Anomaly inflow and RR anomalous couplings*

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Abstract: We review the anomaly inflow mechanism on D-branes and O-planes. In particular, we compute the one-loop world-volume anomalies and derive the RR anomalous couplings required for their cancellation.

1. Anomalies and inflow

It is known that a consistent quantum field theory can happen to admit as vacuum a topological defect carrying chiral zero modes. The anomaly arising on the world-volume must then be canceled by an inflow from the bulk [1]. This is the case of consistent superstring vacua with D-branes and O-planes, where no overall anomaly can arise but zero modes occur. In general, there can be a net world-volume quantum anomaly, but by consistency, this must be canceled by an equal and opposite classical inflow of anomaly.

The W-Z consistency condition implies that any anomaly $\mathcal{A}$ is the descent of some polynomial $I(F,R)$ of the curvatures F and R of the gauge and the tangent bundles. Defining $I = dI^{(0)}$ and $\delta I^{(0)} = dI^{(1)}$: $\mathcal{A} = 2\pi i \int I^{(1)}$. $I(F,R)$ depends on characteristic classes, like (λ_a are the skew-eigenvalues of R)

$$\widehat{A}(R) = \prod_{a=1}^{D/2} \frac{\lambda_a/4\pi}{\sinh \lambda_a/4\pi} \,,$$
$$\widehat{L}(R) = \prod_{a=1}^{D/2} \frac{\lambda_a/2\pi}{\tanh \lambda_a/2\pi} \,,$$
$$e(R) = \prod_{a=1}^{D/2} \lambda_a/2\pi \,,$$
$$\mathrm{ch}(F) = \mathrm{tr}\, \exp iF/2\pi \,.$$

Beside quantum anomalies, arising from the fluctuations of chiral fermions or self-dual tensor fields, also classical anomalies can occur, for instance in magnetic interactions. Consider indeed some defects M_i in spacetime X, with the RR couplings:

$$S = -\sum\nolimits_i \mu_i \int_{M_i} C \wedge Y_i \,,$$

where $C = \sum_p C_{(p)}$ and $Y = Y(F,R)$. This can be written as an integral over X by using the currents τ_{M_i}, which are globally determined by $N(M_i)$ and locally given by $\tau_{M_i} \sim \delta(x^{d_i})\, dx^{d_i} \wedge ... \wedge \delta(x^D)\, dx^D$ [3]. The RR equation of motion and Bianchi identity become then (the bar represents complex conjugation)

$$d^*H = \sum\nolimits_i \mu_i\, \tau_{M_i} \wedge Y_i \,,$$
$$dH = -\sum\nolimits_i \mu_i\, \tau_{M_i} \wedge \bar{Y}_i \,.$$

The modified Bianchi identity implies that $H = dC - \sum_i \mu_i\, \tau_{M_i} \wedge \bar{Y}_i^{(0)}$, and since this must be gauge invariant, C has to transform as $\delta C =$

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$\sum_i \mu_i \, \tau_{M_i} \wedge \bar{Y}_i^{(1)}$. Consequently, the RR couplings give the anomaly

$$\mathcal{A} = -\, i \sum\nolimits_{i,j} \mu_i \, \mu_j \int_X \tau_{M_i} \wedge \tau_{M_j} \wedge \big(Y_i \wedge \bar{Y}_j\big)^{(1)} \ .$$

Using the property $\tau_{M_i} \wedge \tau_{M_j} = \tau_{M_{ij}} \wedge e[N(M_{ij})]$ [3], we see that the magnetic interaction between M_i and M_j is anomalous on the intersection M_{ij}. The classical anomaly inflow on each intersection is $\mathcal{A}_{ij} = 2\pi i \int_{M_{ij}} I_{ij}^{(1)}$, with

$$I_{ij} = -\frac{\mu_i \, \mu_j}{2\pi} \, Y_i \wedge \bar{Y}_j \wedge e[N(M_{ij})] \ . \qquad (1.1)$$

This has to cancel the corresponding quantum anomaly [2, 3] (even if, strictly speaking, subtleties could arise for self-dual objects [3]).

