Correlation functions of chiral primary operators in perturbative $\mathcal{N} = 4$ SYM

S. Penati [a], **A. Santambrogio** [b], **D. Zanon** [c]

[a] *Dipartimento di Fisica dell'Università di Milano–Bicocca, and*
INFN, Sezione di Milano, Via Celoria 16, 20133 Milano, Italy
E-mail: `silvia.penati@mi.infn.it`

[b] *Instituut voor Theoretische Fysica - Katholieke Universiteit Leuven*
Celestijnenlaan 200D B–3001 Leuven, Belgium
E-mail: `alberto.santambrogio@fys.kuleuven.ac.be`

[c] *Dipartimento di Fisica dell'Università di Milano, and*
INFN, Sezione di Milano, Via Celoria 16, 20133 Milano, Italy
E-mail: `daniela.zanon@mi.infn.it`

ABSTRACT: We discuss recent results on two-point functions of chiral primary operators in $\mathcal{N} = 4$ SU(N) supersymmetric Yang-Mills theory. Our results give further support to the belief that such correlators are not renormalized to *all* orders in g and to *all* orders in N.

THE AIM of this talk is to present recent results [1] on the explicit computation of two-point functions of chiral operators $\text{Tr}\Phi^3$ in $\mathcal{N} = 4$ SU(N) supersymmetric Yang-Mills theory to the order g^4 in perturbation theory, using $\mathcal{N} = 1$ superspace techniques. We find that perturbative corrections to the correlators vanish for all N. While at order g^2 the cancellation [2] can be ascribed to the nonrenormalization theorem valid for correlators of operators in the same multiplet as the stress tensor, at order g^4 this argument no longer applies and the actual cancellation occurs in a highly nontrivial way, as will be shown.

The plan of the talk is the following: after a brief introduction to the subject of correlation functions as tools to explore the AdS/CFT correspondence, in section 2 we briefly illustrate the $\mathcal{N} = 4$ theory and give the relevant rules for calculating in $\mathcal{N} = 1$ superspace. In section 3 we present the results of our calculations to order g^4: as a first test of our approach we check that the perturbative corrections to the two-point function with $k = 2$ vanish, after that we consider the two-point correlator with $k = 3$ and we describe all the order g^4 contributions. We do not

give technical details, for which we refer to our paper [1].

1. Introduction

Recently much evidence has been provided in testing the conjectured equivalence of type IIB superstring theory on anti-de-Sitter space (AdS$_5$) times a five–sphere to the $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills conformal field theory living on the boundary, in the large-N limit and at large 't Hooft coupling $\lambda = g^2 N/4\pi$ (g^2 being the Yang-Mills coupling constant) [3]. According to this correspondence correlation functions of operators in the conformal field theory are mapped to appropriate on-shell amplitudes of superstring theory in the bulk AdS background.
$\mathcal{N} = 4$ chiral primary operators

$$\text{Tr}\Phi^k \equiv \text{Tr}\left(\Phi^{\{i_1}(z)\Phi^{i_2}(z)\cdots\Phi^{i_k\}}(z)\right), \quad (1.1)$$

in the symmetric, traceless representation of the R-symmetry group SU(4), play a special role in exploring non-perturbative statements concerning the above mentioned connection. These are local operators of the lowest scaling dimension

in a given irreducible representation of the superconformal algebra $SU(2,2|4)$, and belong to short multiplets which are chiral under a $\mathcal{N} = 1$ subalgebra. In the large-N limit they correspond to Kaluza Klein modes in the AdS supergravity sector. In the special case of $k = 2$, two- and three-point correlators are given by their free-field theory values for any finite N. In this case their form, fixed up to a constant by conformal invariance, is protected by a nonrenormalization theorem [4] valid for two- and three-point functions of operators in the same multiplet as the stress tensor and as the $SU(4)$ flavor currents.

