Solar neutrinos: a VEP laboratory

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Abstract: In this work we use the solar neutrino fluxes as a laboratory to study a possible violation of the equivalence principle (VEP). Using an extended parametrized post-Newtonian formulation, which includes flavor dependent couplings of neutrinos to the gravitational interaction, we analyze the current data on the rates, energy spectrum,and seasonal variations. We find that there are two solutions which can explain the data; one of them corresponds to VEP vacuum oscillations, and the other to a VEP-MSW resonance. Both solutions imply restrictions on the VEP effect whic are several orders of magnitud below the present limits.

1. Introduction

The most famous tests of the weak equivalence principle are the experiments of the Eötvös type [1, 2], which have verified that gravity accelerates macroscopic bodies at the same rate to an accuracy of one part in 10^{12}. The comparison of inertial and passive gravitational masses is inappropriate for particles like photons and neutrinos, since their motion in a gravitational field is not correctly described by Newtonian dynamics [3]. In this case a suitable context is provided by the parametrized post-Newtonian (PPN) formalism [2]. The accuracy of the equivalence principle may then be characterized by limits on the differences of the PPN parameters of a given gravitational theory for different types of particles.

On the other hand, a widely accepted explanation for the discrepancy between the predicted and the observed solar-neutrino fluxes is based on the assumption that nondegenerate massive neutrinos do undergo flavor oscillations, either in vacuum or within the Sun [4]. A less orthodox mechanism for neutrino oscillations, which does not need neutrinos to have a mass, was proposed several years ago [5] and requires the coupling of neutrinos to gravity to be flavor dependent, i.e., a violation of the equivalence principle (VEP) in the neutrino sector. Some phenomenological consequences of this mechanism have been investigated in a number of papers[6, 7].

In this work, we reexamine the possibility of the VEP mechanism as a solution of the solar neutrino problem within the framework of a generalized PPN formalism, with a different set of parameters for each flavor [8]. We show that such a solution is possible both for vacuum and matter oscillations[9]. In addition, the analysis gives us better limits for possible violations of the equivalence principle in the neutrino sector.

2. VEP oscillations

The linearized Dirac equation for a massless neutrino in a static gravitational field reads[10]:

$$i\partial_0 \Psi_\nu = H\Psi_\nu \,, \qquad (2.1)$$

with the Hamiltonian given by

$$H = -i\gamma_0\gamma_i[(1-\frac{1}{2}h^{00})\partial_i - \frac{1}{2}h_{ij}\partial_j] - ih_{0i}\partial_i \,, \qquad (2.2)$$

where the $h^{\mu\nu}$ fields issued from the linealization of the metric, $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$. In writting Eq. (2.1) we have neglect those terms with spatial derivatives of the gravitational potentials,

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which is justified for astrophysical systems. The main effects of the neglected terms are independent of the equivalence principle violation, and have been already analyzed in Ref. [10].

In the solar system there are several sources for the gravitational field, but the dominant contribution is given by the Great Attractor (GA). Its potential U can be approximated by a constant of the order 10^{-5}[11]. The sources can be considered as virialized, that is $\frac{M}{R} \sim \mathrm{w}^2$, where M, R, and w represent estimations of the order of magnitude of the mass, distance and characteristic velocity of their mass distributions. Therefore, we can work within the PPN formalism. Following this approach the dominant contributions to the perturbations of the Minkowskian metric are given by[8, 9]:

$$h_{oo} = 2\gamma' U + \mathcal{O}(\mathrm{w}^4) , \tag{2.3}$$

$$h_{oi} = 0 , \tag{2.4}$$

$$h_{ij} = 2\gamma U \delta_{ij} + \Gamma U_{ij} + \mathcal{O}(\mathrm{w}^4) , \tag{2.5}$$

where the potentials are

$$U = \int \frac{\rho(\mathbf{r}')\, d^3r'}{\mid \mathbf{r} - \mathbf{r}' \mid} , \tag{2.6}$$

$$U_{ij} = \int \frac{\rho(\mathbf{r}')(r_i - r'_i)(r_j - r'_j)\, d^3r'}{\mid \mathbf{r} - \mathbf{r}' \mid^3} . \tag{2.7}$$

Here γ', γ, and Γ are adimensional parameters of the PPN expansion up to order $w^3$and $\rho(\mathbf{r})$ is the mass density of the source of the gravitational field. We are using a system of unities where $G = \hbar = c = 1$.

