The longitudinal fivebrane and tachyon condensation in matrix theory

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ABSTRACT: We study a configuration in matrix theory carying longitudinal fivebrane charge, i.e. a D0-D4 bound state. We calculate the one-loop effective potential between a D0-D4 bound state and a D0–anti-D4 bound state. Next, we identify the tachyonic fluctuations in the D0-D4 and D0–anti-D4 system. We analyse classically the action for these tachyons and find solutions to the equations of motion corresponding to tachyon condensation.

1. Introduction

Matrix theory [1] [2] [3] is the M-theory interpretation of U(N) supersymmetric quantum mechanics which has passed many stringent tests. The brane content of matrix theory was determined in [4]. Amongst other branes, the longitudinal fivebrane was identified [1].

In [10] a first step towards the understanding of Sen's tachyon condensation mechanism [11] in matrix theory was taken, by analyzing the tachyon in the D0-D2 and D0–anti-D2 system. We concentrate on the D0-D4 and D0–anti-D4 system. We identify the tachyonic fluctuations in the D0-D4 and D0–anti-D4 background and analyse the classical action for these fluctuations in the spirit of [10]. We find solutions to the action representing condensation to a vacuum filled with D0-branes and gravitons.

The first section concentrates on a discussion of the classical solution of matrix theory corresponding to a D0-D4 bound state system. In the second section we calculate the effective potential between a D0-D4 and D0–anti-D4 brame. The next section deals with an analysis of the tachyonic fluctuations. Then we analyse possible solutions to the action for the tachyonic fluctuations. Finally, we add remarks on the results and open problems.

[1] The transverse fivebrane remained a puzzle [5].

2. Preliminary discussion of the classical solution

The lagrangian of matrix theory is given by $U(N)$ supersymmetric quantum mechanics, namely the dimensional reduction of tendimensional $\mathcal{N}=1$ $U(N)$ super Yang-Mills theory to $0+1$ dimensions. It reads [1]:

$$\mathcal{L} = \frac{T_0}{2} Tr\big((D_0 X_I)^2 + \frac{1}{2}[X_I, X_J]^2 + 2\theta^T D_0 \theta - 2\theta^T \gamma^I [\theta, X_I]\big) \qquad (2.1)$$

where we take $2\pi\alpha' = 1$ and $T_0 = \frac{\sqrt{2\pi}}{g}$. Furthermore we have $D_0 = \partial_t - i[A_0, .]$ and $I = 1, 2, \ldots, 9$. All fields are in the adjoint of $U(N)$. The fermions are Majorana-Weyl.

We study especially a background configuration ($X_I = B_I$) corresponding to a D0-D4 bound state, or longitudinal fivebrane, satisfying the following commutation rules [4]:

$$[B_1, B_2] = -ic\,\sigma_3 \otimes I_{\frac{N}{2}\times\frac{N}{2}}$$
$$[B_3, B_4] = -ic\,\sigma_3 \otimes I_{\frac{N}{2}\times\frac{N}{2}}, \qquad (2.2)$$

and the other matrices and commutators zero. Here σ_3 is the third Pauli matrix and c is a constant. We take the infinite background matrices to be blockdiagonal such that this configuration solves the equations of motion. it carries longitudinal fivebrane charge in the $1, 2, 3, 4$ directions:

$$q_5 = -\frac{1}{8\pi^2}\epsilon^{IJKL} Tr[B_I B_J B_K B_L] = N\frac{c^2}{4\pi^2}$$

3. Calculating the effective potential in matrix theory at one loop

In this section we calculate the interaction potential between the D0-D4 and the D0–anti-D4 bound state.

Because each object is represented by a 'two-by-two' matrices, we need some conventions and nomenclature, which we will take to be as follows. In this section, the first bound state will have extent n_0, the second N_0. In each 'two-by-two' matrix, the submatrices will have half the extent of the object, e.g. $\frac{n_0}{2}$. We will take the following nomenclature for the different parts of the coordinate matrices:

$$X_I = \begin{pmatrix} \text{block 1} & 0 & \text{sect 13} & \text{sect 14} \\ 0 & \text{block 2} & \text{sect 23} & \text{sect 24} \\ \text{sect 13}^\dagger & \text{sect 23}^\dagger & \text{block 3} & 0 \\ \text{sect 14}^\dagger & \text{sect 24}^\dagger & 0 & \text{block 4} \end{pmatrix}$$

The off-diagonal modes have been divided up into four different sectors.

