

Warped Compactifications and AdS/CFT*

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ABSTRACT: In this talk we discuss two classes of examples of *warped products* of AdS spaces in the context of the AdS/CFT correspondence. The first class of examples appears in the construction of dual Type I' string descriptions to five dimensional supersymmetric fixed points with E_{N_f+1} global symmetry. The background is obtained as the near horizon geometry of the D4-D8 brane system in massive Type IIA supergravity. The second class of examples appears when considering the $\mathcal{N} = 2$ superconformal theories defined on a $3 + 1$ dimensional hyperplane intersection of two sets of M5 branes. We use the dual string formulations to deduce properties of these field theories.

1. Introduction

The consideration of the near horizon geometry of branes on one hand, and the low energy dynamics on their worldvolume on the other hand has lead to conjectured duality relations between field theories and string theory (M theory) on certain backgrounds [1, 2, 3, 4]. The field theories under discussion are in various dimensions, can be conformal or not, and with or without supersymmetry. These properties are reflected by the type of string/M theory backgrounds of the dual description.

In this talk we discuss two classes of examples of *warped products* of AdS spaces in the context of the AdS/CFT correspondence. The warped product structure is a fibration of AdS over some manifold \mathcal{M} [5]. It is the most general form of a metric that has the isometry of an AdS space [6].

The first class of examples [7] appears in the construction of dual Type I' string descriptions to five dimensional supersymmetric fixed points with E_{N_f+1} global symmetry, $E_{N_f+1} = (E_8, E_7, E_6, E_5 = Spin(10), E_4 = SU(5), E_3 = SU(3) \times SU(2), E_2 = SU(2) \times U(1), E_1 = SU(2))$. These fixed points are obtained in the limit of infinite bare coupling

of $\mathcal{N} = 2$ supersymmetric gauge theories with gauge group $Sp(Q_4)$, $N_f < 8$ massless hypermultiplets in the fundamental representation and one massless hypermultiplet in the anti-symmetric representation [8, 9]. These theories were discussed in the context of the AdS/SCFT correspondence in [10]. The dual background is obtained as the near horizon geometry of the D4-D8 brane system in massive Type IIA supergravity. The ten dimensional space is a fibration of AdS_6 over S^4 and has the isometry group $SO(2, 5) \times SO(4)$. This space provides the spontaneous compactification of massive Type IIA supergravity in ten dimensions to the $F(4)$ gauged supergravity in six dimensions [11].

The second class of examples [12] appears when considering the $\mathcal{N} = 2$ superconformal theories defined on a $3 + 1$ dimensional hyperplane intersection of two sets of M5 branes. The $\mathcal{N} = 2$ supersymmetry algebra in four dimensions contains a central extension term that corresponds to string charges in the adjoint representation of the $SU(2)_R$ part of the R-symmetry group. This implies that $\mathcal{N} = 2$ supersymmetric gauge theories in four dimensions can have BPS string configurations at certain regions in their moduli space of vacua. In particular, at certain points in the moduli space of vacua these strings can become tensionless. A brane configuration that exhibits this phenomena consists of two sets of M5

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branes intersecting on $3 + 1$ dimensional hyperplane. The theory on the intersection is $\mathcal{N} = 2$ supersymmetric. One can stretch M2 branes between the two sets of M5 branes in a configuration that preserves half of the supersymmetry. This can be viewed as a BPS string of the four dimensional theory, and we will study such brane configurations using the AdS/CFT correspondence [1, 4].

The talk is organized as follows. In the next two sections we will discuss the first class of examples. In section 2 we will discuss the D4-D8 brane system and its relation to the five dimensional fixed points. In section 3 we will construct the dual string description and use it to deduce some properties of the fixed points. In the following two sections we will discuss the second class of examples. In section 4 we will construct the dual string description of the four dimensional theory on the intersection and discuss the field theory on the intersection. In section 5 we will use the dual string description to deduce some properties of these strings. We will argue that from the four dimensional field theory viewpoint they are simply BPS string configurations on the Higgs branch.

2. The D4-D8 Brane System

We start with Type I string theory on $R^9 \times S^1$ with N coinciding D5 branes wrapping the circle. The six dimensional D5 brane worldvolume theory possesses $\mathcal{N} = 1$ supersymmetry. It has an $Sp(N)$ gauge group, one hypermultiplet in the antisymmetric representation of $Sp(N)$ from the DD sector and 16 hypermultiplets in the fundamental representation from the DN sector. Performing T-duality on the circle results in Type I' theory compactified on the interval S^1/Z_2 with two orientifolds (O8 planes) located at the fixed points. The D5 branes become D4 branes and there are 16 D8 branes located at points on the interval. They cancel the -16 units of D8 brane charge carried by the two O8 planes. The locations of the D8 branes correspond to masses for the hypermultiplets in the fundamental representation arising from the open strings between the D4 branes and the D8 branes. The hypermulti-

plet in the antisymmetric representation is massless.