For any strong-weak coupling duality test it is essential to have quantities that do not acquire radiative corrections as one moves from weak to strong coupling. If an exact computation in the supergravity sector shows agreement with a tree level result in the Yang-Mills sector, then there is an indication of a nonrenormalization theorem at work. This is the case for the three–point correlators $\langle \text{Tr}\Phi^{k_1} \text{Tr}\Phi^{k_2} \text{Tr}\Phi^{k_3} \rangle$ computed in ref. [5] in the large-N limit of $\mathcal{N} = 4$ $SU(N)$ Yang-Mills: the strong limit result $\lambda = g^2 N/4\pi \gg 1$ obtained using type IIB supergravity was shown to agree with the weak 't Hooft coupling limit $\lambda = g^2 N/4\pi \ll 1$ in terms of free fields. According to the AdS/CFT correspondence one concludes that the correlators are independent of λ to leading order in N. A stronger conjecture made in ref. [5] claims that three-point functions might be independent of g for *any* value of N. As emphasized above, for the case $k = 2$ nonrenormalization properties have been proven to be enjoyed by two- and three-point functions of chiral operators. For general k there exists evidence of nonrenormalization based on proofs that rely on reasonable assumptions (analyticity in harmonic superspace [6] and validity of a generalized Adler-Bardeen theorem [7]).

Explicit perturbative calculations in the $\mathcal{N} = 4$ $SU(N)$ Yang-Mills conformal field theory are a way to confirm the conjectures and add insights into potential larger symmetries of the theory. Important steps along this program have been made in [2, 8, 9, 10]. In particular, it has been shown that to order g^2 radiative corrections do not affect the two- and three-point functions of chiral operators [2]. Two-point functions have

been computed for chiral operators with generic k by showing that the order g^2 contributions are proportional to the one for the $k = 2$ case which indeed satisfies the known nonrenormalization theorem mentioned above. Concretely, the cancellation to order g^2 can be traced back to the fact that at this order all the diagrams contain interactions involving at most two matter lines. Clearly this is not true, for example, at order g^4, where diagrams with gluon exchanges among three matter lines appear. Therefore, it is interesting to investigate whether the cancellation shown in [2] for the $k > 2$ case is an accident of order g^2.

In our paper [1] we have addressed the nontrivial test left open at order g^4, by computing the two-point function for the operator $\text{Tr}\Phi^k$ in the case $k = 3$. The analysis for generic k is now under investigation [11]. However, as already mentioned, at order g^4 the $k = 3$ case is a crucial test, being diagrams with interactions involving three matter lines present. We have found that corrections indeed vanish for *all* values of N, then supporting the stronger conjecture of ref. [5].

2. The main features of our calculation

The physical particle content of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory is given by one spin-1 vector, four spin-1/2 Majorana spinors and six spin-0 particles in the **6** of the R-symmetry group $SU(4) \sim SO(6)$. All particles are massless and transform under the adjoint representation of the $SU(N)$ gauge group.

Perturbative calculations are quite difficult to handle using a component field formulation of the theory. (Note that in ref. [2] a component approach was used, but the order g^2 result for the two- and three-point correlators was obtained using a general argumentation based on colour combinatorics. Only a schematic knowledge of the structure of the component action was required.) In general, in order to resum Feynman diagrams at higher-loop orders it is greatly advantageous to work in superspace.

In $\mathcal{N} = 1$ superspace the action can be written in terms of one vector superfield V (real)

and three chiral superfields Φ^i containing the six scalars organized into the $\mathbf{3} \times \bar{\mathbf{3}}$ of $\mathrm{SU}(3) \subset \mathrm{SU}(4)$ (we follow the notations in [12])

$$
\begin{aligned}
S[J,\bar{J}] = {} & \int d^8 z \, \mathrm{Tr} \left(e^{-gV} \bar{\Phi}_i e^{gV} \Phi^i \right) \\
& + \frac{1}{2g^2} \int d^6 z \, \mathrm{Tr} W^\alpha W_\alpha \\
& + \frac{ig}{3!} \mathrm{Tr} \int d^6 z \, \epsilon_{ijk} \Phi^i [\Phi^j, \Phi^k] \\
& + \frac{ig}{3!} \mathrm{Tr} \int d^6 \bar{z} \, \epsilon_{ijk} \bar{\Phi}^i [\bar{\Phi}^j, \bar{\Phi}^k] \\
& + \int d^6 z \, J\mathcal{O} + \int d^6 \bar{z} \, \bar{J}\bar{\mathcal{O}} , \quad (2.1)
\end{aligned}
$$

where $W_\alpha = i\bar{D}^2(e^{-gV} D_\alpha e^{gV})$, and $V = V^a T^a$, $\Phi_i = \Phi_i^a T^a$, T^a being $N \times N$ matrices in the fundamental representation of $\mathrm{SU}(N)$. We have added to the classical action source terms for the chiral primary operators generically denoted by $\mathcal{O}$ since our goal is the computation of their correlators.