2. Anomalies on D-branes and O-planes

Consider two parallel Dp-branes (B) and/or Op-planes (B) on M. The anomalous fields living on their world-volumes can be read from the corresponding potentially divergent one-loop amplitudes: the annulus, the Möbius strip and the Klein bottle for the BB, BO and OO configurations. In the first two cases, one finds chiral R spinors, and in the last case self-dual RR forms. These fields are dimensionally reduced from $D = 10$ to $D = p+1$, and will therefore split into two sets with opposite chirality or self-duality. Anomalies can then arise only when $N(M)$ is non-trivial.

These anomalies are as usual topological indices, which can be computed using index theorems or via a path-integral representation as in [4]. In this second approach, the topological nature of the results is related to supersymmetry, and the tangent, normal and gauge bundle curvatures are realized in terms of fermionic zero modes as ($M, N, ... \in X$; $\mu, \nu, ... \in M$; $i, j, ... \in N$)

$$\begin{aligned} R_{\mu\nu} &= \frac{1}{2} R_{\mu\nu\rho\sigma}(x_0) \, \psi_0^\rho \psi_0^\sigma \ , \\ R'_{ij} &= \frac{1}{2} R_{ij\rho\sigma}(x_0) \, \psi_0^\rho \psi_0^\sigma \ , \\ F &= \frac{1}{2} F_{\mu\nu}(x_0) \, \psi_0^\mu \psi_0^\nu \ . \end{aligned} \qquad (2.1)$$

2.1 Chiral spinors

The anomaly of a chiral spinor reduced from X to M is

$$\mathcal{A} = \lim_{t \to 0} \mathrm{Tr} \left[\Gamma^{D+1} \, \delta \, e^{-t(i\not{D})^2} \right] \ .$$

The trace is over the eigenstates of $i\not{D}$ on M, Γ^{D+1} is the chiral matrix on X, and δ is the operator representing gauge transformations. By exponentiating δ, as in [4], this can be written as $\mathcal{A} = 2\pi i Z^{(1)}$, where

$$Z = \lim_{t \to 0} \mathrm{Tr} \left[\Gamma^{D+1} \, e^{-t(i\not{D})^2} \right] \ .$$

Mathematically, Z is the index of a twisted spin complex: $Z = \mathrm{index}(i\not{D})$. The original chiral or anti-chiral spinor on X is a section of $S^\pm_{T(X)}$. On $M \subset X$, the tangent bundle decomposes into tangent and normal components and these spin bundles reduce to $E^\pm = \big(S^\pm_{T(M)} \otimes S^+_{N(M)}\big) \oplus \big(S^\mp_{T(M)} \otimes S^-_{N(M)}\big)$. Considering also a gauge bundle V, one has then the two-term complex

$$i\not{D} : \ \Gamma\big[M, E^+ \otimes V\big] \to \Gamma\big[M, E^- \otimes V\big] \ .$$

The index theorem for this spin complex reads

$$\mathrm{index}(i\not{D}) = \int_M \mathrm{ch}(V) \left[\mathrm{ch}(E^+) - \mathrm{ch}(E^-)\right] \frac{\mathrm{Td}[T(M^C)]}{e[T(M)]} \ ,$$

and explicit evaluation yields [3]

$$Z = \int_M \mathrm{ch}(F) \wedge \frac{\widehat{A}(R)}{\widehat{A}(R')} \wedge e(R') \ . \qquad (2.2)$$

Physically, Z can be interpreted as a partition function. More precisely, for a super quantum mechanics (SQM) with $Q = i\not{D}$ and $(-1)^F = \Gamma^{D+1}$, Z would becomes the Witten index [5]:

$$Z = \mathrm{Tr} \left[(-1)^F \, e^{-tH} \right] \ .$$

The appropriate SQM is obtained by dimensional reduction of the supersymmetric non-linear sigma model (SNSM) form $D = 1+1$ to $D = 0+1$ with Neumann and Dirichlet boundary conditions $\|$ and $\perp$ to M: $x^i = 0$, $\psi_1^\mu = \psi_2^\mu$, $\psi_1^i = -\psi_2^i$. The Lagrangian is:

$$\begin{aligned} L = \ & \frac{1}{2} g_{\mu\nu} \, \dot{x}^\mu \dot{x}^\nu + \frac{i}{2} \psi_\mu \Big(\dot{\psi}^\mu + \omega_\rho{}^\mu{}_\nu \, \dot{x}^\rho \, \psi^\nu \Big) \\ & + \frac{i}{2} \psi_i \Big(\dot{\psi}^i + \omega_\rho{}^i{}_j \, \dot{x}^\rho \, \psi^j \Big) \\ & + \frac{1}{4} R_{\mu\nu ij} \, \psi^\mu \psi^\nu \psi^i \psi^j + ... \end{aligned}$$

where the dots stand for standard terms accounting for the gauge background. Due to $(-1)^F$, all the fields are periodic and

$$Z = \int_P \mathcal{D}x^\mu \int_P \mathcal{D}\psi^\mu \int_P \mathcal{D}\psi^i \, e^{-S(t)} \; .$$

For $t \to 0$, Z is dominated by constant paths with only small fluctuations: $x^\mu = x_0^\mu + \xi^\mu$, $\psi^\mu = \psi_0^\mu + \lambda^\mu$, $\psi^i = \psi_0^i + \lambda^i$. It is enough to keep quadratic interactions and only terms with the maximum number of ψ_0's, and one finds

$$\begin{aligned} L^{eff} &= \frac{1}{2}\Big[\dot\xi_\mu \dot\xi^\mu + i\lambda_\mu \dot\lambda^\mu + i\lambda_i \dot\lambda^i \\ &\quad + iR_{\mu\nu}\,\dot\xi^\mu \xi^\nu + R'_{ij}\,\lambda^i\lambda^j\Big] \\ &\quad + \frac{1}{2} R'_{ij}\,\psi_0^i \psi_0^j + iF \; , \end{aligned}$$

with R, R' and F given by (2.1). The path-integral gives then

$$\begin{aligned} Z = &\int dx_0^\mu \int d\psi_0^\mu \,\mathrm{tr}\exp\{iFt\} \\ &\frac{\det_P(i\eta_{\mu\nu}\partial_\tau)}{\det_P(\eta_{\mu\nu}\partial_\tau^2 + iR_{\mu\nu}\partial_\tau)} \det_P(i\eta_{ij}\partial_\tau + R'_{ij}) \\ &\int d\psi_0^i \,\exp\left\{\frac{t}{2}R'_{ij}\psi_0^i\psi_0^j\right\} \; . \end{aligned}$$

Evaluating the determinants, one recovers finally (2.2) [6].

2.2 Self-dual tensors

The anomaly of a self-dual tensor reduced from X to M can be written as

$$A = \frac{1}{4} \lim_{t\to 0} \mathrm{Tr}\left[I *_D \delta\, e^{-t\mathcal{D}^2}\right] ,$$

where $*_D$ is the Hodge operator on X and the trace is over the eigenstates of $\mathcal{D} = d + d^\dagger$. The dynamics is constrained to $M \subset X$ thanks to the transverse reflection I. As before, this can be written as $\mathcal{A} = 2\pi i Z^{(1)}$, with

$$Z = -\frac{1}{8} \lim_{t\to 0} \mathrm{Tr}\left[I *_D e^{-t\mathcal{D}^2}\right] .$$

Mathematically, Z is in this case a G-index of the usual signature complex. Indeed, $Z = -1/8\,\mathrm{index}(\mathcal{D}_+^G)$, where

$$\begin{aligned} &\mathcal{D}_+ : \; \Gamma\big[X, {}^+\!\wedge T^*X\big] \longrightarrow \Gamma\big[X, {}^-\!\wedge T^*X\big] \; , \\ &G : X \longrightarrow X \; \big(I : (x^\mu, x^i) \longrightarrow (x^\mu, -x^i)\big) \; . \end{aligned}$$