Consider first $N = 1$, namely one D4 brane. The worldvolume gauge group is $Sp(1) \simeq SU(2)$. The five-dimensional vector multiplet contains as bosonic fields the gauge field and one real scalar. The scalar parametrizes the location of the D4 branes in the interval, and the gauge group is broken to $U(1)$ unless the D4 brane is located at one of the fixed points. A hypermultiplet contains four real (two complex) scalars. The N_f massless matter hypermultiplets in the fundamental and the antisymmetric hypermultiplet (which is a trivial representation for $Sp(1)$) parametrize the Higgs branch of the theory. It is the moduli space of $SO(2N_f)$ one-instanton. The theory has an $SU(2)_R$ R-symmetry. The two supercharges as well as the scalars in the hypermultiplet transform as a doublet under $SU(2)_R$. In addition, the theory has a global $SU(2) \times SO(2N_f) \times U(1)_I$ symmetry. The $SU(2)$ factor of the global symmetry group is associated with the massless antisymmetric hypermultiplet and is only present if $N > 1$, the $SO(2N_f)$ group is associated with the N_f massless hypermultiplets in the fundamental and the $U(1)_I$ part corresponds to the instanton number conservation.

Consider the D8 brane background metric. It takes the form

$$\begin{aligned} ds^2 &= H_8^{-1/2}(-dt^2 + dx_1^2 + \dots + dx_8^2) \\ &\quad + H_8^{1/2} dz^2, \\ e^{-\phi} &= H_8^{5/4}. \end{aligned} \tag{2.1}$$

H_8 is a harmonic function on the interval parametrized by z . Therefore H_8 is a piecewise linear function in z where the slope is constant between two D8 branes and decreases by one unit for each D8 brane crossed. Thus,

$$H_8(z) = c + 16 \frac{z}{l_s} - \sum_{i=1}^{16} \frac{|z - z_i|}{l_s} - \sum_{i=1}^{16} \frac{|z + z_i|}{l_s}, \tag{2.2}$$

where the z_i denote the locations of the 16 D8 branes.

Denote the D4 brane worldvolume coordinates by $t, x_1 \dots x_4$. The D4 brane is located at some point in $x_5 \dots x_8$ and z . We can consider it to be a probe of the D8 branes background. The

gauge coupling g of the D4 brane worldvolume theory and the harmonic function H_8 are related as can be seen by expanding the DBI action of a D4 brane in the background (2.2). We get

$$g^2 = \frac{l_s}{H_8}, \quad (2.3)$$

where $g_{cl}^2 = \frac{c}{l_s}$ corresponds to the classical gauge coupling. In the field theory limit we take $l_s \rightarrow 0$ keeping the gauge coupling g fixed, thus,

$$g^2 = \text{fixed}, \quad \Rightarrow \quad \phi = \frac{z}{l_s^2} = \text{fixed}, \quad l_s \rightarrow 0. \quad (2.4)$$

In this limit we have

$$\frac{1}{g^2} = \frac{1}{g_{cl}^2} + 16\phi - \sum_{i=1}^{16} |\phi - m_i| - \sum_{i=1}^{16} |\phi + m_i|, \quad (2.5)$$

where the masses $m_i = \frac{z_i}{l_s^2}$. Note that in the field theory limit we are studying the region near $z = 0$. The coordinate ϕ takes values in R^+ and parametrizes the field theory Coulomb branch.

Seiberg argued [8] that the theory at the origin of the Coulomb branch obtained in the limit $g_{cl} = \infty$ with $N_f < 8$ massless hypermultiplets is a non trivial fixed point. The restriction $N_f < 8$ can be seen in the supergravity description as a requirement for the harmonic function H_8 in equation (2.2) to be positive when $c = 0$ and $z_i = 0$. At the fixed point the global symmetry is enhanced to $SU(2) \times E_{N_f+1}$. The Higgs branch is expected to become the moduli space of E_{N_f+1} one-instanton.

The generalization to $N = Q_4$ D4 branes is straightforward [9]. The gauge group is now $Sp(Q_4)$ and the global symmetry is as before. The Higgs branch is now the moduli space of $SO(2N_f)$ Q_4 -instantons. At the fixed point the global symmetry is enhanced as before and the Higgs branch is expected to become the moduli space of E_{N_f+1} Q_4 -instanton. Our interest in the next section will be in finding dual string descriptions of these fixed points.

3. The Dual String Description

The low energy description of our system is given by Type I' supergravity [13]. The region between two D8 branes is, as discussed in [14], described

by massive Type IIA supergravity [15]. The configurations that we will study have N_f D8 branes located at one O8 plane and $16 - N_f$ D8 branes at the other O8 plane. Therefore we are always between D8 branes and never encounter the situation where we cross D8 branes, and the massive Type IIA supergravity description is sufficient.

The bosonic part of the massive Type IIA action (in string frame) including a six-form gauge field strength which is the dual of the RR four-form field strength is

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} (e^{-2\Phi} (R + 4\partial_\mu \Phi \partial^\mu \Phi) - \frac{1}{2 \cdot 6!} |F_6|^2 - \frac{1}{2} m^2) \quad (3.1)$$

where the mass parameter is given by

$$m = \sqrt{2}(8 - N_f) \mu_8 \kappa_{10} = \frac{8 - N_f}{2\pi l_s}. \quad (3.2)$$

We use the conventions $\kappa_{10} = 8\pi^{7/2} l_s^4$ and $\mu_8 = (2\pi)^{-9/2} l_s^{-5}$ for the gravitational coupling and the D8 brane charge, respectively.