Although in (2.1) the $\mathcal{N} = 4$ supersymmetry invariance is realized only non linearly, the main advantage offered by a $\mathcal{N} = 1$ formulation of the theory resides in the fact that a straightforward off-shell quantum formulation is available. Thus if the aim is to perform higher-loop perturbative calculations this is the most suited approach to follow. Feynman rules are by now standard (we refer to appendix B of [1] for a complete list).

We will now focus on the two-point supercorrelator for the operator $\mathcal{O} = \mathrm{Tr}(\Phi^{\{i} \Phi^j \Phi^{k\}})$. As in ref. [2], we consider the $\mathrm{SU}(3)$ highest weight Φ^1 field and compute $\langle \mathrm{Tr}(\Phi^1)^3 \mathrm{Tr}(\bar{\Phi}^1)^3 \rangle$. This is not a restrictive choice since all the other primary chiral correlators can be obtained from this one by $\mathrm{SU}(3)$ transformations. What we gain is that we have no flavour combinatorics and we are left to deal with the colour combinatorics only.

We work in euclidean space, with the generating functional defined as

$$
W[J,\bar{J}] = \int \mathcal{D}\Phi \, \mathcal{D}\bar{\Phi} \, \mathcal{D}V \, e^{S[J,\bar{J}]} . \qquad (2.2)
$$

Thus the two-point function is given by

$$
\langle \mathrm{Tr}(\Phi^1)^3(z_1) \mathrm{Tr}(\bar{\Phi}^1)^3(z_2) \rangle = \left. \frac{\delta^2 W}{\delta J(z_1) \delta \bar{J}(z_2)} \right|_{J,\bar{J}=0}
$$

$$(2.3)$$

where $z \equiv (x, \theta, \bar{\theta})$. We use perturbation theory to evaluate the contributions to $W[J,\bar{J}]$ which are quadratic in the sources, i.e. of the form

$$
\int d^4 x_1 \, d^4 x_2 \, d^4 \theta \, J(x_1, \theta, \bar{\theta}) \frac{F(g^2, N)}{(x_1 - x_2)^6} \bar{J}(x_2, \theta, \bar{\theta}) ,
$$

$$(2.4)$$

where the x-dependence of the result is fixed by the conformal invariance of the theory, and the function $F(g^2, N)$ is what we want to determine up to order g^4. We will find a result valid for any N.

In order to perform the calculation we have found it convenient to work in momentum space, using dimensional regularization and minimal subtraction scheme. In n dimensions, with $n = 4 - 2\epsilon$, the Fourier transform of the power factor $(x_1 - x_2)^{-6}$ in (2.4) is given by

$$
\frac{1}{(x^2)^3} = \frac{\pi^{-2+\epsilon}}{64} \frac{\Gamma(-1-\epsilon)}{\Gamma(3)} \int d^n p \, \frac{e^{-ipx}}{(p^2)^{-1-\epsilon}} .
$$

$$(2.5)$$

The presence of the singular factor $\Gamma(-1-\epsilon) \sim 1/\epsilon$ signals, in momentum space and in dimensional regularization, the UV divergence of the correlation function in (2.4) associated to the short-distance behaviour for $x_1 \sim x_2$. It follows that performing perturbative calculations in momentum space it is sufficient to look for all the contributions to (2.4) that behave like $1/\epsilon$, therefore disregarding finite contributions. In fact, once the divergent terms are determined at a given order in g, using (2.5) one can reconstruct an x-space structure as in (2.4) with a nonvanishing contribution to $F(g^2, N)$. Finite contributions in momentum space would correspond in x-space to terms proportional to ϵ which give rise only to contact terms [13].