Notice that $G = \mathbf{Z}_2$ is orientation-preserving, as it should be, since D and d must be even. It leaves $M \subset X$ fixed and acts as $+\mathbf{1}$ in $T(M)$ and $-\mathbf{1}$ in $N(M)$. The G-signature theorem gives then

$$\begin{aligned} \mathrm{index}(\mathcal{D}_+^G) = &\int_M \big[\mathrm{ch}(E^+) - \mathrm{ch}(E^-)\big] \\ &\big[\mathrm{ch}(F^+) - \mathrm{ch}(F^-)\big]\,\mathrm{ch}^{-1}(F) \\ &\frac{\mathrm{Td}[T(M^C)]}{e[T(M)]} \; , \end{aligned}$$

with the definitions $E^\pm = {}^\pm\!\wedge T^*M$, $F^\pm = {}^\pm\!\wedge N^*M$ and $F = \oplus_i(-1)^i \wedge^i N^*M$. One finds finally [6]

$$Z = -\frac{1}{8}\int_M \frac{\widehat{L}(R)}{\widehat{L}(R')} \wedge e(R') \; . \qquad (2.3)$$

Physically, Z looks again like a partition function, and for a SQM with $H = \mathcal{D}^2$ and a symmetry $\Omega = *_D$, it would becomes the supersymmetric index

$$Z = -\frac{1}{8}\mathrm{Tr}\left[I\,\Omega\, e^{-tH}\right] .$$

The appropriate SQM is known [5] to be the trivial dimensional reduction of the SNSM from $D = 1+1$ to $D = 0+1$ ($\Omega : (\psi_1, \psi_2) \to (-\psi_1, \psi_2)$):

$$\begin{aligned} L = &\frac{1}{2} g_{MN}(x)\,\dot x^M \dot x^N \\ &+ \frac{i}{2}\sum_{\alpha=1,2} \psi_{\alpha M}\left(\dot\psi_\alpha^M + \omega_P{}^M{}_N(x)\,\psi_\alpha^N\,\dot x^P\right) \\ &+ \frac{1}{4} R_{MNPQ}(x)\,\psi_1^M\psi_1^N\psi_2^P\psi_2^Q \; . \end{aligned}$$

Due to $I\,\Omega$, the fields acquire non-standard periodicities and

$$\begin{aligned} Z = -\frac{1}{8} &\int_P \mathcal{D}x^\mu \int_A \mathcal{D}x^i \int_P \mathcal{D}\psi_1^\mu \int_A \mathcal{D}\psi_1^i \\ &\int_A \mathcal{D}\psi_2^\mu \int_P \mathcal{D}\psi_2^i \, e^{-S(t)} \; . \end{aligned}$$

For $t \to 0$, Z is again dominated by constant paths with small fluctuations: $x^\mu = x_0^\mu + \xi^\mu$, $x^i = \xi^i$, $\psi_1^\mu = \psi_0^\mu + \lambda_1^\mu$, $\psi_1^i = \lambda_1^i$, $\psi_2^\mu = \lambda_2^\mu$, $\psi_2^i = \psi_0^i + \lambda_2^i$. As before, it is enough to keep terms quadratic in the fluctuations and with a maximum number of fermionic zero modes. One finds (with R, R' as in (2.1))

$$
\begin{aligned}
L^{eff} = \frac{1}{2}\Big[& \dot\xi_\mu \dot\xi^\mu + \dot\xi_i \dot\xi^i + i\lambda_{1\mu}\dot\lambda_1^\mu + i\lambda_{1i}\dot\lambda_1^i \\
& + i\lambda_{2\mu}\dot\lambda_2^\mu + i\lambda_{2i}\dot\lambda_2^i \\
& + R_{\mu\nu}\left(i\,\dot\xi^\mu \xi^\nu + \lambda_2^\mu \lambda_2^\nu\right) \\
& + R'_{ij}\left(i\,\dot\xi^i \xi^j + \lambda_2^i \lambda_2^j\right)\Big] \\
& + \frac{1}{2} R'_{ij}\,\psi_0^i \psi_0^j \,.
\end{aligned}
$$

The path-integral yields then

$$
\begin{aligned}
Z = -\frac{1}{8} & \int dx_0^\mu \int d\psi_0^\mu \\
& \frac{\det_P(i\eta_{\mu\nu}\partial_\tau)\,\det_A(i\eta_{\mu\nu}\partial_\tau + R_{\mu\nu})}{\det_P(\eta_{\mu\nu}\partial_\tau^2 + iR_{\mu\nu}\partial_\tau)} \\
& \frac{\det_A(i\eta_{ij}\partial_\tau)\,\det_P(i\eta_{ij}\partial_\tau + R'_{ij})}{\det_A(\eta_{ij}\partial_\tau^2 + iR'_{ij}\partial_\tau)} \\
& \int d\psi_0^i \, \exp\left\{\frac{t}{2} R'_{ij}\psi_0^i \psi_0^j\right\} \,.
\end{aligned}
$$

Finally, evaluating the determinants one recovers (2.3) [6].