The Einstein equations derived from (3.1) read (in Einstein frame metric)

$$\begin{aligned} 2R_{ij} &= g_{ij} \left(R - \frac{1}{2} |\partial\Phi|^2 - \frac{e^{-\Phi/2}}{2 \cdot 6!} |F_6|^2 - \frac{m^2}{2} e^{5\Phi/2} \right) \\ &\quad + \partial_i \Phi \partial_j \Phi + \frac{e^{-\Phi/2}}{5!} F_{im_1 \dots m_5} F_j^{m_1 \dots m_5}, \\ 0 &= \nabla^j \partial_j \Phi - \frac{5m^2}{4} e^{5\Phi/2} + \frac{e^{-\Phi/2}}{4 \cdot 6!} |F_6|^2, \\ 0 &= \nabla^i \left(e^{-\Phi/2} F_{im_1 \dots m_5} \right). \end{aligned} \quad (3.3)$$

Except for these formulas we will be using the string frame only.

For identifying the solutions of D4 branes localized on D8 branes it is more convenient to start with the conformally flat form of the D8 brane supergravity solution. It takes the form [13]

$$\begin{aligned} ds^2 &= \Omega(z)^2 (-dt^2 + \dots + dx_4^2 \\ &\quad + d\tilde{r}^2 + \tilde{r}^2 d\Omega_3^2 + dz^2), \\ e^\Phi &= C \left(\frac{3}{2} C m z \right)^{-\frac{5}{6}} \Omega(z) = \left(\frac{3}{2} C m z \right)^{-\frac{1}{6}}. \end{aligned} \quad (3.4)$$

In these coordinates the harmonic function of Q_4 localized D4 branes in the near horizon limit derived from (3.3) reads

$$H_4 = \frac{Q_4}{l_s^{10/3} (\tilde{r}^2 + z^2)^{5/3}}. \quad (3.5)$$

This localized D4-D8 brane system solution, in a different coordinate system, has been constructed in [16]. One way to determine the harmonic function (3.5) of the localized D4 branes is to solve the Laplace equation in the background of the D8 branes.

It is useful to make a change of coordinates $z = r \sin \alpha$, $\tilde{r} = r \cos \alpha$, $0 \leq \alpha \leq \pi/2$. We get

$$\begin{aligned} ds^2 &= \Omega^2 (H_4^{-\frac{1}{2}} (-dt^2 + \dots dx_4^2) \\ &\quad + H_4^{\frac{1}{2}} (dr^2 + r^2 d\Omega_4^2)) , \\ e^\Phi &= C \left(\frac{3}{2} C m r \sin \alpha \right)^{-\frac{5}{6}} H_4^{-\frac{1}{4}} , \\ \Omega &= \left(\frac{3}{2} C m r \sin \alpha \right)^{-\frac{1}{6}} , \\ F_{01234r} &= \frac{1}{C} \partial_r (H_4^{-1}) , \end{aligned} \quad (3.6)$$

where C is an arbitrary parameter of the solution [13] and

$$d\Omega_4^2 = d\alpha^2 + (\cos \alpha)^2 d\Omega_3^2 . \quad (3.7)$$

The background (3.6) is a solution of the massive Type IIA supergravity equations (3.3).

The metric (3.6) of the D4-D8 system can be simplified to

$$\begin{aligned} ds^2 &= \left(\frac{3}{2} C m \sin \alpha \right)^{-\frac{1}{3}} (Q_4^{-\frac{1}{2}} r^{\frac{4}{3}} dx_{\parallel}^2 \\ &\quad + Q_4^{\frac{1}{2}} \frac{dr^2}{r^2} + Q_4^{\frac{1}{2}} d\Omega_4^2) \end{aligned} \quad (3.8)$$

where $dx_{\parallel}^2 \equiv -dt^2 + \dots + dx_4^2$. Define now the energy coordinate U by $r^2 = l_s^5 U^3$. That this is the energy coordinate can be seen by calculating the energy of a fundamental string stretched in the r direction or by using the DBI action as in the previous section. In the field theory limit, $l_s \rightarrow 0$ with the energy U fixed, we get the metric in the form of a *warped product* [5] of $AdS_6 \times S^4$

$$\begin{aligned} ds^2 &= l_s^2 \left(\frac{3}{4\pi} C (8 - N_f) \sin \alpha \right)^{-\frac{1}{3}} (Q_4^{-\frac{1}{2}} U^2 dx_{\parallel}^2 \\ &\quad + Q_4^{\frac{1}{2}} \frac{9dU^2}{4U^2} + Q_4^{\frac{1}{2}} d\Omega_4^2) , \end{aligned} \quad (3.9)$$

and the dilaton is given by

$$e^\Phi = Q_4^{-\frac{1}{4}} C \left(\frac{3}{4\pi} C (8 - N_f) \sin \alpha \right)^{-\frac{5}{6}} . \quad (3.10)$$