The one stated above is the basic rule of our strategy that we can summarize as follows:

- consider all the two-point diagrams from $W[J,\bar{J}]$ with J and $\bar{J}$ on the external legs,

- evaluate all the factors coming from combinatorics of the diagram and compute the colour structure,

- perform the superspace D-algebra following standard techniques,

- reduce the result to a multi-loop momentum integral,

3

- compute its $1/\epsilon$ divergent contribution.

This last step, i.e. the calculation of the divergent part of the various integrals we have achieved using the method of uniqueness [14] and various rules and identities [13, 15] that we have collected in appendix B of [1]. Since the theory is at its conformal point, it is not affected by IR divergences. Therefore, even if we work in a massless regularization scheme, we never worry about the IR behavior of our integrals. Moreover, since the theory is finite, the diagrams that we consider do not possess UV divergent subdiagrams. Finally, as a general remark we observe that gauge-fixing the classical action requires the introduction of corresponding Yang-Mills ghosts. However they only couple to the vector multiplet and do not enter our specific calculation.

In the next section we will apply the general procedure just described to the analysis of the two-point function $\left\langle \mathrm{Tr}(\Phi^1)^k \mathrm{Tr}(\bar\Phi^1)^k \right\rangle$ with $k = 3$ to order g^4.

3. Correlation functions to order g^4

Before coming to our main calculation, the $k = 3$ case, we will first sketch how our formalism works in a simpler case, the order g^4 calculation of the two-point correlator with $k = 2$. As previously discussed, in this case we already know that perturbative corrections should not be there: this simpler calculation is then a non trivial test of our techniques.

The two-point correlator we are interested in is obtained from $W[J, \bar J]$ inserting in the action (2.1) the chiral operators $\mathcal{O} = \mathrm{Tr}(\Phi^1)^2$ and $\bar{\mathcal{O}} = \mathrm{Tr}(\bar\Phi^1)^2$. As outlined in the previous section, the relevant contribution is obtained from the generating functional isolating terms of the form

$$\int d^4x_1 \, d^4x_2 \, d^4\theta \, J(x_1, \theta, \bar\theta) \frac{E(g^2, N)}{(x_1 - x_2)^4} \bar J(x_2, \theta, \bar\theta) \,. \tag{3.1}$$

The general form of (3.1) is fixed by conformal invariance, while the function $E(g^2, N)$ is the unknown to be determined. Fourier transforming from x-space to momentum space

$$\frac{1}{(x^2)^2} = \frac{\pi^{-2+\epsilon}}{16} \frac{\Gamma(-\epsilon)}{\Gamma(2)} \int d^n p \, \frac{e^{-ipx}}{(p^2)^{-\epsilon}} \tag{3.2}$$
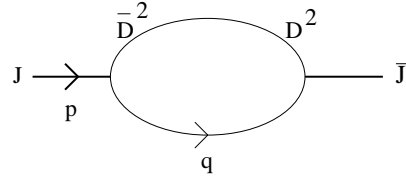

Figure 1: Tree-level contribution to $\langle \mathrm{Tr}(\Phi^1)^2 \mathrm{Tr}(\bar\Phi^1)^2 \rangle$.

makes it clear that non-trivial contributions to the generating functional are given by the divergent part of our Feynman diagrams.

To start with we consider the tree-level contribution corresponding to the graph in figure 1. The calculation in this case is very simple [1]: its contribution to the two-point function is given by

$$\frac{1}{(4\pi)^2} \, 2(N^2 - 1) \frac{1}{\epsilon} \int d^4p \, d^4\theta \, J(-p, \theta, \bar\theta) \bar J(p, \theta, \bar\theta) \,. \tag{3.3}$$

The order g^2 contribution, once evaluated in superspace gives immediately a zero result: the diagrams one would need to consider are shown in figure 2. Diagram 2a does not contribute since the one-loop correction to the chiral propagator vanishes due to a complete cancellation between vector and chiral loops [16]. Diagram 2b, after completion of the D-algebra leads to a finite momentum integral.

Now we consider the order g^4 contributions: they are shown in figure 3.

