The Holographic RG flow to conformal and non-conformal theories

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Abstract: We review some aspects of the AdS supergravity description of RG flows. The case of a flow to an IR CFT can be rigorously studied within the framework of supergravity. Here we discuss various central charges of the conformal theory (included the usually neglected ones) and we compare them with QFT expectations. The case of flows to non-conformal theories is more problematic in that one usually encounters a naked singularity. We discuss the properties of these solutions and we briefly comment on the fate of the singularity.

1. Introduction

The AdS/CFT correspondence has deserved some surprises when extended outside the realm of strictly conformal invariant theories. The study of the supergravity dual of RG flows has flourished, both in the concrete application to SYM theories and in a general setting [1]-[14]. Asymptotically AdS_{d+1} backgrounds, breaking the full $O(d,2)$ invariance but preserving at least d-dimensional Poincaré invariance, describe RG flows for a d-dimensional CFT. These supergravity solutions with an asymptotic AdS region have a double QFT interpretation: deformations of an UV fixed point versus the same theory in a different vacuum [15, 16]. Both cases have been extensively studied. Many results have been obtained upon reduction to a $d+1$-dimensional effective theory, where the RG flow can be studied in terms of a theory of scalar fields coupled to gravity. In this simple set-up, the RG flows are identified as domain-walls interpolating between AdS_{d+1} vacua (or approaching infinity on one side), and general results are very easy to obtain. The correspondence defines a *holographic* scheme, where beta and c-functions have a natural definition. A c-theorem, for example, can be easily proven [1, 8]. Also, the quantum field theory RG equations can be obtained from supergravity [14][1].

The study of RG flows between CFTs (at large N and strong coupling) can be rigorously performed using supergravity. The phase space of massive deformations of the N=4 SYM theory have been throughly investigated and several IR fixed points have been found [1, 2, 3, 4, 8]. The results are on solid grounds because supergravity is valid all along the RG flow. Still problematic is the precise mapping of some QFT couplings to supergravity quantities. For example, it is still unclear what in supergravity corresponds to the gauge coupling running.

Most of the unsolved problems arise for flows to non-conformal theories, where supergravity is invalidated by a (typically naked) singularity in the IR region of the flow. Solutions flowing to infinity for a generic $5d$-Lagrangian are certainly a dense set in the space of solutions. The full recipe for selecting the *physical* ones is still unclear [2]. The distinction between deformations and vacua of an UV fixed point helps but does not solve the problem. Supersymmetric and supersymmetric-inspired solutions however are uniquely selected

[1] Notice that the *holographic* beta and c-functions do not need to coincide with analogous functions defined in schemes that are more natural from the QFT point of view [17].

[2] A criterion for selecting physical solutions has been recently proposed in [18].

because the equations of motion can be reduced to first order ones [8, 19]. N=4 Coulomb branch solutions have been studied in [9, 10, 20] and a flow to N=1 SYM in [11].

Since singularities are apparently unavoidable in interesting supergravity solutions, it is mandatory to understand their fate in the full string theory, where they must be resolved. Available options are the chance that the singularity is an artifact of the dimensional reduction to 5 dimensions, mechanisms such that proposed in [21] and, more generally, some help from string corrections.

The supergravity solutions with an asymptotic AdS region certainly have many other applications. Relaxing the d-Poincaré invariance, we have examples of RG flow due to finite temperature. This is indeed the firstly proposed method for discussing non-conformal theories from AdS [22] and the one not suffering from unpleasant singularities. Cutting the AdS-boundary, we can describe CFTs coupled to gravity and make contact with the large extra-dimension scenario [23]. We will not discuss this issue here, but we simply notice that singular solutions have been recently considered in this context.

2. RG Flow from 5d Supergravity

In general, we interpret the $(d+1)$-th coordinate y of AdS_{d+1} as an energy scale [24, 25]: $y \to \infty$ corresponds to the UV regime while $y \to -\infty$ to the IR.

Then RG flows in QFT correspond to type II or M-theory supergravity solutions interpolating (along y) between these two asymptotic regions. Flows between CFTs are given by solutions interpolating between $\mathrm{AdS}_{d+1} \times_W H$ vacua. Since along the flow conformal invariance is lost, one has to look at supergravity solutions that are only asymptotically AdS, but that still preserve d-dimensional Poincaré invariance.