The ten dimensional space described by (3.9) is a fibration of AdS_6 over S^4 . It is the most general form of a metric that has the isometry of an AdS_6 space [6]. The space has a boundary at $\alpha = 0$ which corresponds to the location of the O8 plane ($z = 0$). The boundary is of the form $AdS_6 \times S^3$. In addition to the $SO(2,5)$ AdS_6 isometries, the ten dimensional space has also $SO(4)$ isometries associated with the spherical part of the metric (3.9). In general S^4 has the $SO(5)$ isometry group. However, this is reduced due to the warped product structure. As is easily seen from the form of the spherical part (3.7), only transformations excluding the α coordinate are isometries of (3.9). We are left with an $SO(4) \sim SU(2) \times SU(2)$ isometry group.

The two different viewpoints of the D4-D8 brane system, the near horizon geometry of the brane system on one hand, and the low energy dynamics on the D4 branes worldvolume on the other hand suggest a duality relation. Namely, *Type I' string theory compactified on the background (3.9), (3.10) with a 4-form flux of Q_4 units on S^4 is dual to an $\mathcal{N} = 2$ supersymmetric five dimensional fixed point*. The fixed point is obtained in the limit of infinite coupling of $Sp(Q_4)$ gauge theory with N_f hypermultiplets in the fundamental representation and one hypermultiplet in the antisymmetric representation, where $m \sim (8 - N_f)$ as in (3.2). The $SO(2,5)$ symmetry of the compactification corresponds to the conformal symmetry of the field theory. The $SU(2) \times SU(2)$ symmetry of the compactification corresponds $SU(2)_R$ R-symmetry and to the $SU(2)$ global symmetry associated with the massless hypermultiplet in the antisymmetric representation.

At the boundary $\alpha = 0$ the dilaton (3.10) blows up and Type I' is strongly coupled. In the weakly coupled dual heterotic string description this is seen as an enhancement of the gauge symmetry to E_{N_f+1} . One can see this enhancement of the gauge symmetry in the Type I' description by analysing the D0 brane dynamics near the orientifold plane [17, 18, 19]. This means that we have E_{N_f+1} vector fields that propagate on the $AdS_6 \times S^3$ boundary, as in the Horava-Witten picture [20]. The scalar curvature of the back-

ground (3.9), (3.10)

$$\mathcal{R}l_s^2 \sim (C(8 - N_f))^{1/3} Q_4^{-1/2} (\sin \alpha)^{-5/3}, \quad (3.11)$$

blows up at the boundary as well. In the dual heterotic description the dilaton is small but the curvature is large, too. For large Q_4 there is a region, $\sin \alpha \gg Q_4^{-3/10}$, where both curvature (3.11) and dilaton (3.10) are small and thus we can trust supergravity.

The AdS_6 supergroup is $F(4)$. Its bosonic subgroup is $SO(2, 5) \times SU(2)$. Romans constructed an $\mathcal{N} = 4$ six dimensional gauged supergravity with gauge group $SU(2)$ that realizes $F(4)$ [11]. It was conjectured in [10] that it is related to a compactification of the ten dimensional massive Type IIA supergravity. Indeed, we find that the ten dimensional background space is the warped product of AdS_6 and S^4 (3.9) (with $N_f = 0$). The reduction to six dimensions can be done in two steps. First we can integrate over the coordinate α . This yields a nine dimensional space of the form $AdS_6 \times S^3$. We can then reduce on S^3 to six dimensions, while gauging its isometry group. Roman's construction is based on gauging an $SU(2)$ subgroup of the $SO(4)$ isometry group.

The massive Type IIA supergravity action in the string frame goes like

$$l_s^{-8} \int \sqrt{-g} e^{-2\Phi} \mathcal{R} \sim Q_4^{5/2}, \quad (3.12)$$

suggesting that the number of degrees of freedom goes like $Q_4^{5/2}$ in the regime where it is an applicable description. Terms in the Type I' action coming from the D8 brane DBI action turn out to be of the same order. Viewed from M theory point of view, we expect the corrections to the supergravity action to go like $l_p^3 \sim l_s^3 e^\Phi \sim 1/Q_4$, where l_p is the eleven-dimensional Planck length. This seems to suggest that the field theory has a $1/Q_4$ expansion at large Q_4 . For example, the one-loop correction of the form

$$l_s^{-8} \int \sqrt{-g} l_s^6 \mathcal{R}^4 \sim Q_4^{1/2} \quad (3.13)$$

is suppressed by Q_4^2 compared to the tree-level action.