In figure 3a we have the insertion of a two-loop propagator correction, while in figure 3b a one-loop vertex correction appears [16]. Note that a diagram with a vector propagator corrected at order g^2 is absent since at one-loop order there is a complete cancellation among chiral, vector and ghost contributions [16].

The graph in figure 3a is easy to compute: with an overall factor

$$\frac{16}{(4\pi)^6} g^4 \, N^2(N^2 - 1)$$

$$\times \int d^4p \, d^4\theta \, J(-p, \theta, \bar\theta) \bar J(p, \theta, \bar\theta) \tag{3.4}$$

one obtains the following divergent contribution

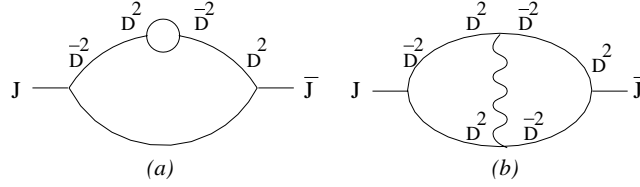$$\text{figure } 3a \rightarrow \zeta(3)\frac{1}{\epsilon} \,. \tag{3.5}$$

Figure 2: g^2-order contribution to $\langle \mathrm{Tr}(\Phi^1)^2 \mathrm{Tr}(\bar{\Phi}^1)^2 \rangle$.
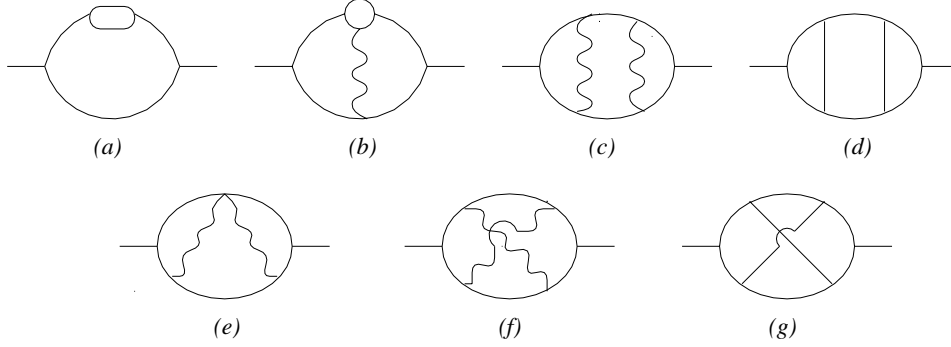

Figure 3: g^4-order contribution to $\langle \mathrm{Tr}(\Phi^1)^2 \mathrm{Tr}(\bar{\Phi}^1)^2 \rangle$.

For figure 3b, with the same overall factor as in (3.4), one obtains

$$\text{figure } 3b \to -2\zeta(3)\frac{1}{\epsilon}. \qquad (3.6)$$

A rather straightforward computation of the D-algebra for the diagrams in figures 3c, 3d and 3e allows to conclude that the corresponding momentum integrals are actually all finite and, as previously observed, not relevant for our purpose.

Finally, for the graphs in figure 3f and in figure 3g, factoring out the same overall quantity we have

$$\text{figure } 3f \to \frac{1}{2}\zeta(3)\frac{1}{\epsilon}, \qquad (3.7)$$

and

$$\text{figure } 3g \to \frac{1}{2}\zeta(3)\frac{1}{\epsilon}. \qquad (3.8)$$

It is a trivial matter to sum up the contributions listed in (3.5), (3.6), (3.7) and (3.8) and obtain a vanishing result, as expected from the nonrenormalization theorem.

We note that the diagrams in figures 3f and 3g lead to planar contributions, i.e. with exactly the same N dependence from colour combinatorics as the other diagrams (the N dependence is the one shown in the common overall factor (3.4)): indeed to this order nonplanar diagrams are absent. In the $k = 3$ case we will be confronted with a more complicated situation.

Let us now come to the computation of the two-point function for the chiral operator $\mathcal{O} = \mathrm{Tr}(\Phi^1)^3$. To this end we go back to (2.4) and compute the perturbative contributions to the function $F(g^2, N)$. As previously emphasized, making use of (2.5) we write Feynman diagrams in momentum space and isolate the $1/\epsilon$ poles.