The very first example of RG flow in the AdS/CFT correspondence is manifest in the multi-centre supergravity solution for D3-branes [24]. This represents the Coulomb branch of N=4 SYM. Given two sets of N and M branes at different points, the near-horizon geometry is AdS_5 with radius $\sim \sqrt{N+M}$ far from both sets of branes, and AdS_5 with radius $\sim \sqrt{N}$ near one set. In QFT this is the RG flow between the $U(N+M)$ N=4 CFT in the UV, where the Higgs VEVs can be neglected, and the $U(N)$ N=4 CFT in the IR. A more sophisticated example was found in [26]. A supergravity solution interpolating between $\mathrm{AdS}_5 \times S^5/Z_2$ and $\mathrm{AdS}_5 \times T^{1,1}$ was also interpreted on the QFT side as a RG flow between CFTs. It is a supersymmetric massive deformation of the N=2 $SU(N) \times SU(N)$ theory corresponding to a Z_2 orbifold of N=4 SYM which flows to an N=1 IR fixed point. Many successful checks of this interpretation have been performed [26, 27, 28, 29].

However, interpolating 10d backgrounds are difficult to find. Sometimes dimensional reduction to 5 dimensions helps.

The RG flow has a natural description in 5d. Consider a certain UV CFT and suppose we have the corresponding 5d Lagrangian and that it contains all the fields/modes we are interest in. The effective 5d Lagrangian we need is just the most general Lagrangian for scalars coupled to gravity

$$L = \sqrt{-g}\left[-\frac{R}{4} + \frac{1}{2} g^{IJ} \partial_I \lambda_a \partial_J \lambda_b G^{ab} + V(\lambda)\right]. \tag{2.1}$$

The scalars λ can either be the *massless* modes or Kaluza-Klein modes of the compactification to 5 dimensions. The form of the potential depends on the particular case we are considering. We may have, for example, N=8 gauged supergravity, which describes N=4 SYM and most of its bilinear relevant operators (almost all of the masses for scalars and fermions). Or we may have an N=4 theory describing the orbifold R^4/Z_2 and the supersymmetric mass term that drives the theory to an N=1 IR fixed point. Or else we may have the Lagrangian for the some of the KK modes. The interactions among the modes in the graviton multiplet in 5d can be found using supersymmetry. In particular, for the N=4 SYM case, the 5d Lagrangian for the *massless* modes is uniquely fixed by supersymmetry in the form of the N=8 gauged supergravity [30]. All the mass terms for the scalars and the fermions contained in the KK spectrum are associated to modes in the gauged supergravity. 5d supersymmetric Lagrangians have been discussed also for less supersymmetry, but the uniqueness of N=8 supergravity is lost and interesting modes are split into various vector, tensor and hyper-multiplets. One needs some help from QFT intuition in identifying the right potential. In principle, $V(\lambda)$ can be obtained for all modes (often with non-trivial effort) by dimensional reduction from 10 dimensions.

If the UV CFT perturbed by a particular operator O_λ flows in the IR to another CFT, the potential V must have a critical point for non-zero value of the scalar field λ. Analogously, the dual of the flow to a non-conformal field theory is given by the flow from one minimum of the potential to infinity.

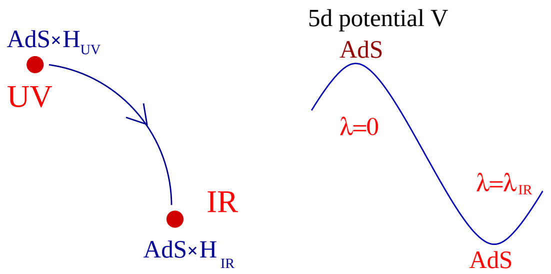

Figure 1: Schematic picture of the RG flow.

2.1 Flow between conformal field theories

The $5d$ description of the RG flow between conformal theories is a kink solution, which interpolates between two critical points. The ansatz for a $4d$ Poincaré invariant metric is

$$ds^2 = dy^2 + e^{2\phi(y)} dx^\mu dx_\mu, \quad \mu = 0, 1, 2, 3. \quad (2.2)$$

AdS corresponds to $\phi = y/R$. We then look for solutions with asymptotics: $\phi(y) \to y/R_{UV,IR}$ for $y \to \pm\infty$; $\lambda(y) \to 0$ for $y \to \infty$, while $\lambda(y) \to \lambda_{IR}$ for $y \to -\infty$. We associate larger energies with increasing y.