The technique to calculate the one-loop effective potential between two objects in matrix theory is standard by now [6] [7]. To calculate the potential, we determine the spectrum of the off-diagonal fluctuations corresponding to strings stretching from one object to the other. Their mass matrix is easily determined by expanding the action of matrix theory around the relevant background. This is slightly more involved when objects are represented by two-by-two matrices, but the general formulae in for instance [9] [12] can easily be adapted to our case, essentially because the background matrices are block diagonal. We do not give the details of the calculation, but summarize the end result.

The D0–anti-D4 will be at a distance b of the D0-D4 in some transverse direction ("8") and it will be moving with a velocity v relative to it in another transverse direction ("9"). This is incorporated by choosing the background matrices corresponding to these transverse coordinates to be:

$$B_8 = \begin{pmatrix} 0_{n_0\times n_0} & 0 \\ 0 & b\, I_{N_0\times N_0} \end{pmatrix}$$
$$B_9 = \begin{pmatrix} 0_{n_0\times n_0} & 0 \\ 0 & vt\, I_{N_0\times N_0} \end{pmatrix} \quad (3.1)$$

Finally, to make the interaction energies finite, we wrap the fourbranes on a four-torus. This hardly influences the calculation. It is moreover convenient to take the four-torus to have self-dual radii $R_i = \sqrt{\alpha'}$. It is straightforward to again add in the dependence on the compactification radii in the final formulae. See for instance [9].

The background matrices are:

$$\begin{aligned} [P_1, Q_1] &= -ic_1 \\ [P_2, Q_2] &= -ic_1 \\ [P_3, Q_3] &= -ic_3 \\ [P_4, Q_4] &= -ic_3 \\ B^1 &= \begin{pmatrix} P_1 & 0 & 0 & 0 \\ 0 & P_1 & 0 & 0 \\ 0 & 0 & -P_3 & 0 \\ 0 & 0 & 0 & -P_3 \end{pmatrix} \\ B^2 &= \begin{pmatrix} Q_1 & 0 & 0 & 0 \\ 0 & -Q_1 & 0 & 0 \\ 0 & 0 & Q_3 & 0 \\ 0 & 0 & 0 & -Q_3 \end{pmatrix} \\ B^3 &= \begin{pmatrix} P_2 & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 \\ 0 & 0 & P_4 & 0 \\ 0 & 0 & 0 & P_4 \end{pmatrix} \\ B^4 &= \begin{pmatrix} Q_2 & 0 & 0 & 0 \\ 0 & -Q_2 & 0 & 0 \\ 0 & 0 & Q_4 & 0 \\ 0 & 0 & 0 & -Q_4 \end{pmatrix} \end{aligned} \quad (3.2)$$

We find four sectors of extent $\frac{n_0}{2} \times \frac{N_0}{2}$, all with identical spectra, when we ignore the origin in terms of the different coordinates [2]. The relevant hamiltonian is:

$$\begin{aligned} H^{(13)} &= (P_1 - P_3)^2 + (Q_1 - Q_3)^2 + (P_2 + P_4)^2 \\ &\quad + (Q_2 - Q_4)^2 + b^2 + v^2 t^2, \end{aligned}$$

corresponding to a system of two harmonic oscillators. We will always suppose that $c_1 - c_3$ is positive, the other case being fully equivalent. The mass operators are for each sector for the bosons $4 : H \pm 2iv$; $4 : H \pm 2(c_1 + c_3)$; $4 : H \pm 2(c_1 - c_3)$;

[2] We can do so for calculating the effective potential, but in section 4 we need the precise origin of the tachyonic modes in terms of the coordinate matrices. We return there to this point.

