CP violation in SUSY

A. Masiero and O. Vives
SISSA, Via Beirut 2-4, 34013 Trieste, Italy
and
INFN, sez. di Trieste, Trieste, Italy
E-mail: masiero@sissa.it

Abstract: Supersymmetry exhibits new sources of CP violation. We discuss the implications of these new contributions to CP violation both in the K and B physics. We show that CP violation puts severe constraints on low energy SUSY, but it represents also a promising ground to look for signals of new physics.

1. CP Violation in SUSY

CP violation has major potentialities to exhibit manifestations of new physics beyond the standard model. Indeed, it is quite a general feature that new physics possesses new CP violating phases in addition to the Cabibbo–Kobayashi–Maskawa (CKM) phase ($\delta_{\text{CKM}}$) or, even in those cases where this does not occur, $\delta_{\text{CKM}}$ shows up in interactions of the new particles, hence with potential departures from the SM expectations. Moreover, although the SM is able to account for the observed CP violation in the kaon system, we cannot say that we have tested so far the SM predictions for CP violation. The detection of CP violation in $B$ physics will constitute a crucial test of the standard CKM picture within the SM. Again, on general grounds, we expect new physics to provide departures from the SM CKM scenario for CP violation in $B$ physics. A final remark on reasons that make us optimistic in having new physics playing a major role in CP violation concerns the matter– antimatter asymmetry in the universe. Starting from a baryon–antibaryon symmetric universe, the SM is unable to account for the observed baryon asymmetry. The presence of new CP–violating contributions when one goes beyond the SM looks crucial to produce an efficient mechanism for the generation of a satisfactory $\Delta B$ asymmetry.

The above considerations apply well to the new physics represented by low–energy super–symmetric extensions of the SM. Indeed, as we will see below, supersymmetry introduces CP violating phases in addition to $\delta_{\text{CKM}}$ and, even if one envisages particular situations where such extra–phases vanish, the phase $\delta_{\text{CKM}}$ itself leads to new CP–violating contributions in processes where SUSY particles are exchanged. CP violation in $B$ decays has all potentialities to exhibit departures from the SM CKM picture in low–energy SUSY extensions, although, as we will discuss, the detectability of such deviations strongly depends on the regions of the SUSY parameter space under consideration.

In any MSSM, at least two new “genuine” SUSY CP–violating phases are present. They originate from the SUSY parameters $\mu$, $M$, $A$ and $B$. The first of these parameters is the dimensionful coefficient of the $H_uH_d$ term of the superpotential. The remaining three parameters are present in the sector that softly breaks the $N=1$ global SUSY. $M$ denotes the common value of the gaugino masses, $A$ is the trilinear scalar coupling, while $B$ denotes the bilinear scalar coupling. In our notation, all these three parameters are dimensionful. The simplest way to see which combinations of the phases of these four parameters are physical is to notice that for vanishing values of $\mu$, $M$, $A$ and $B$ the theory possesses two additional symmetries $\mathbb{R}$. Indeed, letting $B$ and $\mu$ vanish, a $U(1)$ Peccei–Quinn symmetry originates, which in particular rotates $H_u$ and $H_d$. If
If $\varphi_A = \varphi_B = 0$, then the novelty of SUSY in CP violating contributions merely arises from the presence of the CKM phase in loops where SUSY particles run. The crucial point is that the usual GIM suppression, which plays a major role in evaluating $\varepsilon_K$ and $\varepsilon'/\varepsilon$ in the SM, in the MSSM case (or more exactly in the CMSSM) is replaced by a super–GIM cancellation which has the same “power” of suppression as the original GIM (see previous section). Again, also in the CMSSM, as it is the case in the SM, the smallness of $\varepsilon_K$ and $\varepsilon'/\varepsilon$ is guaranteed not by the smallness of $\delta_{\text{CKM}}$, but rather by the small CKM angles and/or small Yukawa couplings. By the same token, we do not expect any significant departure of the CMSSM from the SM predictions also concerning CP violation in $B$ physics. As a matter of fact, given the large lower bounds on squark and gluino masses, one expects relatively tiny contributions of the SUSY loops in $\varepsilon_K$ or $\varepsilon'/\varepsilon$ in comparison with the normal $W$ loops of the SM. Let us be more detailed on this point.

