

Trilinear Neutral Gauge Boson Couplings.

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ABSTRACT: We list all possible operators up to dimension 8 which induce the triple neutral gauge boson couplings $ZZ\gamma$, $Z\gamma\gamma$, and ZZZ within the electroweak linear (decoupling) effective Lagrangian approach. Attention is given to those that contribute to the ZZ and $Z\gamma$ production form factors.

1. Introduction

The present agreement between experiment and the standard model (SM) suggests that the energy scale associated with any new physics should be large compared to the electroweak scale $v = 246\text{GeV}$. In order to infer the existence of new particles through virtual effects, effective Lagrangian techniques have been used to study quantities that are forbidden or unobservably small within the SM [1, 2, 3]. In particular, it has been pointed out that the self-couplings of the electroweak gauge bosons constitute a sensitive probe of nonstandard interactions [4]. While experimental limits on possible anomalous $W^+W^-Z(\gamma)$ couplings have reached an accuracy of the few percent level in both hadronic [5] and leptonic [6] colliders the situation is not that good for possible trilinear neutral gauge boson (TNGB) couplings [7], which are absent in the SM at the tree level.

Our aim is to study TNGB couplings in the framework of the linearly realized $SU(2) \times U(1)$ effective Lagrangian approach. We will maintain both generality and gauge invariance and itemize all non-equivalent gauge-invariant operators of dimension 8.

Previous studies of TNGB couplings have pointed

out that these interactions are not generated by dimension six operators, but they have used a parametrization that is only $U(1)_{EM}$ gauge-invariant [8].

There is one motivation for studying TNGB interactions with as much generality as we can. Since persuasive theoretical arguments indicate that these couplings unlikely to be larger than one percent [9, 10], and the Large Hadron Collider (LHC) and the planned Next Linear Collider (NLC) are expected to achieve limits of order 10^{-3} - 10^{-5} on these couplings [4, 8], a first concern will be to understand in a systematical and model independent way how TNGB interactions are generated. These anomalous couplings arise at one loop in the SM as well as in the MSSM, and typical sizes are of order 10^{-4} [11, 12]. We would like to compare with the expected values from dim 8 operators.

2. Characterization of operators

We will find out all dimension 8 C-odd operators that generate one or more of the vertices ZZZ , $ZZ\gamma$ or $Z\gamma\gamma$. As it turns out, no dimension 6 operator gives rise to TNGB couplings.

Any $SU(2) \times U(1)$ gauge invariant operator is constructed out of the appropriate combinations of the following building blocks:

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Number	Operator
T.1	$W_{\mu\nu}^i$
T.2	$B_{\mu\nu}$
T.3	$\Phi^\dagger\Phi$
T.4	$\Phi^\dagger\sigma^i\Phi$
T.5	$\Phi^\dagger D_\mu\Phi$
T.6	$\Phi^\dagger\sigma^i D_\mu\Phi$
T.7	$\Phi^\dagger(D_\mu D_\nu + D_\nu D_\mu)\Phi$
T.8	$\Phi^\dagger\sigma^i(D_\mu D_\nu + D_\nu D_\mu)\Phi$

Besides the well known SM interactions, operators of dimension 6, 8 and higher can be built by Lorentz contraction of any number of the terms above. At the dimension 6 level there are a total of 12 independent purely bosonic operators, half of them generating trilinear $W^+W^-Z(\gamma)$ couplings but none of them generating any TNGB coupling [2]. Therefore, to generate this vertex one has to look into the dimension 8 level.

Because of Bose symmetry and the antisymmetry of $V_{\mu\nu} = -V_{\nu\mu}$, any lorentz contraction of the $W_{\mu\nu}^i$ and/or the $B_{\mu\nu}$ tensor fields with ordinary derivatives will not generate any TNGB coupling. It is thus necessary to include the Higgs doublet in our search. Moreover, we can already see that the terms T.3 and T.4 will not be useful; the reason is the following: suppose there was a dimension 8 operator that contained T.3, since this is a scalar we could just remove it without destroying the Lorentz nor the $SU(2) \times U(1)$ invariance. Then we would end up with a dimension 6 operator; but we know there are no dimension 6 operators that generate TNGB couplings.

We are now constrained to use the T.5 and T.6 terms along with the tensor fields T.1 and T.2 plus one ordinary derivative to get enough Lorentz indices to contract.