$4 : H$ and for the fermions $16 : H \pm iv \pm (c_1 + c_3) \pm (c_1 - c_3)$. The potential is then :

$$\begin{aligned}\mathcal{V} &= \frac{2\mathcal{N}}{\sqrt{\pi}} \int_0^\infty \frac{ds \qquad e^{-b^2 s}}{4s^{3/2}\sinh{(c_1-c_3)s}\sinh{(c_1+c_3)s}} \\ &\quad \times(2 + 2\cos 2vs + 2\cosh 2(c_1 - c_3)s \\ &\quad +2\cosh 2(c_1+c_3)s - 8\cos vs \\ &\quad \cosh{(c_1+c_3)s}\cosh{(c_1-c_3)s}) \\ &\approx \frac{n_4 N_4}{b^3} + \frac{(n_0 N_4 + N_0 n_4)v^2}{4b^3} + \frac{n_0 N_0 v^4}{16 b^3}\end{aligned}$$

The interaction potential is non-trivial at zero velocity and the background fully breaks supersymmetry. The end result can be reproduced by a supergravity calculation [8]. Clearly, the formula for the potential breaks down at small distances $b^2 \leq 2c_3$. Then there is a tachyon in the spectrum of the bosons since the lowest energy mode has mass: $E = (c_1 - c_3) + (c_1 + c_3) + b^2 - 2(c_1 + c_3) = b^2 - 2c_3$. We will treat the system at short distances in section 4.

4. The action for the tachyonic fluctuations

4.1 The action

From now on, we will consider the D0-D4 system and the D0–anti-D4 system to lie on top of each other, so we put the background matrices B_8 and B_9 (3.1) to zero. We compute the mass matrix for the fluctuations in the coordinate matrices X_1 and X_2, and find the tachyonic fluctuations

$$\begin{aligned}\phi &= \frac{y_2^{(13)} - iy_1^{(13)}}{\sqrt{2}} \\ \phi' &= \frac{y_2^{(24)} + iy_1^{(24)}}{\sqrt{2}} \\ \chi &= \frac{y_4^{(14)} - iy_3^{(14)}}{\sqrt{2}} \\ \chi' &= \frac{y_4^{(23)} + iy_3^{(23)}}{\sqrt{2}} \qquad (4.1)\end{aligned}$$

Next we turn to the analysis of the action for the tachyonic fluctuations in the spirit of [10]. We expand the classical action around the D0-D4 and D0–anti-D4 background, only keeping track of the tachyonic fluctuations and the gauge fields of the unbroken gauge group $U(1)^4$ under which the tachyons are charged. For simplicity, we take the number of D0-D4 bound states and D0-anti-D4 bound states to be equal, i.e. $c_1 = c_3 = c$. We will use a representation in terms of gauge fields [13]. Under the preceding assumptions, the coordinate matrices are given by:

$$X^1 = c\begin{pmatrix} -i\nabla_{x_1}^{(1)} & 0 & i\frac{\phi}{\sqrt{2c}} & 0 \\ 0 & -i\nabla_{x_1}^{(2)} & 0 & -i\frac{\phi'}{\sqrt{2c}} \\ -i\frac{\phi^*}{\sqrt{2c}} & 0 & -i\nabla_{y_1}^{(3)} & 0 \\ 0 & i\frac{\phi'^*}{\sqrt{2c}} & 0 & -i\nabla_{y_1}^{(4)}\end{pmatrix}$$

And similarly for X^2, X^3 and X^4. We defined $\nabla_{x_i}^{(m)} = \partial_{x_i} + iA_{x_i}^{(m)} + ia_{x_i}^{(m)}$ where A is the background gauge field and a the gauge field fluctuation. The background is invariant under $U(1)^4$, each $U(1)$ has its own upper index. We choose the background gauge fields such that the appropriate commutation relations between the background matrices are satisfied:

$$\begin{aligned}A_{x_2}^{(1)} = -A_{x_2}^{(2)} &= \frac{x_1}{c} \\ A_{x_4}^{(1)} = -A_{x_4}^{(2)} &= \frac{x_3}{c} \\ A_{y_2}^{(3)} = -A_{y_2}^{(4)} &= -\frac{y_1}{c} \\ A_{y_4}^{(3)} = -A_{y_4}^{(4)} &= \frac{y_3}{c}, \qquad (4.2)\end{aligned}$$

and the rest zero. Each tachyonic mode is charged under two of the abelian gauge symmetries, with opposite charges, as can easily be seen by looking at the transformation properties of the full coordinate matrix.