If we take the z-axis along the direction determined by the solar system and the gravitational source, we then have $U_{zz} \sim U$. The components U_{xx}, U_{yy} and U_{xy} are proportional to $(\Delta\theta)^2 U$, where $\Delta\theta$ is the angular size of the source, while U_{xz} and U_{yz} are of the order $\Delta\theta\, U$. Considering that the GA is a rather extended object with an angular size of the order of 10^{-1}, we see that in the case of the sun there are only three relevant types of U_{ji} contributions: those coming from our galaxy, which are of the order of 10^{-6}, a longitudinal component from the GA, of order of $U_{zz} \simeq U \simeq 10^{-5}$, and transverse-longitudinal components also produced by the GA, of the same order as the galactic contributions, $U_{xz} \simeq U_{yz} \simeq 10^{-6}$.

For simplicity, in what follows we consider that there are only two neutrino flavors, ν_e and ν_μ. In our VEP scenario they are assumed to be linear superpositions of the gravitational eigenstates ν_1^g and ν_2^g, with a mixing angle θ_g. Each gravitational eigenstate is characterized by a different set of PPN parameters, $\{\gamma^a, \gamma'^a, \Gamma^a\}$ $(a = 1, 2)$. This leads to different dispersion relations for the ν_a^g. If the initial state corresponds to a pure electron neutrino, for a constant gravitational field the survival probability after traveling a distance $L = t - t_0$ from the production point t_0 is

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_g \, \sin^2 \frac{\pi L}{\lambda_g} , \tag{2.8}$$

with $\lambda_g = 2\pi/|\Delta_0|$. Here,

$$\Delta_0 = -E_\nu \left[(\delta\gamma' + \delta\gamma) U \; + \delta\Gamma\, U_{ij} \frac{p_i p_j}{E^2} \right] , \tag{2.9}$$

where $E_\nu \simeq p$ is the neutrino energy, $\delta\gamma = \gamma^2 - \gamma^1, \delta\gamma' = \gamma'^2 - \gamma'^1$, and $\delta\Gamma = \Gamma^2 - \Gamma^1$. In contrast to the ordinary vacuum oscillations induced by a mass difference, where $\lambda_m = 4\pi E/\delta m^2$, the effect we are considering here has an oscillation length that goes with E_ν^{-1}. This leads to observable distinctions between both mechanisms and makes the gravitational induced oscillations suitable to be observed with higher energy neutrinos. Note that even though the overall sign of the gravitational potential is irrelevant for oscillations, the relative signs among differences of the PPN parameters are very significant. If we assume that these differences are all of the same order, then the most important directional effect would be given by the quadrupolar contribution corresponding to U_{zz}. This effect could be of the order of the dipolar one originated by the elliptical orbit of the Earth, but the latter only depends on the eccentricity of the orbit , whereas the gravitational one depends on the energy of the neutrinos and their direction with respect to the Great Attractor. Therefore, in principle, the effects could be discriminated.

Taking into account the dominant contributions due to U and $U_{zz} \simeq U$, the coefficient Δ_0 can be written as follows

$$\Delta_0 = -E\, U \delta\bar{\gamma} \left[1 + \left(\cos^2 (\alpha - A) - \frac{1}{2} \right) \delta \right] , \tag{2.10}$$

where $\delta\bar{\gamma} = (\delta\gamma + \delta\gamma')/(1-\delta/2)$ and $\delta = \delta\Gamma\cos^2 D\ /\delta\bar{\gamma}$. We take δ positive, because (δ, A) is equivalent to ($-\delta$, $A + \pi/2$). Here α is the right ascension of the Sun, and A and D are the right ascension and declination of the GA in ecliptic coordinates. The second term in Δ_0 arises from the quadrupolar potential of the gravitational source and generates a seasonal dependence in the oscillation wavelength, as first discussed in Ref. [8]. This effect went unnoticed in previous work on the subject [7], where only the contribution coming from the Newtonian gravitational potential was considered. As we see, meditions of the flux of solar neutrinos can give us information about the following combinations of parameters: $\delta\gamma + \delta\gamma', \delta\Gamma\cos^2 D$, and A.

When neutrinos propagate through matter, under favorable conditions an enhancement of flavor transformations may occur. This is the MSW effect whose consequences in astrophysics and cosmology has been extensively investigated. In normal matter, as in the case of the Sun, there exists a resonant flavor conversion when $\sqrt{2}G_F\ N_e(t_R) = \Delta_0\ \cos 2\theta_g$, and the mixing in matter θ_m is maximal. G_F is the Fermi constant and $N_e(t)$ denotes the electron number density along the neutrino path. The efficiency of the conversion mechanism depends on the adiabaticity of the process. For a constant gravitational field, the average probability for a ν_e produced in the Sun to reach the Earth reads

$$\bar{P}(\nu_e \to \nu_e) = \frac{1}{2} + \frac{1}{2}(1 - 2P_c)\cos 2\theta_m^0 \cos 2\theta_g \ , \tag{2.11}$$

with $\theta_m^0 = \theta_m(t_0)$. The function P_c represents the probability of transition between the instantaneous energy eigenstates. It embodies the total correction to the adiabatic result for $\bar{P}_{\nu_e}$, which corresponds to $P_c = 0$.