The equations of motion for the scalars and the metric read

$$\ddot{\lambda}_a + 4\dot{\phi}\dot{\lambda} = \frac{\partial V}{\partial \lambda_a},$$
$$6(\dot{\phi})^2 = \sum_a (\dot{\lambda}_a)^2 - 2V. \quad (2.3)$$

With the above boundary conditions and a reasonable shape for the potential, a kink interpolating between critical points always exists [1].

As an example of flows between conformal field theories, we can discuss the mass deformations of N=4 SYM. These can be studied in the context of N=8 gauged supergravity, where the form of the potential V is known. N=8 gauged supergravity [30] is the low energy effective action for the "massless" modes of the compactification of Type IIB on $AdS_5 \times S_5$. It is believed to be a consistent truncation of type IIB on S^5 in the sense that every solution of the 5d theory can be lifted to a consistent 10d type IIB solution. Five-dimensional gauged supergravity has 42 scalars, which transform under the N=4 YM R-symmetry $SU(4)$ as $\underline{1}, \underline{20}, \underline{10}$. The singlet is associated with the margi- nal deformation corresponding to a shift in the coupling constant of the N=4 theory. The mode in the $\underline{20}$ has mass square $M^2 = -4$ and is associated with a symmetric traceless mass term for the scalars $\mathrm{Tr}\,\phi_i\phi_j, i, j = 1, ..., 6$ $(\Delta = 2)$. The $\underline{10}$ has mass square $M^2 = -3$ and corresponds to the fermion mass term $\mathrm{Tr}\,\lambda_A\lambda_B, A, B = 1, ..., 4$, of dimension 3. Thus the scalar sector of N=8 gauged supergravity is enough to discuss at least all mass deformations that have a supergravity description[3].

The scalar potential V (eq.(2.1) is known and it turns out to have only isolated minima (apart from one flat direction, given by the dilaton). Up to now, all critical points with at least $SU(2)$ symmetry have been classified [3].There is a central critical point with $SO(6)$ symmetry and with all the scalars λ_a vanishing: it corresponds to the unperturbed N=4 YM theory. There are three N=0 theories with residual symmetry $SU(3) \times U(1)$, $SO(5)$ and $SU(2) \times U(1)^2$. They correspond to non-zero VEV for some of the scalars in the $\underline{10}$, $\underline{20}$, and $\underline{10} + \underline{20}$ respectively. Then there is N=2 point with symmetry $SU(2) \times U(1)$, obtained giving VEV to scalars in the $\underline{10} \oplus \underline{20}$ [3]. According to the AdS/CFT correspondence, these other minima should correspond to IR conformal field theories [4]. The

[3]The only missing state is Tr $\sum_i^6 \phi_i^2$, the prototype of a stringy states in the correspondence. Even without this state, we can study almost all massive deformations of the N=4 theory and all these deformations can be described by just the Lagrangian for the massless multiplet.

[4]The symmetries of the field theories can be read from those of the supergravity minima according to the correspondence : gauge symmetry in supergravity ↔ global

following IR CFT theories can be obtained as mass deformations of N=4 SYM:

- Three N=0 theories with symmetry $SU(3)\times U(1)$, $SO(5)$ and $SU(2)\times U(1)^2$. All these theories are unstable and correspond to non-unitary CFTs. A natural question arises: do all the N=0 critical points are unstable?

- A stable N=1 point with symmetry $SU(2)\times U(1)$. It corresponds to the N=4 theory deformed with a mass for one of the three N=1 chiral superfields. Results and supergravity description [4, 8] are almost identical to the $T^{1,1}$ case, which is just a Z_2 projection of this example.

2.2 Central charges

In a supersymmetric gauge field theory in $4d$, the trace anomaly and the R-symmetry anomaly are given by [31]

$$\begin{aligned} T^\mu_\mu &= \frac{\tilde\beta}{2g^2}F^2_{\mu\nu} + \frac{c}{16\pi^2}W^2_{\mu\nu\rho\sigma} - \frac{a}{16\pi^2}\tilde R^2_{\mu\nu\rho\sigma} \\ &+ \frac{c}{6\pi^2}V^2_{\mu\nu} + \frac{b}{32\pi^2}B^2_{\mu\nu} \qquad (2.4) \\ \partial_\mu\sqrt{g}R^\mu &= -\frac{\tilde\beta}{3g^2}F_{\mu\nu}\tilde{F^{\mu\nu}} - \frac{a-c}{24\pi^2}R_{\mu\nu\rho\sigma}\tilde{R^{\mu\nu\rho\sigma}} \\ &+ \frac{5a-3c}{9\pi^2}V_{\mu\nu}\tilde V^{\mu\nu} - \frac{b}{48\pi^2}B_{\mu\nu}\tilde{B^{\mu\nu}} \qquad (2.5) \end{aligned}$$