In figure 4 we have drawn the tree-level contribution. With an overall factor

$$\frac{3}{(4\pi)^4}\frac{(N^2-1)(N^2-4)}{N}$$
$$\times \int d^4p\, d^4\theta\, J(-p,\theta,\bar{\theta})\bar{J}(p,\theta,\bar{\theta}) \qquad (3.9)$$

we obtain

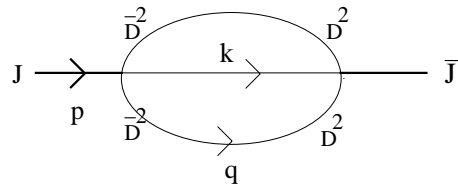$$\text{figure } 4 \to -\frac{1}{4\epsilon}p^2. \qquad (3.10)$$

Figure 4: Tree-level contribution to $\langle \mathrm{Tr}(\Phi^1)^3 \mathrm{Tr}(\bar{\Phi}^1)^3 \rangle$.

5

S. Penati [a], A. Santambrogio [b], D. Zanon [c]

The result in x-space is readily recovered using formula (2.5).

The superspace diagrams that enter the order g^2 computation are shown in figure 5. They are nothing but the ones that appear in figure 2 with one line added from the chiral external vertices. One proves that their contributions vanish with exactly the same reasoning outlined previously. As found in ref. [2] to order g^2 the vanishing of the correlator is due to the fact that it is proportional to the correlator of $\mathcal{O} = \text{Tr}(\Phi^1)^2$ for which the nonrenormalization theorem is valid. However, this is no longer true at order g^4 to which we turn now.

The diagrams contributing to g^4-order are collected in figure 6. The ones in figure $6a$–$6g$ are the same as in figure 3 with one extra line added from the chiral external vertices. From the result obtained in the previous case at order g^4, we would be tempted to believe that these diagrams still sum up to zero. However this would be a wrong conclusion. In fact, what makes things different is that in $6f$ and $6g$ the addition of the extra line changes completely the topology of the diagrams which become really *nonplanar*. As a consequence, their colour combinatorics changes and their N-dependence is distinct from the remaining planar diagrams $6a$– $6e$. More specifically, in this case it turns out that the nonplanar diagrams $6f$ and $6g$ lead to a vanishing colour combinatorics factor.

The evaluation of the colour coefficient for the other nonplanar diagram in figure $6h$ reveals again a vanishing contribution. The fact that the nonplanar diagrams do not contribute indicates that the final answer is going to be valid for all values of N, independently of any large-N limit. In light of this result it becomes challenging to prove the cancellation of nonplanar diagrams to all orders in the Yang-Mills coupling. Moreover it is natural to ask if this mechanism of cancellation is still valid for two-point correlation functions of the form $\langle \text{Tr}(\Phi^1)^k \text{Tr}(\bar{\Phi}^1)^k \rangle$, with $k > 3$. However, a simple direct analysis shows that this is not true [11].

Going back to figure 6, one easily convinces oneself that for the graphs in figures $6c$, $6d$ and $6e$ the same analysis as in the previous section applies. In this case the addition of the chiral line simply adds a $D^2 \bar{D}^2$ factor which accounts for the D-algebra of one added loop; performing the D-algebra in the diagrams one is left with finite integrals.

We note that at this order diagrams containing the scalar superpotential vertex

$$\epsilon_{ijk} \text{Tr}(\Phi^i [\Phi^j, \Phi^k]) \qquad (3.11)$$

do not contribute.

We are left with the contributions from figures $6a$, $6b$, $6i$ and $6j$. We will find that a highly nontrivial cancellation occurs.