Except for regions close to the center and the surface, the electron density in the Sun is well approximated by an exponential profile [12]. For such profile [13]

$$P_c = \frac{\exp\left[\pi\kappa\left(\frac{\cos 2\theta_g}{1-\cos 2\theta_g}\right)\right] - 1}{\exp\left[\pi\kappa\left(\frac{2\cos 2\theta_g}{\sin^2 2\theta_g}\right)\right] - 1} \ , \tag{2.12}$$

where the adiabatic parameter κ is

$$\kappa = |\Delta_0| \frac{\sin 2\theta_g \tan 2\theta_g}{\frac{1}{N_e(t_R)}\left|\frac{dN_e(t)}{dt}\right|_{t_R}} \ . \tag{2.13}$$

Non adiabatic effects become important when $\kappa \lesssim 1$, provided that the neutrinos go through a resonance. If $b(t_0) < \Delta_0 \cos 2\theta$, level crossing cannot occur, $P_c = 0$ and neutrino propagation will be always adiabatic. An effective way to account for this situation is to multiply the expression of Eq. (2.12) by the step function $\Theta(b(t_0) - \Delta_0\cos 2\theta)$, so that the transition probability vanishes when neutrinos are produced below the resonance.

3. Analysis with solar neutrino data

In the presence of neutrino oscillations, the capture rate for the radiochemical experiments, such as ^{37}Cl and ^{71}Ga, is given by:

$$R = g(t)\sum_k \int_0^\infty dE_\nu\ \Phi_k(E_\nu)\,\langle P_{\nu_e}\rangle\,\sigma(E_\nu)\ , \tag{3.1}$$

where $\sigma(E_\nu)$ is the cross section for neutrino capture and $\Phi_k(E_\nu)$ is the k-component of neutrino flux spectrum. The geometrical factor $g(t)$ is due to the Earth's orbit eccentricity and $\langle P_{\nu_e}\rangle$ is the survival probability averaged over the production regions for the different neutrino sources.

For neutrino-electron scattering experiments, such as Superkamiokande (SK), the solar neutrino induced event rate can be written:

$$\begin{aligned} R = g(t)\int_{-\infty}^{\infty} dE_e\ \Xi(E_e, E)\int_{E_{\nu\,\min}}^{\infty} dE_\nu\ \Phi(E_\nu) \\ \times\left[\langle P_{\nu_e}\rangle \frac{d\sigma_e(E_\nu, E_e)}{dE_e}\right. \\ \left. + (1 - \langle P_{\nu_e}\rangle)\frac{d\sigma_\mu(E_\nu, E_e)}{dE_e}\right]\ , \end{aligned} \tag{3.2}$$

where E_ν is the energy of the incident neutrino, E_e is the electron kinetic energy, and $\Phi(E_\nu)$ gives the flux spectrum of ^8B neutrinos. The function $\Xi(E_e, E)$ characterizes the SK efficiency to measure the energy of the scattered electrons. The differential cross section $d\sigma_\ell/dE_e$ ($\ell = e, \mu$) for the $\nu_\ell - e$ elastic scattering can be calculated from the standard electroweak theory.

The VEP mechanism begins to be significant when half of an oscillation is about equal to the Sun-Earth distance. For a 10 MeV neutrino this correspondsto $|U\delta\bar{\gamma}| \approx 10^{-25}$, in which case we have pure vacuum oscillations. For larger values of $|U\delta\bar{\gamma}|$ the oscillation wavelength shortens, and when it becomes smaller than the solar radius the effect of the background matter turns out to be relevant through the MSW effect. To compute the event rate we follow in general the scheme of Ref. [4]. The matter effects on the calculation of $\langle P_{\nu_e}\rangle$ were incorporated by applying the analytic formula given by Eqs. (2.11) and (2.12). The electron density is given in Refs. [12] and [14], while the neutrino fluxes were obtained from Refs. [12] and [15].