Here $W_{\mu\nu\rho\sigma}$ and $R_{\mu\nu\rho\sigma}$ are the Weyl and curvature tensors for an external metric $g_{\mu\nu}$ that couples to the energy-momentum tensor $T_{\mu\nu}$. Similarly $V_{\mu\nu}$ and $B^{\mu\nu}$ are the field strengths of the external sources V_μ, B_μ that couple to the R-symmetry currents and to the flavour currents, respectively. $F_{\mu\nu}$ is the gauge field strength and $\tilde\beta$ is the denominator of the exact beta-function [32].

The external anomaly coefficients a and c have a straightforward interpretation in the dual supergravity theory.

c is the central charge of the CFT, and it is associated with the cosmological constant at the critical points. From eq. (2.1), we can see by a simple scaling that, at least at the fixed points, where $ds^2 = R^2[dy^2 + \exp(2y)\sum_i dx_i^2]$,

$$\langle T(x)T(0)\rangle = \frac{c}{|x|^8} \quad \to c\sim R^3 \sim (\Lambda)^{-3/2}. \quad (2.6)$$

This scaling reproduces the known results for c [33, 27]. More interestingly, one can prove that for the class of field theory that have a supergravity dual a c-theorem exists. Indeed we can exhibit a c-function that is monotonically decreasing along the flow [1, 8]. The c-function

$$c(y) \sim (T_{yy})^{-3/2}, \qquad (2.7)$$

is constructed with the y component of the stress-energy tensor

$$T_{yy} = 6(\dot\phi)^2 = \sum_a(\dot\lambda_a)^2 - 2V. \qquad (2.8)$$

At the critical points, where $\dot\lambda_a = 0$,

$$c(y) = c_{UV,IR} \sim (-V)^{-3/2}_{UV,IR} \sim \Lambda^{-3/2}_{UV,IR}, \quad (2.9)$$

and using the equations of motion ($\ddot\phi < 0$) and the boundary conditions one can easily check that $c(y)$ is monotonic [1, 8].

Let us consider a. AdS computations [33] showed that $a = c$ for all CFTs that have an AdS dual.

It is then natural to ask what can AdS/CFT correspondence say about the coefficient b[5]. The coefficient b is related to the two-point function of the flavour (global) symmetry currents [31]. According to AdS/CFT correspondence the R-symmetry and flavour currents are associated to the gauge fields of the SUGRA Lagrangian

$$J_\mu, R_\mu \longleftrightarrow A_\mu. \qquad (2.10)$$

One should then be able to read the b (and a) coefficient from the kinetic terms of the corresponding SUGRA modes. The generic $5d$-Lagrangian we are interested in has the following structure

$$L = \sqrt{-g}\left[-\frac{R}{4} + \Lambda + fF^2_{\mu\nu} + f_R F^2_{\mu\nu R}\right]. \qquad (2.11)$$

Here $F_{\mu\nu R}$ and $F_{\mu\nu}$ represent the kinetic terms for the fields corresponding to the R-symmetry and flavour symmetry currents, respectively. At

symmetry in field theory, supersymmetry in supergravity $\leftrightarrow$ superconformal symmetry in field theory

[5] These results have been obtained in collaboration with D. Anselmi, L. Girardello and M. Petrini.

the critical points (or generically for a metric of the form (2.2)), one obtains by scaling

$$\langle J(x)J(0)\rangle = \frac{b}{|x|^6} \;\rightarrow b \sim fR \sim fc^{1/3}. \quad (2.12)$$

A similar behaviour is obtained for the R-symmetry currents. In this case, supersymmetry [6] implies $b = c$, and the previous equation can be used as a check of the consistency of the procedure.