For every diagram we need compute the specific combinatorics, the various factors from vertices and propagators and the colour structure. Then we have to perform the D-algebra in the loops and finally evaluate the momentum integrals. We factorize for each contribution the same quantity

$$\frac{9}{(4\pi)^8} g^4 \ N(N^2 - 4)(N^2 - 1)$$
$$\times \int d^4p \, d^4\theta \, J(-p, \theta, \bar{\theta}) \bar{J}(p, \theta, \bar{\theta}) \, . \ (3.12)$$

The diagram in figure $6a$ which contains the two-loop propagator correction, gives

$$\text{figure } 6a \rightarrow -\frac{3}{2}\zeta(3)\frac{1}{\epsilon}\, p^2 \, . \qquad (3.13)$$

The diagram in figure $6b$ contains the one-loop vertex correction. In this case the resulting contribution is given by

$$\text{figure } 6b \rightarrow 3\zeta(3)\frac{1}{\epsilon}\, p^2 \, . \qquad (3.14)$$

In the same way for the graph in figure $6i$ one has

$$\text{figure } 6i \rightarrow -5\zeta(5)\frac{1}{\epsilon}\, p^2 \, . \qquad (3.15)$$

Finally we concentrate on the diagram in Fig. $6j$. The evaluation of the corresponding momentum integral is highly nontrivial and we refer to our paper [1] for all the technical details. The result is given by

$$\text{figure } 6j \rightarrow \left[5\zeta(5) - \frac{3}{2}\zeta(3)\right]\frac{1}{\epsilon}\, p^2 \, . \qquad (3.16)$$

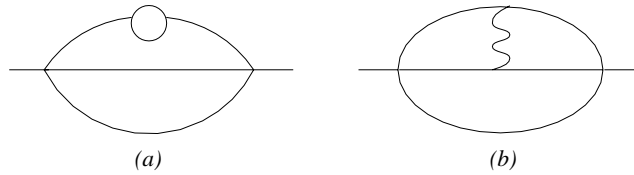At this point it is simple to add the four contributions in (3.13), (3.14), (3.15) and (3.16) and

Figure 5: g^2-order contribution to $\langle \mathrm{Tr}(\Phi^1)^3 \mathrm{Tr}(\bar{\Phi}^1)^3 \rangle$.
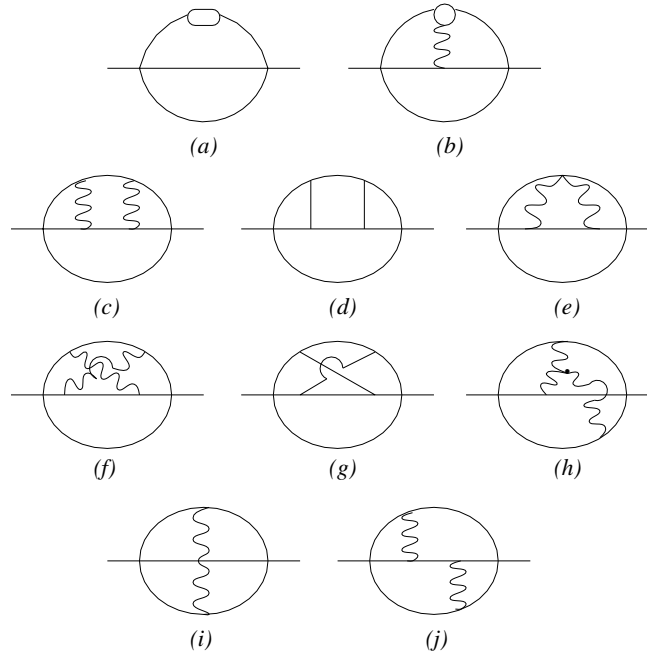

Figure 6: g^4-order contribution to $\langle \mathrm{Tr}(\Phi^1)^3 \mathrm{Tr}(\bar{\Phi}^1)^3 \rangle$.

check the complete cancellation of the $1/\epsilon$ terms. It is interesting to note that, while the diagrams 6a, 6b only contribute with a divergent term proportional to $\zeta(3)$ and the diagram 6i gives only a $\zeta(5)$-term, from the diagram 6j both terms arise with the correct coefficients to cancel completely the divergence.

4. Conclusions

We have discussed the calculation of the two-point correlation function for the chiral primary operator $\mathrm{Tr}\Phi_1^3$ in $\mathcal{N} = 4$ SU(N) SYM theory up to g^4-order. We have found a complete cancellation of quantum corrections for any finite N. Our result represents the first $\mathcal{O}(g^4)$ direct check of the nonrenormalization theorem conjectured on the basis of the AdS/CFT correspondence [5]. It supports also the stronger claim [5] that there might be no quantum corrections at all, for any finite N.