3. Anomalous couplings

Using the results (2.2) and (2.3), the quantum anomalies on n parallel Dp-branes and/or Op-planes are found to be

$$
\begin{aligned}
I_{BB} &= \mathrm{ch}_{\mathbf{n}\otimes\bar{\mathbf{n}}}(F) \wedge \frac{\widehat{A}(R)}{\widehat{A}(R')} \wedge e(R') \,, \\
I_{BO} &= \mathrm{ch}_{\mathbf{n}\oplus\bar{\mathbf{n}}}(2F) \wedge \frac{\widehat{A}(R)}{\widehat{A}(R')} \wedge e(R') \,, \\
I_{OO} &= -\frac{1}{8}\,\frac{\widehat{L}(R)}{\widehat{L}(R')} \wedge e(R') \,. \qquad (3.1)
\end{aligned}
$$

On the other hand, assigning the anomalous couplings

$$
S_{B,O} = \sqrt{2\pi}\int C \wedge Y_{B,O} \,, \qquad (3.2)
$$

one gets, according to (1.1), the classical inflows

$$
\begin{aligned}
I_{BB} &= -Y_B \wedge \bar Y_B \wedge e(R') \,, \\
I_{BO} &= -\big(Y_B \wedge \bar Y_O + Y_O \wedge \bar Y_B\big) \wedge e(R') \,, \\
I_{OO} &= -Y_O \wedge \bar Y_O \wedge e(R') \,. \qquad (3.3)
\end{aligned}
$$

Due to the property $\sqrt{\widehat{A}(R)}\sqrt{\widehat{L}(R/4)} = \widehat{A}(R/2)$, the relevant (D+2)-form component of (3.1) and (3.3) are compatible, and anomaly cancellation requires

$$
\begin{aligned}
Y_B &= \mathrm{ch}_{\mathbf{n}}(F) \wedge \sqrt{\frac{\widehat{A}(R)}{\widehat{A}(R')}} \,, \\
Y_O &= -2^{p-4}\sqrt{\frac{\widehat{L}(R/4)}{\widehat{L}(R'/4)}} \,. \qquad (3.4)
\end{aligned}
$$

Notice that for non-trivial embeddings and multiple branes, the pulled-back curvatures depend also on the gauge connection (see for instance [12]).

The presence of anomalous couplings of the form (3.2) for D-branes and O-planes has been also predicted in particular cases using string dualities [7, 8]. Their actual occurrence in the form (3.4) has been demonstrated in [9] through a direct string theory computation, by factorizing RR magnetic interactions between D-branes and O-planes, encoded in one-loop amplitudes on the annulus, Möbius strip and Klein bottle surfaces. These couplings have been also checked through tree-level computations on the disk and the crosscap [10, 11].

4. String theory computation

Interestingly, the anomaly inflow mechanism on D-branes and O-planes can be analyzed directly in string theory, where tadpole cancellation guarantees overall finiteness and implies anomaly cancellation. Recall that one can compute anomalies by evaluating amplitudes with external photons and/or gravitons, one of them being pure gauge. This measures the clash of gauge invariance and gives directly the anomaly. Only the CP-odd part of potentially divergent diagrams can contribute. In string theory, these are the annulus, the Möbius strip and Klein bottle amplitudes in the RR odd spin-structure.