The values of the coefficients f and f_R depend on the particular model under consideration. Consider for example the massive deformations of N=4 SYM, for which we have the dual supergravity Lagrangian: that of N=8 gauged supergravity. In this case, the kinetic term for the gauge fields is expressed in terms of the vielbein that parametrise the scalar coset-manifold [34]. To determine f and f_R we have then to evaluate the contractions of the vielbein and therefore these coefficients depend on the critical point and on the way the UV $SU(4)$ group is broken (e.g. $SU(4) \rightarrow SU(3) \times U(1)_R$, $SU(4) \rightarrow SU(2) \times U(1)_R$, ...). We now want to compute the charge b for the global non-abelian symmetry group preserved along the flow (e.g. $SU(3)$, $SU(2)$, ...). The computation of the coefficients f can be performed using the results of [34] for most of the critical points. Alternatively, using the parametrisation in appendix A of [8], it is easy to convince themselves that

$$f = e^{4\alpha}. \quad (2.13)$$

Here α is the scalar in the $\underline{20}$ of $SU(4)$ corresponding to a mass term for the scalars in N=4 SYM [8]. The value of the scalar α and c for the various fixed points can be found in [3, 8, 34]. One then gets the following results for the coefficient b [35]:

- N=1 point with symmetry $SU(2) \times U(1)$. $\frac{b_{IR}}{b_{UV}} = \frac{3}{2}$. This is the only case where comparison with field theory is possible. Consider a set of N=1 chiral superfields X_i in the representation R_i of the gauge group and in the representation T_i of the flavour symmetry group. Then, because of supersymmetry, the following formula holds [31]

$$b_{UV} - b_{IR} = 3\sum_{ij} (dim R_i)\left[\left(r_i - \frac{2}{3}\right) T_i^j T_j^i\right], \quad (2.14)$$

  where r_i is IR R-symmetry charge of the field X_i and T_i^j are the generators of the flavour group in the representation T_i. It is straightforward to check that the supergravity and the field theory computations agree.

- N=0 theories. For the $SU(3)\times U(1)$, $SO(5)$ and $SU(2) \times U(1)^2$ symmetric points, we have $\frac{b_{IR}}{b_{UV}} = \frac{2\sqrt{2}}{3}$, $\frac{b_{IR}}{b_{UV}} = \sqrt{2}$ and $\frac{b_{IR}}{b_{UV}} = 2$, respectively.

In [36] it was observed that for several examples of supersymmetric gauge theory b increases going from the UV to the IR. This was suggestive of possible anti-b-theorem. The same authors however pointed out that for non-supersymmetric gauge theories b has no universal behaviour, and that also a large class of supersymmetric theories violates the relation $b_{IR}/b_{UV} > 1$. Then it is not possible to state any anti-b-theorem in field theory. It is interesting to see what are the supergravity results. Consider first the non-supersymmetric cases. For the point $SU(3)\times U(1)$ we have $b_{IR}/b_{UV} < 1$, which violates the anti-b-theorem. The situation is different for the supersymmetric point $SU(2)\times U(1)$. In this case the coefficient b increases along the flow. The same analysis carried on for the massive flow to N=1 super Yang-Mills (see section 4) or for the Coulomb branch of N=4 SYM [9] seems to indicate a similar behaviour.

Notice that the theories that have a supergravity dual represent a very restricted class of gauge theories. First of all these theories always have $a = c$, which is in general not the case in field theory. It has been argued that the requirement $a = c$ simplifies the structure and OPEs of a CFT, making it most similar to a two dimensional conformal field theory [37]. Secondly it has been suggested (see [8] and next section) that all these theories could be characterised by having a pre-potential. It could then be possible, and interesting to check, whether an anti-b-theorem

[6] The R-symmetry currents are in the same multiplets as the energy-momentum tensor.

could hold for this particular class of gauge theories.

The previous results on b could have been obtained from the analysis of the Chern-Simons terms of the N=8 Lagrangian, which contain all informations about global anomalies [38, 39]. In particular, b can be read from the $SU(2)^2\times U(1)_R$ anomaly coefficient, which can be extracted from the Chern-Simon terms. It is easy to check, using the results in [8], that the result for b coincides with the previously obtained one[7]. Notice that the Chern-Simon terms uniquely determine the form of a supersymmetric gauge supergravity. From the knowledge of the global anomaly, we should be able to reconstruct the entire AdS Lagrangian for *massless* modes for a given supersymmetric CFT fixed point [39].