According to the AdS/CFT correspondence the spectrum of chiral primary operators of the

fixed point theory can be derived from the spectrum of Kaluza-Klein excitations of massive Type IIA supergravity on the background (3.9). We will not carry out the detailed analysis here but make a few comments. The operators fall into representations of the $F(4)$ supergroup. As in the case of the six dimensional (0, 1) fixed point with E_8 global symmetry [21] we expect E_{N_f+1} neutral operators to match the Kaluza-Klein reduction of fields in the bulk geometry, and E_{N_f+1} charged operators to match the Kaluza-Klein reduction of fields living on the boundary. Among the E_{N_f+1} neutral operators we expect to have dimension $3k/2$ operators of the type $Tr \phi^k$ where ϕ is a complex scalar in the hypermultiplet, which parametrize the Higgs branch of the theory. Like in [21] we do not expect all these operators to be in short multiplets. We expect that those in long multiplets will generically receive $1/Q_4$ corrections to their anomalous dimensions. Unlike the hypermultiplet, the vector multiplet in five dimensions is not a representation of the superconformal group $F(4)$. Therefore, we do not expect Kaluza-Klein excitations corresponding to neutral operators of the type $Tr \varphi^k$ where φ is a real scalar in the vector multiplet, which parametrizes the Coulomb branch of the theory. Among the E_{N_f+1} charged operators we expect to have the dimension four E_{N_f+1} global symmetry currents that couple to the massless E_{N_f+1} gauge fields on the boundary.

4. The M5-M5' Brane System

We denote the eleven dimensional space-time coordinates by $(x_{||}, \vec{x}, \vec{y}, \vec{z})$, where $x_{||}$ parametrize the (0, 1, 2, 3) coordinates, $\vec{x} = (x_1, x_2)$ the (4, 5) coordinates, $\vec{y} = (y_1, y_2)$ the (6, 7) coordinates and $\vec{z} = (z_1, z_2, z_3)$ the (8, 9, 10) coordinates.

Consider two sets of fivebranes in M theory: N_1 coinciding M5 branes and N_2 coinciding M5' branes. Their worldvolume coordinates are $M5 : (x_{||}, \vec{y})$ and $M5' : (x_{||}, \vec{x})$. Such a configuration preserves eight supercharges. The eleven-dimensional supergravity background takes the form [22, 23, 24]

$$ds_{11}^2 = (H_1 H_2)^{2/3} [(H_1 H_2)^{-1} dx_{||}^2 + H_2^{-1} d\vec{x}^2 + H_1^{-1} d\vec{y}^2 + d\vec{z}^2], \quad (4.1)$$

with the 4-form field strength \mathcal{F}

$$\mathcal{F} = 3(*dH_1 \wedge dy^1 \wedge dy^2 + *dH_2 \wedge dx^1 \wedge dx^2) \quad (4.2)$$

where $*$ defines the dual form in the three dimensional space (z_1, z_2, z_3) . When the M5 and M5' branes are only localized along the overall transverse directions \vec{z} the harmonic functions are $H_1 = 1 + l_p N_1 / 2|\vec{z}|$, $H_2 = 1 + l_p N_2 / 2|\vec{z}|$, where l_p is the eleven dimensional Planck length. This case has been discussed in [25]. The near horizon geometry in this case does not have the AdS isometry group and thus cannot describe a dual SCFT. Moreover, there does not seem to be in this case a theory on the intersection which decouples from the bulk physics.

Consider the semi-localized case when the M5 branes are completely localized while the M5' branes are only localized along the overall transverse directions. When the branes are at the origin of (\vec{x}, \vec{z}) space the harmonic functions in the near core limit of the M5' branes take the form [26, 27]

$$H_1 = 1 + \frac{4\pi l_p^4 N_1 N_2}{(|\vec{x}|^2 + 2l_p N_2 |\vec{z}|)^2}, \quad H_2 = \frac{l_p N_2}{2|\vec{z}|}. \quad (4.3)$$

The numerical factors in (4.3) are determined by the requirement that the integral of \mathcal{F} yields the appropriate charges. This solution can also be obtained from the localized D2-D6 brane solution of [28] by a chain of dualities.

It is useful to make a change of coordinates $l_p z = (r^2 \sin^2 \alpha) / 2N_2$, $x = r \cos \alpha$, $0 \leq \alpha \leq \pi/2$. In the near-horizon limit we want to keep the energy $U = \frac{r}{l_p}$ fixed. This implies, in particular, that membranes stretched between the M5 and M5' branes in the \vec{z} direction have finite tension $|\vec{z}|/l_p^3$. Since we are smearing over the \vec{y} directions we should keep \vec{y}/l_p fixed. It is useful to make a change of variables $\vec{y}/l_p \rightarrow \vec{y}$. The near horizon metric is of the form of a *warped product* of AdS_5 and a six dimensional manifold \mathcal{M}_6

$$ds_{11}^2 = l_p^2 (4\pi N_1)^{-1/3} (\sin^{2/3} \alpha) \left(\frac{U^2}{N_2} dx_{||}^2 + \frac{4\pi N_1}{U^2} dU^2 + d\mathcal{M}_6^2 \right), \quad (4.4)$$

with

$$d\mathcal{M}_6^2 = 4\pi N_1 (d\alpha^2 + \cos^2 \alpha d\theta^2 + \frac{\sin^2 \alpha}{4} d\Omega_2^2)$$

$$+ \frac{N_2}{\sin^2 \alpha} (dy^2 + y^2 d\psi^2). \quad (4.5)$$

The metric (4.4) has the AdS_5 isometry group [6]. Therefore, in the spirit of [1], M theory on the background (4.4), (4.5) should be dual to a four dimensional $\mathcal{N} = 2$ SCFT.