The amplitudes we want to compute have the form

$$
\mathcal{A} = \int_0^\infty dt\,\big\langle\, V_1^{phy.} \ldots V_n^{phy.}\, V^{unphy.}\,(T_F + \tilde T_F)\,\big\rangle \,.
$$

The insertion of $T_F + \tilde T_F$ is due to the gravitino zero mode, and the vertices must have total superghost charge -1. Take all the $V^{phy.}$'s in the 0-picture, with an arbitrary transverse polarisation ξ_M or ξ_{MN}, and $V^{unphy.}$ in the -1-picture, with a longitudinal polarisation given by $\xi_M = p_M \eta$ or $\xi_{MN} = p_M \eta_N + p_N \eta_M$. Interesting enough, the latter can then be written as a supersymmetry variation, $V^{unphy.} = \big[Q+\tilde{Q}, \hat{V}^{unphy.}\big]$. Using standard arguments, one can then move $Q+\tilde{Q}$ onto the other operators in the correlation. One gets no effect on the $V^{phy.}$'s, since they are supersymmetric, but the supercurrent is changed to the energy-momentum tensor, $\big[Q+\tilde{Q}, T_F + \tilde{T}_F\big] = T_B + \tilde{T}_B$. The net effect of $T_B + \tilde{T}_B$ is to take the derivative of the remaining correlation with respect to the modulus t, and one is then left with a total derivative in moduli space:

$$\mathcal{A} = \int_0^\infty dt \, \frac{d}{dt} \left\langle \, V_1^{phy.} \ldots V_n^{phy.} \, \hat{V}^{unphy.} \, \right\rangle \, . \quad (4.1)$$

In consistent models, this total anomaly has to vanish, reflecting a cancellation between one-loop anomalies and tree-level inflows associated to the same surface. At finite p's, only the ultraviolet boundary $t \to 0$ can contribute and has to vanish by itself. The computation is still difficult, but fortunately, to get a field theory interpretation, it is enough to restrict to the leading order in $p \to 0$. In this limit, the correlation becomes t-independent and yields at the same time the anomaly and the inflow. Moreover, since the correlation vanishes unless all the fermionic zero modes are inserted, one can use [9, 6]

$$\begin{aligned} V_\gamma^{eff.} &= \oint d\tau \, F \, , \\ V_g^{eff.} &= \oint d^2 z \, R_{MN} \Big[X^M (\partial + \bar{\partial}) X^N \qquad (4.2) \\ &\qquad\qquad + (\psi - \tilde{\psi})^M (\psi - \tilde{\psi})^N \Big] \, . \end{aligned}$$

This holds both for physical and unphysical vertices, with

$$\begin{aligned} F^{phys.} &= \frac{1}{2} F_{\mu\nu} \, \psi_0^\mu \psi_0^\nu \, , \\ R_{MN}^{unphys.} &= \frac{1}{2} R_{MN\mu\nu} \, \psi_0^\mu \psi_0^\nu \, , \\ F^{unphys} &= \eta \, , \\ R_{MN}^{unphys.} &= p_M \eta_N + p_N \eta_M \, . \end{aligned}$$

The generating functional of (4.1) is a partition function twisted by the interactions (4.2) in the backgrounds $F + \eta$ and $R_{MN} + p_M \eta_N + p_N \eta_M$. The correct number of physical vertices is automatically selected, the unphysical one being obtained by restricting to the term linear in η. Not too surprisingly, the only role of the unphysical vertex is to take the descent of the remaining partition function, and the anomaly polynomial is given by $I = Z'$ [13].

It is straightforward to applying this general result to standard D-branes and O-planes. One finds ($\Omega_I = \Omega \, I$ is the T-dual of the world-sheet parity Ω)

$$\begin{aligned} I_{BB} = Z'_A &= \frac{1}{4} \, \mathrm{Tr}'_R \left[(-1)^F \, e^{-tH} \right] \, , \\ I_{BO} = Z'_M &= \frac{1}{4} \, \mathrm{Tr}'_R \left[\Omega_I \, (-1)^F \, e^{-tH} \right] \, , \\ I_{OO} = Z'_K &= \frac{1}{8} \, \mathrm{Tr}'_{RR} \left[\Omega_I \, (-1)^{F+\tilde{F}} \, e^{-tH} \right] \, . \end{aligned}$$

These are supersymmetric indices, and only massless modes do contribute. Effectively, one recovers precisely the SQM models seen before, reproducing therefore the same results for the anomalies and the anomalous couplings.

One can apply this general approach also in more complicated cases, like for instance D-branes, O-planes and fixed-points in orientifold models [13, 14]. This provides an efficient tool to analyse in detail the complicated G-S mechanism of anomaly cancellation in this kind of models.

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