We find three regions in the $|U\delta\bar{\gamma}|$-$\sin^2 2\theta$ parameter space for the VEP induced oscillations that are compatible with the measured total rates in all the experiments. Two of them correspond to MSW-enhanced VEP oscillations, whereas the third one is associated to vacuum VEP oscillations. To identify these regions we use a standard χ^2 analysis of the data from all the solar neutrino experiments, taking into account both the experimental and theoretical errors. For the MSW VEP oscillations the allowed regions at 99% c.l. are: a small mixing angle region, with $3.2\times10^{-3} \lesssim \sin^2(2\theta_g) \lesssim 5.7\times10^{-3}$ and $|U\delta\bar{\gamma}| \simeq 3.2\times10^{-19}$; and a large mixing angle region, with $0.6 \lesssim \sin^2(2\theta_g) \lesssim 1$ and $10^{-22} \lesssim |U\delta\bar{\gamma}| \lesssim 4\times10^{-21}$. The best fit for the small mixing solution is obtained with $\sin^2(2\theta_g) = 4\times10^{-3}$, whereas in the case of the large mixing it occurs at $|U\delta\bar{\gamma}| = 1.58\times10^{-22}$ and $\sin^2(2\theta_g) = 0.87$. At 94 c.l. the small mixing region disappears, and only the large mixing region remains.

The MSW VEP solutions are consistent with those already found using the Newtonian approximation for the gravitational interaction[7]. However, contrary to what is argued there, our analysis reveals that there is another VEP solution which corresponds to long-wavelength vacuum oscillations. At 99% c.l. the main region is bound by $0.75 \lesssim \sin^2(2\theta_g) \lesssim 1$ and $10^{-24} \lesssim |U\delta\bar{\gamma}| \lesssim 10^{-22}$. The values of the parameters for the best-fit point are $|U\delta\bar{\gamma}| = 1.82\times10^{-24}$ and $\sin^2(2\theta_g) = 1$. Besides the total rate, the SK collaboration has provided spectral information on the ^{8}B solar neutrinos [16]. The small-angle MSW solution is excluded by the energy spectrum at 99% c.l., while both the vacuum solution and the large-angle MSW solution are allowed at 90% c.l.

The eccentricity of the Earth's orbit produces a geometrical 7% variation of the neutrino flux. Due to the dependence of $\langle P_{\nu_e}\rangle$ on distance, an anomalous additional effect can be caused by the presence of the usual vacuum oscillations between massive neutrinos. Both effects are characterized by a one year period. Indications of a seasonal variation in the neutrino flux from the Sun has already been seen in the Gallex and Homestake experiments [17]. In Ref.[16], SK has also presented preliminary results that slightly favor a seasonal variation of the solar neutrino flux for $E_e > 11.5$ MeV, in addition to the geometric variation. Within the present VEP oscillation scheme a non-geometrical seasonal variation of the flux is caused by the presence of the anisotropic term proportional to $\delta\Gamma$ in Δ_0, which produces a six month period variation. As a consequence, in contrast with the usual mass mechanism, the effect should be observed even in the case of MSW transformations.

From the whole set of data from Sage, Gallex, Homestake and SK, and using a χ^2 analysis, we have identified the best fit solutions, taking for $\delta\bar{\gamma}$ and θ_g the values obtained from the mean solar neutrino flux. The best fit solution for the VEP vacuum oscillations is $U\delta\Gamma\cos^2 D \sim 1.1\times10^{-24}$ and $\delta A \sim 0.3435$, while for the VEP-MSW solution we obtain $U\delta\Gamma\cos^2 D \sim -2.68\times10^{-22}$ and $\delta A \sim -0.1162$. δA denotes the difference between the perihelion right ascension of the Sun and A (modulo π). For both solutions, taking into account the poor angular resolution of the data and the uncertainty concerning the position of the GA, δA is consistent with its position. If this is effectively the case, we can take $\cos D \sim 1$, and therefore conclude that the VEP effects for the monopolar and cuadrupolar components of the gravitational field are of the same order ($\delta\bar{\gamma} \sim \delta\Gamma$).

4. Last remarks

More accurate data are necessary to properly establish the viability of the VEP mechanism as an adequate explanation of the solar neutrino deficit. Nevertheless, the present analysis is sufficient to set new boundaries on the PPN coefficients that parametrize the violation of the equivalence principle. Assuming that the GA is the main gravitational source, we have $\delta\bar{\gamma} \sim \delta\Gamma \sim 10^{-19}$ for the VEP vacuum oscillations, while in the case of the VEP-MSW solution we have $\delta\bar{\gamma} \sim -\delta\Gamma \sim 10^{-16}$. Both set of values are several orders of magnitude below the already stated limits. An improved time resolution in the measurements is required to establish the existence of a six-month period variation in the ^8B neutrino flux, which is a signature of the VEP mechanism and makes a clear difference with other possible solutions, such as the standard vacuum oscillations of massive neutrinos [18].

5. Acknowledgments

This work was partially supported by CONICET-Argentina and CONACYT-México, and by the Universidad Nacional Autónoma de México under Grants DGAPA-IN117198 and DGAPA-IN100 397.

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