To represent the action in terms of an integral over the worldvolume of the branes, we use the rules of [4], improved in [13] and elaborated upon in [10]. The following definitions come in handy in writing down the endresult. The non-center-of-mass coordinates are:

$$u_i = \frac{x_i + y_i}{2}. \qquad (4.3)$$

Covariant derivatives and field strengths are defined as (Upper indices label the gauge symmetries, lower indices $w_i = (x_i, y_i)$ label coordinates.) :

$$\begin{aligned}\nabla_{w_i}^{(\pm m)} &= \partial_{w_i} \pm iA_{w_i}^{(m)} \pm ia_{w_i}^{(m)} \\ F_{w_i w_j}^{(m)} &= i\left[\nabla_{w_i}^{(m)}, \nabla_{w_j}^{(m)}\right]\end{aligned}$$

$$\begin{aligned}
\nabla_{u_i}^{(m,\pm n)} &= \nabla_{x_i}^{(m)} + \nabla_{y_i}^{(\pm n)} \\
F_{u_i u_j}^{(m,\pm n)} &= i\left[\nabla_{u_i}^{(m,\pm n)}, \nabla_{u_j}^{(m,\pm n)}\right] \\
&= F_{x_i x_j}^{(m)} \pm F_{y_i y_j}^{(n)} \qquad (4.4)
\end{aligned}$$

By a small f we will denote the field strength F without the background gauge fields contribution. The relevant part of the action for the fluctuations that we consider is then given by $S = \int d^4u\,\mathcal{L}$, and the lagrangian by (up to an overall factor) :

$$\begin{aligned}
-\mathcal{L} =\ & (\frac{c^2}{2} f_{u_1u_2}^{(1,-3)} - c + |\phi|^2)^2 \\
& +(\frac{c^2}{2} f_{u_3u_4}^{(1,-4)} - c + |\chi|^2)^2 \\
& +(\frac{c^2}{2} f_{u_1u_2}^{(2,-4)} + c - |\phi'|^2)^2 \\
& +(\frac{c^2}{2} f_{u_3u_4}^{(2,-3)} + c - |\chi'|^2)^2 \\
& +\frac{c^2}{2}\Big(|(\nabla_{u_2}^{(1,-3)} + i\nabla_{u_1}^{(1,-3)})\phi|^2 \\
& +2|\nabla_{u_3}^{(1,-3)}\phi|^2 + 2|\nabla_{u_4}^{(1,-3)}\phi|^2 \\
& +|(\nabla_{u_4}^{(1,-4)} + i\nabla_{u_3}^{(1,-4)})\chi|^2 \\
& +2|\nabla_{u_1}^{(1,-4)}\chi|^2 + 2|\nabla_{u_2}^{(1,-4)}\chi|^2 \\
& +|(\nabla_{u_2}^{(2,-4)} - i\nabla_{u_1}^{(2,-4)})\phi'|^2 \\
& +2|\nabla_{u_3}^{(2,-4)}\phi'|^2 + 2|\nabla_{u_4}^{(2,-4)}\phi'|^2 \\
& +|(\nabla_{u_4}^{(2,-3)} - i\nabla_{u_3}^{(2,-3)})\chi'|^2 \\
& +2|\nabla_{u_1}^{(2,-3)}\chi'|^2 + 2|\nabla_{u_2}^{(2,-3)}\chi'|^2\Big) \\
& +\frac{c^4}{4}\Big(f_{u_1u_3}^{(1,3)^2} + f_{u_1u_3}^{(1,-3)^2} + f_{u_1u_3}^{(2,4)^2} + f_{u_1u_3}^{(2,-4)^2} \\
& +f_{u_1u_3}^{(1,3)^2} + f_{u_2u_3}^{(1,-3)^2} + f_{u_2u_3}^{(2,4)^2} + f_{u_2u_3}^{(2,-4)^2} \\
& +f_{u_1u_4}^{(1,3)^2} + f_{u_1u_4}^{(1,-3)^2} + f_{u_1u_3}^{(2,4)^2} + f_{u_1u_4}^{(2,-4)^2} \\
& +f_{u_2u_4}^{(1,3)^2} + f_{u_2u_4}^{(1,-3)^2} + f_{u_2u_4}^{(2,4)^2} + f_{u_2u_4}^{(2,-4)^2} \\
& +f_{u_1u_2}^{(1,3)^2} + f_{u_1u_2}^{(2,4)^2} + f_{u_3u_4}^{(1,4)^2} + f_{u_3u_4}^{(2,3)^2}\Big) \\
& +|(\phi\chi'^* - \chi\phi'^*)|^2 + |(\phi\chi^* - \chi'\phi'^*)|^2 \quad (4.5)
\end{aligned}$$