We thus find the following list of 10 operators, 5 CP odd and 5 CP even:

$$\begin{aligned}
\mathcal{O}_{BB1} &= i\partial^\lambda(T5)_\mu B_{\lambda\nu} B^{\mu\nu} + h.c. \\
\mathcal{O}_{BB2} &= i(T5)_\mu B_{\lambda\nu} \partial^\lambda B^{\mu\nu} + h.c. \\
\mathcal{O}_{WW} &= i\partial_\lambda(T5)^\mu W_{\mu\nu}^i W^{i\lambda\nu} + h.c. \\
\mathcal{O}_{WB1} &= i(T6)_\lambda^i W_{\mu\nu}^i \partial^\lambda B^{\mu\nu} + h.c. \\
\mathcal{O}_{WB2} &= i(T6)_\lambda W_{\mu\nu}^i \partial^\mu B^{\lambda\nu} + h.c.
\end{aligned}$$

$$\mathcal{O}_{B\tilde{B}1} = i\partial^\lambda(T5)_\mu B_{\lambda\nu} \tilde{B}^{\mu\nu} + h.c. \quad (2.1)$$

$$\mathcal{O}_{B\tilde{B}2} = i(T5)_\mu B_{\lambda\nu} \partial^\lambda \tilde{B}^{\mu\nu} + h.c.$$

$$\mathcal{O}_{\tilde{W}W} = i\partial_\lambda(T5)^\mu \tilde{W}_{\mu\nu}^i W^{i\lambda\nu} + h.c.$$

$$\mathcal{O}_{\tilde{W}B1} = i(T6)_\lambda \tilde{W}_{\mu\nu}^i \partial^\lambda B^{\mu\nu} + h.c.$$

$$\mathcal{O}_{W\tilde{B}2} = i(T6)_\lambda W_{\mu\nu}^i \partial^\mu \tilde{B}^{\lambda\nu} + h.c.$$

There is one more operator that generates a ZZZ coupling:

$$\Phi^\dagger(D_\mu D_\nu + D_\nu D_\mu)\Phi \partial^\mu \Phi^\dagger D^\nu \Phi + h.c.. \quad (2.2)$$

Another similar operator, with T.6 and T.8, could be written which turns out to be equivalent to this one. However, these operators only generate scalar couplings which cannot be measured in production processes.

The previous list of operators Eq. (2.1) is complete in the sense that any other dimension 8 operator that generates TNGB couplings will be related to them via partial integration or equations of motion. We have avoided operators with terms like $\partial^\nu B_{\mu\nu}$ that can be related to other dimension 8 operators with fermions using the eqs. of motion. We have also avoided operators with divergences like $\partial^\mu Z_\mu$ because these are known to be redundant as well [13]. Hence, in this work we are considering all nonredundant purely bosonic operators that can generate TNGB couplings at the lowest dimension possible in the linear $SU(2) \times U(1)_Y$ effective Lagrangian.

In conclusion, we have provided a list of 10 independent dimension 8 $SU(2) \times U(1)$ invariant operators that generate TNGB couplings. This itemization can now be used to perform a systematic study, in the context of a linearly realized effective Lagrangian, that includes effects on the low energy observables and LEP/SLC measurements as is done for the charged boson couplings[2, 15].

Five of the 10 operators: \mathcal{O}_{BB2} , \mathcal{O}_{WB1} , \mathcal{O}_{WB2} , $\mathcal{O}_{\tilde{W}W}$ and $\mathcal{O}_{B\tilde{B}1}$ generate scalar couplings and cannot be measured at the colliders. Their effects could only be probed, if at all, through virtual effects. The other five: \mathcal{O}_{WW} , \mathcal{O}_{BB1} , $\mathcal{O}_{B\tilde{B}2}$, $\mathcal{O}_{\tilde{W}B1}$ and $\mathcal{O}_{W\tilde{B}2}$ generate non-scalar couplings. We will now focus our attention on these. The effective dimension 8 Lagrangian to be considered is:

$$\mathcal{L}^{(8)} = \frac{a_1}{\Lambda^4} \mathcal{O}_{\widetilde{W}B1} + \frac{a_2}{\Lambda^4} \mathcal{O}_{W\widetilde{B}2} + \frac{a_3}{\Lambda^4} \mathcal{O}_{B\widetilde{B}2} + \frac{a^+}{\Lambda^4} \mathcal{O}^{(+)} + \frac{a^-}{\Lambda^4} \mathcal{O}^{(-)}. \quad (2.3)$$