Note that the curvature of the metric diverges for small α as

$$\mathcal{R} \sim \frac{1}{l_p^2 N_1^{2/3} \sin^{8/3} \alpha}. \quad (4.6)$$

Away from $\alpha = 0$, eleven-dimensional supergravity can be trusted for large N_1 . The singularity at $\alpha = 0$ is interpreted as a signal that some degrees of freedom have been effectively integrated and are needed in order to resolve the singularity. These presumably correspond to membranes that end on the M5' branes.

The above M5-M5' brane system can be understood as up lifting to eleven dimensions of an elliptic brane system of Type IIA. It consists of N_2 NS5-branes with worldvolume coordinates $(0, 1, 2, 3, 4, 5)$ periodically arranged in the 6-direction and N_1 D4-branes with worldvolume coordinates $(0, 1, 2, 3, 6)$ stretched between them as in figure 1. When we lift this brane configuration to eleven dimensions, we delocalize in the eleven coordinate (which in our notation is 7) and since we have delocalization in coordinate 6 as well, we end up with the semi-localized M5-M5' brane system.

The four dimensional theory at low energies on the D4-branes worldvolume is an $SU(N_1)^{N_2}$ gauge theory with matter in the bi-fundamentals. The metric (4.4) can be viewed as providing the eleven-dimensional supergravity description of an M5 brane with worldvolume $R^4 \times \Sigma$ [29], where Σ is the Seiberg-Witten holomorphic curve (Riemann surface) associated with the four dimensional SCFT at the origin of the moduli space of vacua [30]. In this brane set-up the R-symmetry group $SU(2)_R \times U(1)_R$ is realized as the rotation group $SU(2)_{8910} \times U(1)_{45}$. The dimensionless gauge coupling of each $SU(N_1)$ part of the gauge group is $g_{YM}^2 \sim g_s N_2$.

The ten dimensional metric describing the elliptic Type IIA brane configuration is

$$ds_{10}^2 = H_4^{-1/2} dx_{||}^2 + H_4^{1/2} d\vec{x}^2 + H_4^{-1/2} H_{NS} dy^2$$

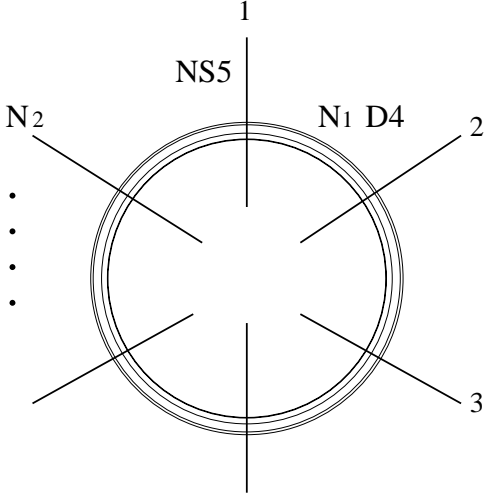


Figure 1: The Type IIA elliptic brane system corresponding to the semi-localized M5-M5' brane configuration. It consists of N_2 NS5-branes with world-volume coordinates $(0, 1, 2, 3, 4, 5)$ periodically arranged in the 6-direction and N_1 D4-branes with world-volume coordinates $(0, 1, 2, 3, 6)$ stretched between them.

$$+ H_4^{1/2} H_{NS} dz^2, \quad (4.7)$$

where

$$H_4 = 1 + \frac{4\pi g_s l_s^4 N_1 N_2}{[|\vec{x}|^2 + 2l_s N_2 |\vec{z}|]^2}, \quad H_{NS} = \frac{l_s N_2}{2|\vec{z}|}. \quad (4.8)$$

Its near-horizon limit is

$$ds_{10}^2 = l_s^2 \left(\frac{U^2}{R^2} dx_{||}^2 + R^2 \frac{dU^2}{U^2} + d\mathcal{M}_5^2 \right), \quad (4.9)$$

where

$$d\mathcal{M}_5^2 = R^2 (d\alpha^2 + \cos^2 \alpha d\theta^2 + \frac{\sin^2 \alpha}{4} d\Omega_2^2) + \frac{N_2^2}{R^2 \sin^2 \alpha} dy^2, \quad (4.10)$$

and $R^2 = (4\pi g_s N_1 N_2)^{1/2}$. The curvature of the metric is $\mathcal{R} \sim \frac{1}{l_s^2 R^2}$ and it has a dilaton $e^\Phi = \frac{g_s N_2}{R} \sin \alpha^{-1}$. This supergravity solution can be trusted when both the curvature and the dilaton are small. Away from $\alpha = 0$ we need large N_2 and $N_2 \ll N_1^{1/3}$. When the latter condition is not satisfied the dilaton is large and we should consider the eleven-dimensional description.