where all fields only depend on the non-center-of-mass coordinates. Note that it is the Lagrangian you expect, with the usual kinetic terms for the gauge fields, the appropriate covariant derivatives hitting the tachyons and a Higgs potential for the tachyons. There are some interactions between the tachyons and the gauge fields [10], and an interaction potential between the different tachyons.

4.2 Boundary conditions

The non-zero background gauge fields appearing in the covariant derivatives in the kinetic terms for the tachyons are:

$$\begin{aligned}
\mathcal{A}_{u_2}^{(1,-3)} &= \frac{2u_1}{c} \\
\mathcal{A}_{u_2}^{(2,-4)} &= -\frac{2u_1}{c} \\
\mathcal{A}_{u_4}^{(2,-3)} &= -\frac{2u_3}{c} \\
\mathcal{A}_{u_4}^{(1,-4)} &= \frac{2u_3}{c} \qquad (4.6)
\end{aligned}$$

Taking the background gauge fields to live on a four-torus with radii R_{u_i}, they satisfy 't Hooft's twisted boundary conditions [14]. They read in direction u_1 :

$$\begin{aligned}
\mathcal{A}_{u_i}(R_{u_1}, u_2, u_3, u_4) =\ & -i\Omega_{u_1}\partial_{u_i}\Omega_{u_1}^{-1} \\
& +\Omega_{u_1}\mathcal{A}_{u_i}(0, u_2, u_3, u_4)\Omega_{u_1}^{-1}
\end{aligned}$$

and analogous for the other directions, where Ω_{u_i} are the transition functions. The transition functions can be choosen to be:

$$\begin{aligned}
\Omega_{u_1} &= \exp\left[-iu_2\frac{R_{u_1}}{c}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right] \\
\Omega_{u_2} &= 1 \\
\Omega_{u_3} &= \exp\left[-iu_4\frac{R_{u_3}}{c}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\right] \\
\Omega_{u_4} &= 1 \qquad (4.7)
\end{aligned}$$

These boundary conditions are due to the presence of the *background* field, i.e. due to the magnetic field made up of the D0-branes, representing the background objects. For the full background matrix this implies:

$$B_I(R_{u_1}, u_2, u_3, u_4) = \Omega_{u_1}B_I(0, u_2, u_3, u_4)\Omega_{u_1}^{-1}$$

and analogously for the other directions.

The boundary conditions for the tachyons that are trivial *with respect to the background* are:

$$\begin{aligned}
\phi(u_1 = R_1) &= \phi(u_1 = 0)e^{-2iu_2R_1/c} \\
\phi'(u_1 = R_1) &= \phi'(u_1 = 0)e^{2iu_2R_1/c} \\
\chi(u_3 = R_3) &= \chi(u_3 = 0)e^{-2iu_4R_3/c} \\
\chi'(u_3 = R_3) &= \chi'(u_3 = 0)e^{2iu_4R_3/c} \qquad (4.8)
\end{aligned}$$

and the other background boundary conditions are trivial.