Where we have defined

$$\mathcal{O}^{(\pm)} \equiv \mathcal{O}_{WW} \pm \mathcal{O}_{BB1} \quad (2.4)$$

for convenience: it turns out that $\mathcal{O}^{(+)}$ generates $\gamma\gamma Z$ and ZZZ couplings, whereas $\mathcal{O}^{(-)}$ generates $ZZ\gamma$ only. This Lagrangian provides the size of the h and f coefficients for new physics TNGB effects that come from purely bosonic operators:

$$\begin{aligned} h_1^\gamma &= \frac{-v^2 m_Z^2 a^+}{s_w c_w \Lambda^4} \\ h_1^Z &= \frac{-v^2 m_Z^2 a^-}{\Lambda^4} \\ h_3^\gamma &= \frac{v^2 m_Z^2}{\Lambda^4} \left(a_2 - \frac{c_w}{s_w} a_3 \right) \\ h_3^Z &= \frac{v^2 m_Z^2}{4s_w c_w \Lambda^4} \left((1 + c_w^2) a_1 + (c_w^2 - s_w^2) a_2 + 2s_w c_w a_3 \right) \\ f_4^\gamma &= \frac{v^2 m_Z^2 a^-}{\Lambda^4} \\ f_4^Z &= \frac{v^2 m_Z^2 a^+}{s_w c_w \Lambda^4} \\ f_5^\gamma &= \frac{v^2 m_Z^2}{4s_w c_w \Lambda^4} \left((1 + c_w^2) a_1 + (s_w^2 - c_w^2) a_2 - 2s_w c_w a_3 \right) \\ f_5^Z &= \frac{v^2 m_Z^2}{2\Lambda^4} \left(a_2 + \frac{c_w}{s_w} a_3 \right). \end{aligned} \quad (2.5)$$

It should be noted that there are two form factors h_2^V and h_4^V that do not appear at this (dim 8) level, but at the next higher (dim 10) level [4, 8].

3. Constraints from precision measurements.

As mentioned before, precision observable effects of trilinear couplings of the charged W^\pm bosons, $WW\gamma$ and WWZ have been studied in the effective Lagrangian approach for both the decoupling as well as the nondecoupling case[2,

4]. In particular, the anomalous magnetic moment δa_μ of the muon has been used to probe the effects of trilinear couplings[15]. For $WW\gamma(Z)$ couplings, dimension 6 operators in the linear case and dimension 4 operators in the nonlinear chiral Lagrangian appear. In the linear case, assuming a cut-off scale Λ of order 1 TeV the contributions of the dimension 6 operators turn out to be of the order of $(0.5 - 1.0) \times 10^{-9} \alpha$ with α being the dimensionless coefficient of the operator. Since the goal for experimental sensitivity is of the order of $\delta a_\mu \simeq 5 \times 10^{-10}$ [16] we conclude that α would have to be of the order of 1 to have any chance of being probed. Such value is highly unexpected because these dim 6 bosonic operators must come from at least one-loop-level effects in the underlying theory, and this means $\alpha \simeq g/16\pi^2$. (A dim 8 operator can arise at tree level, and its effects could then be similar or even higher[4].) Moreover, even in the case of a higher than expected value for α , the contribution of a bosonic operator to δa_μ is through loop diagrams, and it has to compete with the direct contribution of a fermionic operator like $\overline{\psi} \sigma^{\mu\nu} \psi B_{\mu\nu}$. Seeing the remarkable difficulties to probe decoupling physics effects in trilinear $WW\gamma(Z)$ vertices, we can already expect a no less pessimistic conclusion for TNGB couplings: that their detection is very hard if physics beyond the SM is of a decoupled nature.

Concerning bounds from electroweak precision observables (oblique corrections), it has been shown that the limits on the κ and λ form factors for $WW\gamma(Z)$ couplings are around 0.05 and 0.1 respectively [2]. These bounds are two orders of magnitude above possible effects from dim 6 operators (see Table (1)). The situation is probably the same for the neutral boson couplings.

The prospects of direct measurement of the TNGB form factors (Eq. (2.5)) have been studied in the literature[8]. The best resolution possible is achieved by the 14TeV LHC collider, for which coefficients like h_3 and f_4 can be probed down to 10^{-3} . These figures are still higher than the SM (and MSSM with light enough superpartners) values [12], which could be similar to the expected effects from dim 8 operators. These results are summarized in Table (1).

In conclusion, if new physics comes from a

decoupled theory whose effects below a cut-off scale Λ can be parameterized by higher dimension $SU(2) \times U(1)$ operators, it will hardly be seen through trilinear gauge boson couplings.

	SM	Eff. Lag.	obl. corr.
f_4^Z	-	10^{-4}	-
f_5^Z	1×10^{-4}	10^{-4}	-
f_4^γ	-	10^{-4}	-
f_5^γ	1×10^{-4}	10^{-4}	-
h_1^Z	-	10^{-4}	-
h_3^Z	1×10^{-4}	10^{-4}	-
h_1^γ	-	10^{-4}	-
h_3^γ	1×10^{-4}	10^{-4}	-
$\Delta\kappa^Z$	-	10^{-4}	0.06
$\Delta\kappa^\gamma$	0.008	10^{-4}	0.05
λ^Z	-	10^{-4}	0.1
λ^γ	0.002	10^{-4}	0.1

Table 1: SM value and New Physics estimate ($\Lambda = 1\text{TeV}$) for trilinear gauge boson couplings.

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