We have performed the calculation in $\mathcal{N} = 1$ superspace using dimensional regularization. The loop-integrals have been evaluated in momentum space with the method of uniqueness [14, 15]. In momentum space nontrivial, potential contributions appear as local divergent terms that are easily isolated and evaluated. Finite contributions would correspond to contact terms and can be neglected.

Our procedure is applicable to the perturbative analysis of more complicated cases. Two-point functions for $\mathrm{Tr}\Phi^k$, $k > 3$, three-point functions and extremal correlators for chiral primary operators are now under investigation [11].

Acknowledgments

References

[1] S. Penati, A. Santambrogio and D. Zanon, *Two-point functions of chiral operators in $\mathcal{N} = 4$ SYM at order g^4*, *J. High Energy Phys.* **12** (1999) 006 [`hep-th/9910197`].

[2] E. D'Hoker, D.Z. Freedman and W. Skiba, *Field theory tests for correlators in the AdS/CFT correspondence*, *Phys. Rev.* **D 59** (1999) 045008 [`hep-th/9807098`].

[3] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [`hep-th/9711200`];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [`hep-th/9802109`];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [`hep-th/9802150`].

[4] S.S. Gubser and I.R. Klebanov, *Absorption by branes and Schwinger terms in the world volume theory*, *Phys. Lett.* **B 413** (1997) 41 [`hep-th/9708005`];
D. Anselmi, D.Z. Freedman, M.T. Grisaru and A.A. Johansen, *Nonperturbative formulas for central functions of supersymmetric gauge theories*, *Nucl. Phys.* **B 526** (1998) 543 [`hep-th/9708042`].

[5] S. Lee, S. Minwalla, M. Rangamani and N. Seiberg, *Three-point functions of chiral operators in $D = 4$, $N = 4$ SYM at large-N*, *Adv. Theor. Math. Phys.* **2** (1998) 697 [`hep-th/9806074`].

[6] B. Eden, P.S. Howe and P.C. West, *Nilpotent invariants in $N = 4$ sym*, *Phys. Lett.* **B 463** (1999) 19 [`hep-th/9905085`];
P.S. Howe, C. Schubert, E. Sokatchev and P.C. West, *Explicit construction of nilpotent covariants in $N = 4$ SYM*, `hep-th/9910011`.

[7] A. Petkou and K. Skenderis, *A non-renormalization theorem for conformal anomalies*, *Nucl. Phys.* **B 561** (1999) 100 [`hep-th/9906030`].

[8] W. Skiba, *Correlators of short multi-trace operators in $N = 4$ supersymmetric Yang-Mills*, *Phys. Rev.* **D 60** (1999) 105038 [`hep-th/9907088`].

[9] E. D'Hoker, D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Extremal correlators in the AdS/CFT correspondence*, `hep-th/9908160`.

[10] M. Bianchi and S. Kovacs, *Non-renormalization of extremal correlators in $N = 4$ SYM theory*, *Phys. Lett.* **B 468** (1999) 102 [`hep-th/9910016`].

[11] S. Penati, A. Santambrogio and D. Zanon, *Work in progress.*

[12] S.J. Gates, M.T. Grisaru, M. Roček and W. Siegel, *Superspace*, Benjamin-Cummings, Reading, MA, 1983.

[13] K.G. Chetyrkin and F.V. Tkachev, *Integration by parts: the algorithm to calculate beta functions in 4 loops*, *Nucl. Phys.* **B 192** (1981) 159.

[14] D.I. Kazakov, *The method of uniqueness, a new powerful technique for multiloop calculations*, *Phys. Lett.* **B 133** (1983) 406; *Analytical methods for multiloop calculations: Two lectures on the method of uniqueness*, Dubna preprint JINR-E2-84-410 (1984).

[15] K.G. Chetyrkin, A.L. Kataev and F.V. Tkachev, *New approach to evaluation of multiloop Feynman integrals: the Gegenbauer polynomial X-space technique*, *Nucl. Phys.* **B 174** (1980) 345.

[16] M.T. Grisaru, W. Siegel and M. Rocek, *Improved methods for supergraphs*, *Nucl. Phys.* **B 159** (1979) 429.