5. A solution to the equations of motion

First we look for a solution to the equations of motion where the total Lagrangian (4.5) vanishes and the background boundary conditions are satisfied. We make the following ansatz:

$$\begin{aligned} \phi &= \phi'^{*}(u_1, u_2) \\ \chi &= \chi'^{*}(u_3, u_4). \end{aligned} \tag{5.1}$$

Then we find we can take:

$$\begin{aligned} a^{(1)}_{u_{1,2}} &= -a^{(2)}_{u_{1,2}} = -a^{(3)}_{u_{1,2}} = a^{(4)}_{u_{1,2}} \\ a^{(1)}_{u_{3,4}} &= -a^{(2)}_{u_{3,4}} = a^{(3)}_{u_{3,4}} = -a^{(4)}_{u_{3,4}}. \end{aligned} \tag{5.2}$$

The remaining non-trivial equations are:

$$\begin{aligned} \frac{c^2}{2} f^{(1,-3)}_{u_1 u_2} - c + |\phi|^2 &= 0 \\ (\nabla^{(1,-3)}_{u_2} + i\nabla^{(1,-3)}_{u_1})\phi &= 0 \end{aligned} \tag{5.3}$$

and similar equations for χ. Under the assumption (5.1), we get two copies of the Bogomolny equations. These have been studied in the context of Chern-Simons theory in detail [15] [16] and we only summarize some main features. We can find magnetic soliton solutions to these equations with the background boundary conditions (4.8). Since the spatial worldvolume of the D4-brane is fourdimensional, and the tachyons have non-trivial winding number around a circle at infinity, the magnetic solitons are twodimensional. The boundary conditions are treated in detail in [10]. Using the solutions, we calculate the D0-brane charge from the worldvolume action of the D4-branes:

$$\begin{aligned} N &= \frac{1}{8\pi^2} \int d^4u \big(F^{(1)}F^{(1)} + F^{(2)}F^{(2)} \\ &\quad - F^{(3)}F^{(3)} - F^{(4)}F^{(4)}\big) \\ &= \frac{A_4}{c^2\pi^2}, \end{aligned} \tag{5.4}$$

which is the original D0-brane charge. The D0 charge is concentrated at the intersections of the orthogonal twodimensional solitons. Moreover, from (5.2) we find that the D2-brane charge cancels. This is consistent with the fact that we find from the supersymmetry variations

$$\delta\theta = \frac{1}{2}\left(D_0 X^I \gamma_I + \frac{1}{2}\left[X^I, X^J\right]\gamma_{IJ}\right)\epsilon + \epsilon'$$

that the tachyon condensation restores all dynamical supersymmetry. We conclude that the end products after tachyon condensation are the original D0-branes, and extra gravitons as argued in [10].

6. Remarks and conclusion

In the previous section, we considered tachyon condensation where the tachyons had trivial boundary conditions relative to the background. We can consider more general possibilities, where the tachyons satisfy different boundary conditions. In the case of a membrane–anti-membrane configuration, this amounts to the following. By choosing the topological sector of the tachyon on the D2-brane anti-D2-brane to be non-trivial, one can add or subtract D0-brane charge. After condensation, this gives an arbitrary number of D0-branes. Technically, this is a trivial extension of [10]. In particular, the approximate solution to the equations of motion in [10] remains practically unchanged. In the case of the D0-D4 and D0-anti-D4, we have more possibilities. For instance, by changing the topological sectors of the four tachyons simultaneously, we can modify the amount of D0-brane charge in the end product in a fairly obvious manner (keeping the condition (5.1)). It is clear that for a more general choice of topological sectors, the end product will have D2-brane charge. It would be interesting to study such condensation in detail.

In this paper we have studied the interactions between a D0-D4 bound state and a D0-anti-D4 bound state in matrix theory. First, we calculated the interaction potential at large distances. Next, we looked at a coinciding D0-D4 and D0–anti-D4 bound state system and identified the tachyonic fluctuations. We derived the classical action for these tachyonic fluctuations and found solutions to the equations of motion corresponding to tachyon condensation to D0-branes.

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