2.3 Vacua and deformations

We end this section with a brief discussion of a general point that will play an important role in our analysis, namely the fact that supergravity solutions can represent both deformations of a CFT and different vacua of the same theory [15, 16]. The running of coupling constants and parameters along the RG flow can be induced in the UV theory in two different ways: by deforming the CFT with a relevant operator, or by giving a nonzero VEV to some operators. The asymptotic UV behaviour discriminates between the two options. In the asymptotic AdS-region, we just need a linearised analysis. A scalar fluctuation $\lambda(y)$ in the asymptotically AdS background must satisfy

$$\ddot{\lambda} + 4\dot{\lambda} = M^2\lambda, \tag{2.15}$$

where the dot means the derivative with respect to y. The previous equation has a solution depending on two arbitrary parameters

$$\lambda(y) = Ae^{-(4-\Delta)y} + Be^{-\Delta y}, \tag{2.16}$$

where Δ is the dimension of the operator, $M^2 = \Delta(\Delta-4)$ [40, 38]. We are interested in the case of relevant operators, where $\Delta \leq 4$. From the basic prescription of the AdS/CFT, we associate solutions behaving as $e^{-(4-\Delta)y}$ with deformations of the N=4 theory with the operator O_λ. On the other hand, solutions asymptotic to $e^{-\Delta y}$ (the subset with $A = 0$) are associated with a different vacuum of the UV theory, where the operator O_λ has a non-zero VEV [8]. [15, 16].

Since in general the UV-IR interpolating solution is not known, it is not even obvious whether a particular solution corresponds to a deformation or to a different vacuum. For many problems, we may invoke supersymmetry. It helps in finding the solution all along the flow and in unambiguously identifying the UV behaviour. In ref. [8, 19] the conditions for a supersymmetric flow were found. As usual, a solution for which the fermionic shifts vanish, automatically satisfies the equations of motion. Moreover, this shortcut reduces the second order equations to first order ones. For a supersymmetric solution, the potential V can be written in terms of a superpotential W as

$$V = \frac{1}{8}\sum_{a=1}^{n}\left|\frac{\partial W}{\partial\lambda_a}\right|^2 - \frac{1}{3}\left|W\right|^2, \tag{2.17}$$

where W is one of the eigenvalues of the tensor W_{ab} defined in [34]. The equations of motion reduce to

$$\begin{aligned}\dot{\lambda}_a &= \frac{1}{2}\frac{\partial W}{\partial\lambda_a},\\ \dot{\phi} &= -\frac{1}{3}W.\end{aligned} \tag{2.18}$$

It is easy to check that a solution of eq.(2.18) satisfies also the second order equations (2.3).

It is quite plausible and generally assumed that all the supergravity flows connecting fixed points correspond to deformations of the UV fixed point.

3. Confining Solutions

Solutions flowing to infinity represent RG flows to non-conformal theories, which may exist in various phases in the IR. These kinds of solution are difficult to classify. In many cases the asymptotic IR behaviour is known, but the entire solution along the flow can not be found. Typically,

[7] It is crucial to pay attention to normalisations and the definition of $U(1)_R$, which varies from UV to IR.

[8] We are not careful about subtleties for particular values of Δ [16].

we encounter a singularity somewhere along the flow. Many solutions exhibit a logarithmic divergence at finite y_0 for the scalar fields $\lambda_a \sim B_a \log|y-y_0|$ and the metric $\phi \sim A\log|y-y_0|$. There are many criteria for studying the IR properties and the phase of these solutions. One of them, the Wilson loop, will be discussed later. The spectrum can be determined also from two-point functions, where physical bound states appear as poles. Poles in the two-point function corresponding to a minimally coupled scalar, for example, correspond to F^2 glueball masses in the theory. The analysis of the spectrum can be reduced, as usual in the AdS/CFT correspondence, to the solution of a Schroedinger problem [22, 41]. After a change of variable $y \to z$ to the conformally flat metric $ds^2 = e^{2\phi(z)}((dz)^2 + (dx)^2)$ and a field redefinition $\Phi_k(z) = e^{-3\phi(z)/2}\psi(z)$, the $5d$ equation for a minimally coupled scalar $\Phi(x,y) = e^{-ikx}\Phi_k(y)$ takes the Schroedinger form

$$(-\partial_z^2 + V(z))\psi = E\psi \qquad (3.1)$$

where $V = \frac{3}{2}\phi'' + \frac{9}{4}(\phi')^2$. The eigenvalues E give the poles in the two-point function and the spectrum.

