Z^a \gamma_a + Z^0 \mathrm{Id},$$

where γ_a, $a = 1, \ldots 5$ are the $\mathrm{O}(5)$ γ-matrices and Z^a, Z^0 are real numbers. Z^0 is a singlet under $\mathrm{O}(5)$ and Z^a is a vector. It follows that

$$\mathrm{Tr} Z = 4 Z^0$$
$$(\det Z)^{1/2} = \frac{1}{8}(\mathrm{Tr} Z)^2 - \frac{1}{4}\mathrm{Tr} Z^2 =$$
$$Z^{0^2} - \vec{Z}^2 \tag{3.1}$$

The characteristic equation of Z (or better, its square root) is

$$\sqrt{\det Z - \lambda \mathrm{Id}} =$$
$$\lambda^2 - \frac{1}{2}\mathrm{Tr} Z \, \lambda + (\det Z)^{1/2} = 0,$$

implying that Z has two coinciding eigenvalues (in absolute value) either if

$$\mathrm{Tr} Z = 0 \quad \text{or} \quad \frac{1}{4}(\mathrm{Tr} Z)^2 = 4(\det Z)^{1/2}$$

Using (3.1), the above equation directly implies

$$Z_0 Z_a = 0. \tag{3.2}$$

The eigenvalues are given by

$$\lambda_{1,2} = \frac{1}{2}\left(\frac{1}{2}\mathrm{Tr} Z \pm \sqrt{\mathrm{Tr} Z^2 - \frac{1}{4}(\mathrm{Tr} Z)^2}\right),$$

being the plus sign the mass squared of the BPS state.

The central charge representation of G (and H), singlet + vector, is reducible and the coset representative L_a^Λ splits into blocks

$$\begin{pmatrix} e^{2\sigma} & \\ & e^{-\sigma}\,(\, M \,) \end{pmatrix},$$

where σ parametrizes $\mathrm{O}(1,1)$ and M is in the fundamental representation of $\mathrm{O}(5,n)$. Since $Z_a = q_\Lambda L_a^\Lambda$, then $Z_0 = e^{2\sigma} q_0$. The condition $Z_0 = 0$ implies $q_0 = 0$, which is a singlet of $\mathrm{O}(5+n)$, and then, U-duality invariant.

If Z_I, $I = 1, \ldots n$ are the matter charges associated to the n matter multiplets, we have that, because of (2.6),

$$\nabla_{O(5)} Z_a = \mathrm{Tr}(\gamma_a P_I) Z^I + Z_a d\sigma,$$

therefore $Z_a = 0$ implies $Z_I = 0$. This is also an $\mathrm{O}(5,n)$ invariant statement since, it comes by differentiating the quadratic invariant polynomial

$$I = \sum_{a=1}^{5} Z_a Z^a - \sum_{I=1}^{M} Z_I Z^I.$$

Therefore, $Z_a = Z_I = 0$ implies $q^\Lambda = 0$ where q^Λ, $\Lambda = 1, \ldots 5 + n$, is a fixed charge vector of $\mathrm{O}(5, M)$, as found in [4].

Acknowledgments

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