The supergravity solution with both the M5 branes and the M5' branes being completely localized has not been constructed yet. This fully

localized system has been studied via a matrix description in [31]. In the following we make a few comments on this branes configuration. It can be viewed as a single M5 brane with world-volume $R^4 \times \Sigma$, where Σ is a singular Riemann surface defined by the complex equation

$$x^{N_2} y^{N_1} = 0, \quad (4.11)$$

where $x = x_1 + ix_2, y = y_1 + iy_2$. Consider first the case $N_1 = N_2 = 1$. The complex equation $xy = 0$ can be written by a change of variables as $w_1^2 + w_2^2 = 0$. This equation describes the singular Riemann surface at the point in the Seiberg-Witten solution [32] where a monopole becomes massless. The massless spectrum consists of a $U(1)$ vector field and one hypermultiplet and the theory at this point is free in the IR. Indeed, the chiral ring associated with the singular variety $xy = 0$ consists only of the identity operator. Since the field theory is free in the IR we expect the dual string description to be strongly coupled. That means that the dual background is highly curved and the classical supergravity analysis cannot be trusted. This theory has a dual string formulation [33]. The more general N_1, N_2 case is harder to analyse. The reason being that the singular locus of the variety (4.11) is not isolated and the notion of a chiral ring which is used for an isolated singularity is not appropriate. We do expect a non trivial conformal field theory in this case, but it has not been identified yet.

5. BPS String Solitons

The centrally extended $\mathcal{N} = 2$ supersymmetry algebra in four dimensions takes the form [34]

$$\begin{aligned} \{Q_\alpha^A, \bar{Q}_{\dot{\alpha}B}\} &= \sigma_{\alpha\dot{\alpha}}^\mu P_\mu \delta_B^A + \sigma_{\alpha\dot{\alpha}}^\mu Z_{\mu B}^A, \\ \{Q_\alpha^A, Q_\beta^B\} &= \varepsilon_{\alpha\beta} Z^{[AB]} + \sigma_{\alpha\beta}^{\mu\nu} Z_{\mu\nu}^{(AB)}, \end{aligned} \quad (5.1)$$

where $A, B = 1, 2$ and $Z_{\mu A}^A = 0$. The $SU(2)_R$ part of the R-symmetry group acts on the indices A, B . In addition to the particle charge $Z^{[AB]}$, there are in (5.1) the string charges $Z_{\mu B}^A$ in the adjoint representation of $SU(2)_R$ and the membrane charges $Z_{\mu\nu}^{(AB)}$ in the 2-fold symmetric representation of $SU(2)_R$. Thus, $\mathcal{N} = 2$ supersymmetric gauge theories in four dimensions can

have BPS strings and BPS domain walls in addition to the well studied BPS particles.

The BPS particles and BPS strings are realized by stretching M2 brane between the M5 and M5' branes. When the membrane worldvolume coordinates are $(0, x, y)$ where x is one of the \vec{x} components and y is one of the \vec{y} components we get a BPS particle of the four dimensional theory. It is charged under $U(1)_R$ that acts on \vec{x} and is a singlet under $SU(2)_R$ that acts on \vec{z} . The BPS particles exist on the Coulomb branch of the gauge theory on which $U(1)_R$ acts, as in figure 3, and their mass is given by the Seiberg-Witten solution.

When the membrane worldvolume coordinates are $(0, 1, z)$ where z is one of the \vec{z} components we get a BPS string of the four dimensional theory. It is not charged under $U(1)_R$ and it transforms in the adjoint $SU(2)_R$ since \vec{z} transforms in this representation. The BPS strings exist on the Higgs branch of the gauge theory on which $SU(2)_R$ acts, as in figure 3. The tension of the BPS string is given by $\frac{|\vec{z}|}{l_p^3}$ where $|\vec{z}|$ is the distance between the M5 and M5' branes and it is finite in the field theory limit. It is easy to see that we do not have in this set-up BPS domain walls since stretching a membrane with worldvolume coordinates $(0, 1, 2)$ breaks the supersymmetry completely. One expects BPS domain wall configurations when the moduli space of vacua has disconnected components.

In the Type IIA picture the BPS string is constructed by stretching a D2 brane between the D4 branes and NS5-brane. The separation between these two types of branes in the \vec{z} direction has two interpretations depending on whether the four dimensional gauge group has a $U(1)$ part or not [35]. If it does then the separation is interpreted as the $SU(2)_R$ triplet FI parameters $\vec{\zeta}$. The BPS string tension is proportional to $|\vec{\zeta}|$. If the gauge group does not have a $U(1)$ part the separation between these two types of branes in the \vec{z} direction is interpreted as giving a vev to an $SU(2)_R$ triplet component of the meson $\tilde{Q}Q$ which transforms under $SU(2)_R$ as $\mathbf{2} \times \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$. The BPS string tension is proportional to this vacuum expectation value. At the origin of the moduli space, where we have SCFT, the string

becomes tensionless.