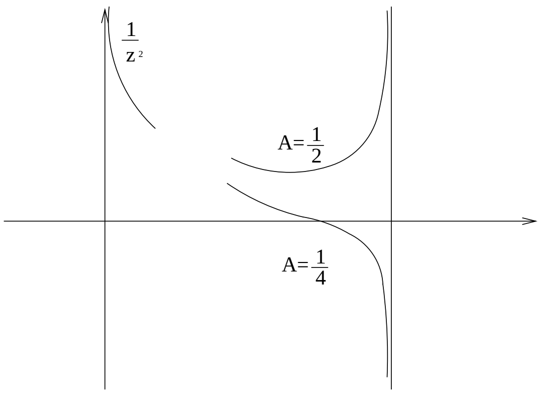

Figure 2: The Schroedinger potential in various cases.

The form of V immediately tells us whether the theory has a mass gap and a discrete spectrum or a continuous one, whether it confines or not. Unfortunately, in very few examples V is known along the entire flow. We can nevertheless extract some information from the IR behaviour. For the logarithmically divergent flows discussed above, if $A < 1$, the singularity is mapped to a finite z_0 and we have

$$V \sim \frac{3A(5A-2)}{4(1-A)^2(z-z_0)^2}. \qquad (3.2)$$

This behaviour looks potentially dangerous, but, as discussed in all quantum mechanics textbooks, $V \sim k/z^2$ has a discrete spectrum bounded from below, provided $k \geq -1/4$. It is easy to check that, for the logarithmically divergent flows, this condition is always satisfied. The value $k = -1/4$ is obtained for $A = 1/4$. This is the value that appears in many solutions where the supergravity potential is irrelevant in the IR [6], but also in one of the examples of N=4 Coulomb branch in [9]. If $A > 1$, the singularity is mapped to $z = \infty$, the potential goes to zero and we may expect portions of continuous spectrum. Clearly, the full knowledge of V is requested for all sensible predictions about the spectrum. The above Schroedinger equation is also the one to be considered in looking at generalisations of the RS scenario.

3.1 Supersymmetric and non-supersymmetric examples

We now briefly discuss few examples in the literature.

In [6], the class of non-supersymmetric solutions where the potential can be neglected in the IR have been discussed. They all have $A = 1/4$. It was argued that they may exhibit a variety of IR behaviours, from confinement to screening, depending on the values of the constants B_a. Since we can not follow the solution from UV to IR, it is difficult to make more meaningful claims. We do not even know whether these solutions correspond to deformations or to different vacua of the UV fixed point.

In the N=4 Coulomb branch solutions discussed in [9], A assumes various values. There is one solution with $A = 1/5$, one with $A = 1/4$ and all the other have $A > 1/4$. The UV behaviour can be unambiguously determined using the first-order equations (2.18). All these solutions correspond to different vacua (Coulomb branch) of the UV fixed point.

The supersymmetric massive flow from N=4 to N=1 SYM was discussed in [11]. It has $A = 1/2$. The qualitative properties of the solution agree with QFT expectations. They are briefly reviewed in the next section.

Due to the IR singularity, not all the previous solutions are expected to be *physical*. A possible criterion for selecting the *physical* solutions has been proposed in [18]. According to this criterion, the supergravity potential must be bounded above along the flow. This seems to eliminate all solutions with $A < 1/4$. The case $A = 1/5$ in the examples of N=4 Coulomb branch is indeed known to correspond to a singular 10d solution with negative tension branes. The criterion can be also understood as follows. It selects solutions for which the IR ambiguities noticed in [6] are absent. The action for a (canonically normalized) scalar $S = \int e^{4\phi}(\partial\lambda)^2$ predicts an IR contribution to the condensate

$$< O_\lambda >= \frac{\delta S}{\delta\lambda} \sim e^{4\phi}\partial\lambda \sim |y - y_0|^{4A-1} \qquad (3.3)$$

for all logarithmic flows. This IR ambiguities diverges when $A < 1/4$. The case $A = 1/4$ is borderline. It is possible that, as noticed in [18], only the $A = 1/4$ solutions representing vacua have a *physical* interpretation.

3.2 The flow to N=1 SYM

We now briefly discuss the example of a holographic RG flow from N=4 SYM to pure N=1 SYM in the IR [11][9]. It is a supersymmetric analytic solution of the supergravity equations of motion for the two scalar fields m and σ. m represents the diagonal supersymmetric mass term for the three chiral fields of N=4 SYM and σ the N=1 gaugino condensate. The solution depends on two parameters C_1 and C_2, determining the position y_0 where the two scalar fields m and σ, respectively, become singular. For $C_2 \leq 3C_1$, m becomes singular first at $y = C_1$ with metric

$$ds^2 = dy^2 + |y - C_1|dx^\mu dx_\mu. \qquad (3.4)$$

corresponding to $A = 1/2$. At this point, σ is still finite and its magnitude determines the gaugino condensate. We can study the UV behaviour and check that m corresponds to a true deformation of the N=4 Lagrangians while σ corresponds to a condensate. The interpretation of the solution is therefore the following: upon perturbation with a mass term for the three chiral fields, the N=4 SYM theory flows in the IR to pure N=1 SYM in a vacuum with a non-zero gaugino condensate.