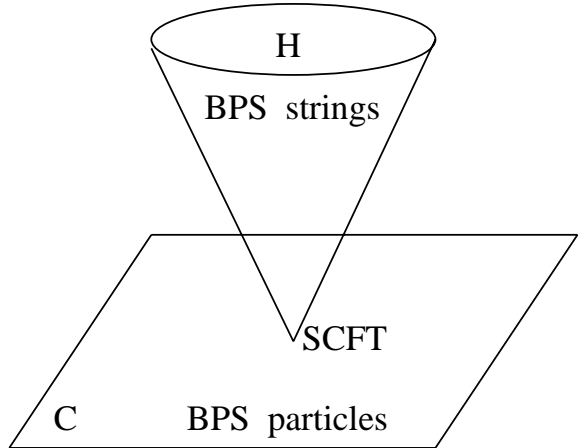


Figure 2: There are BPS particles on the Coulomb branch and BPS strings on the Higgs branch. The SCFT is at the intersection of these two branches.

We can use the AdS/CFT correspondence in order to compute the self energy of a string and a potential between two such strings of opposite orientation [36]. This is obtained, in the supergravity approximation, by a minimization of the M2 brane action

$$S = \frac{1}{(2\pi)^2 l_p^3} \int d\tau d\omega d\sigma \sqrt{\det G_{\mu\nu} \partial_a x^\mu \partial_b X^\nu} \quad (5.2)$$

in the background (4.4), (4.5). Consider a static configuration $x^0 = \tau$, $x^1 = \omega$ and $x^i = x^i(\sigma)$. Then, the energy per unit length of the string is given by

$$E = \int \frac{U \sin \alpha}{4\pi^2 N_2} \sqrt{dU^2 + U^2 d\alpha^2} . \quad (5.3)$$

Thus, the string self-energy

$$E = \mathcal{E}(\alpha) \frac{U^2}{N_2} \sim \mathcal{E}(\alpha) \frac{N_1}{(\delta x_{||})^2} . \quad (5.4)$$

The function \mathcal{E} parametrizes the dependence of the self-energy on the coordinate α . From the field theory point of view, this parametrizes a dependence on the moduli that parametrize the space of vacua. $\delta x_{||}$ is a cut-off. In (5.4) we used the holographic relation between a distance $\delta x_{||}$ in field theory and the coordinate U which in our case reads

$$\delta x_{||} \sim \frac{(N_2 N_1)^{1/2}}{U} . \quad (5.5)$$

This corresponds to the familiar relation [37]

$$\delta x_{||} \sim \frac{(g_{YM}^2 N_1)^{1/2}}{U}. \quad (5.6)$$

Similarly, the potential energy per units length between two strings of opposite orientation separated by distance L is

$$E \sim -\frac{N_1}{L^2}. \quad (5.7)$$

here L is distance between two strings. The results (5.4),(5.7) do not depend on N_2 which means that they do not depend on the gauge coupling of the four dimensional field theory $g_{YM}^2 \sim N_2$. This is not unexpected. The gauge coupling can be viewed as a vacuum expectation value of a scalar in the vector multiplet and does not appear in the hypermultiplet metric. Since the BPS strings exist on the Higgs branch it is natural that their potential and self-energy do not depend on the gauge coupling. However, (5.4) and (5.7) are only the large N_1 results and will presumably have $1/N_1$ corrections.

As noted above, the theory on the intersection of two sets of M5 branes contains BPS particles, which arise from M2 branes stretched in the directions $(0, x, y)$. Minimization of the M2 brane action in this case yields the potential between these two such objects of opposite charge separated by distance L

$$V \sim \frac{(N_1 N_2)^{1/2}}{L}. \quad (5.8)$$

Again, this is expected since for the four dimensional field theory it reads as $V \sim \frac{(g_{YM}^2 N_1)^{1/2}}{L}$.

The 11-dimensional supergravity action goes like

$$l_p^{-9} \int \sqrt{-g} \mathcal{R} \sim N_1^2 N_2 \quad (5.9)$$

suggesting that the number of degrees of freedom goes like $N_1^2 N_2$. This is also deduced from the two-point function of the stress energy tensor. This is, of course, expected for $SU(N_1)^{N_2}$ gauge theory. It is curious to note that there is similar growth (N^3) of the entropy for a system of N parallel M5 branes [38].

The one loop correction has the following form $l_p^{-9} \int \sqrt{-g} l_p^6 \mathcal{R}^4 \sim N_2$ which is suppressed by N_1^2 compared to the tree level action suggesting that the field theory has a $1/N_1$ expansion,

which again is in agreement with the field theory expectation.

The Type IIA background (4.10) is T-dual to Type IIB on $AdS_5 \times S^5/Z_{N_2}$ [30]. The latter is the dual description of the Z_{N_2} orbifold of $\mathcal{N} = 4$ theory [39]. For instance the number of degrees of freedom can be understood as $c(\mathcal{N} = 4)/N_2 \sim N_1^2 N_2$. We can identify the spectrum of chiral primary operators with the supergravity Kaluza-Klein excitations as analysed in [40, 41].

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