For $C_2 > 3C_2$, σ diverges first with a value $A = 1/6$ which does not satisfy the criterion in [18]. For this and other reasons, we regard these solutions as *unphysical*.

Despite the presence of a singularity that invalidates the supergravity approximation in the IR, the qualitative properties of the solution agree with the QFT expectations: quarks confine, monopoles are screened, and there is a gaugino condensate [11]. In supergravity, we have two independent parameters C_1 and C_2. We have a chirally-symmetric vacuum and a continuous degeneracy of vacua with arbitrary small condensate. We certainly expect that the correct treatment of the singularity and its resolution in string theory fixes the relation between C_1 and C_2 in agreement with field theory expectations. Strong coupling QFT results for N=1 SYM have been recently obtained and differ considerably from the weak coupling ones [43]. At weak coupling, the spontaneous breaking of the chiral symmetry Z_N gives N vacua that only differ for the phase of the gaugino condensate $< \lambda\lambda >\sim e^{2\pi ik/N}\Lambda^3_{N=1}$. In the large N limit, we obtain a *circle* of vacua. The magnitude of the gaugino condensate is fixed in terms of the SYM scale $\Lambda_{N=1} \sim me^{-1/3Ng^2}$. At strong coupling instead, it was shown in [43] that there is, at least for $\theta = 0$, a distribution of vacua with condensate $< \lambda\lambda >\sim m^3x^3/j^2$, $j = 1, 2, ...$ with zero phase. The weakly coupled *circle* is lost, the condensate magnitude is not fixed and the vacua have an accumulation point at the origin (zero condensate). We may identify the solution with $C_2 = 3C_1$ with the $j = 1$ vacuum in [43] and the other solutions with $C_2 < 3C_1$ with the $j \neq 1$ vacua. To see how the continuum of vacua in supergravity is reduced to a discrete numerable set, we should understand how to include string corrections in our computation. Notice that the solution with $\sigma = 0$, which is not appealing on the ground of weak coupling intuition, could be nevertheless used as a (reasonable ?) approximation for the many vacua with small condensate at strong coupling.

[9] This example is discussed in details in M. Petrini's talk at this conference.

It was proposed in [18] to fix the relation between C_1 and C_2 by considering the finite temperature version of our solution, where conditions to be imposed at the horizon fix the parameters. One finds $C_2 = 3C_1$. This is the only special value for our parameters, since, exactly for $C_2 = 3C_1$, the two scalars m and σ diverge at the same point in y. In SYM the breaking of supersymmetry will select the vacuum with minimal energy. At weak coupling, where all the vacua have a condensate with the same magnitude, this procedure should give us also the value of the N=1 condensate. At strong coupling, with condensates of almost arbitrary magnitude, this would give information at most about one particular vacuum ($j = 1$?).

3.3 The fate of the singularity

The knowledge of the full 10 dimensional solution would greatly help in understanding the properties of the RG flow and in studying possible resolutions of the singularity. It may even happen that the singularity is an artifact of the dimensional reduction and disappears in $10d$. This happens, for example, in the case of the Coulomb branch of N=4 SYM [9], where the 10 dimensional background is just a regular continuous distribution of D3-branes. However, even in this context, some other equally nice[10] $5d$ solutions have a lift to still singular $10d$ solutions, representing D3-branes with negative tension. The complete ansatz for the $10d$ lifting of $5d$ solutions is known only for a subset of scalars, the $\underline{20}$, coming from the KK modes of the internal metric. This is sufficient to lift all solutions representing the Coulomb branch [9], but it is not of help with the N=1 SYM solution [11], where the modes $\underline{10}$ from the anti-symmetric tensors are excited. A ten dimensional interpretation of the N=1 SYM solution in terms of a background with also D5-branes has been proposed in [42].

Finally, we mention that a mechanism for resolving singularities in distributions of branes which may help, after the $10d$ lifting, has been proposed in [21].

[10] But not satisfying the criterion in [18].

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