In the CMSSM, the gluino exchange contribution to FCNC is subleading with respect to chargino ($\chi^\pm$) and charged Higgs ($H^\pm$) exchanges. Hence, when dealing with CP violating FCNC processes in the CMSSM with $\varphi_A = \varphi_B = 0$, one can confine the analysis to $\chi^\pm$ and $H^\pm$ loops. If one takes all squarks to be degenerate in mass and heavier than $\sim 200$ GeV, then $\chi^\pm - \tilde{q}$ loops are obviously severely penalized with respect to the SM $W^+-q$ loops (remember that at the vertices the same CKM angles occur in both cases).

The only chance for the CMSSM to produce some sizeable departure from the SM situation in CP violation is in the particular region of the parameter space where one has light $\tilde{q}$, $\chi^\pm$ and/or $H^\pm$. The best candidate (indeed the only one unless tan $\beta \sim m_t/m_b$) for a light squark is the stop. Hence one can ask the following question: can the CMSSM present some novelties in CP–violating phenomena when we consider $\chi^\pm - \tilde{t}$ loops with light $\tilde{t}$, $\chi^\pm$ and/or $H^+$?

Several analyses in the literature tackle the above question or, to be more precise, the more general problem of the effect of light $\tilde{t}$ and $\chi^\pm$ on FCNC processes $\tilde{q}, \tilde{t}, \chi^\pm$. A first important observation concerns the relative sign of the $W^+-t$
loop with respect to the $\chi^+-\tilde{t}$ and $H^+-t$ contributions. As it is well known, the latter contribution always interferes positively with the SM one. Interestingly enough, in the region of the MSSM parameter space that we consider here, also the $\chi^+-\tilde{t}$ contribution interferes constructively with the SM contribution. The second point regards the composition of the lightest chargino, i.e. whether the gaugino or higgsino component prevails. This is crucial since the light stop is predominantly $\tilde{t}_R$ and, hence, if the lightest chargino is mainly a wino, it couples to $\tilde{t}_R$ mostly through the LR mixing in the stop sector. Consequently, a suppression in the contribution to box diagrams going as $\sin^4 \theta_{LR}$ is present ($\theta_{LR}$ denotes the mixing angle between the lighter and heavier stops). On the other hand, if the lightest chargino is predominantly a higgsino (i.e. $M_2 \gg \mu$ in the chargino mass matrix), then the $\chi^+-\text{lighter~}\tilde{t}$ contribution grows. In this case, contributions $\propto \theta_{LR}$ become negligible and, moreover, it can be shown that they are independent on the sign of $\mu$. A detailed study is provided in reference \cite{Fayet}. For instance, for $M_2/\mu = 10$, they find that the inclusion of the SUSY contribution to the box diagrams doubles the usual SM contribution for values of the lighter $\tilde{t}$ mass up to 100–120 GeV, using $\tan \beta = 1.8$, $M_{H^+} = 100$ TeV, $m_\chi = 90$ GeV and the mass of the heavier $\tilde{t}$ of 250 GeV. However, if $m_\chi$ is pushed up to 300 GeV, the $\chi^+-\tilde{t}$ loop yields a contribution which is roughly 3 times less than in the case $m_\chi = 90$ GeV, hence leading to negligible departures from the SM expectation. In the cases where the SUSY contributions are sizeable, one obtains relevant restrictions on the $\rho$ and $\eta$ parameters of the CKM matrix by making a fit of the parameters $A$, $\rho$ and $\eta$ of the CKM matrix and of the total loop contribution to the experimental values of $\varepsilon_K$ and $\Delta M_{B_d}$. For instance, in the above mentioned case in which the SUSY loop contribution equals the SM $W^+-t$ loop, hence giving a total loop contribution which is twice as large as in the pure SM case, combining the $\varepsilon_K$ and $\Delta M_{B_d}$ constraints leads to a region in the $\rho$-$\eta$ plane with $0.15 < \rho < 0.40$ and $0.18 < \eta < 0.32$, excluding negative values of $\rho$.

In conclusion, the situation concerning CP violation in the MSSM case with $\varphi_A = \varphi_B = 0$ and exact universality in the soft-breaking sector can be summarized in the following way: the MSSM does not lead to any significant deviation from the SM expectation for CP-violating phenomena as $d_K$, $\varepsilon_K$, $\varepsilon'/\varepsilon$ and CP violation in $B$ physics; the only exception to this statement concerns a small portion of the MSSM parameter space where a very light $\tilde{t}$ ($m_{\tilde{t}} < 100$ GeV) and $\chi^+$ ($m_\chi \sim 90$ GeV) are present. In this latter particular situation, sizeable SUSY contributions to $\varepsilon_K$ are possible and, consequently, major restrictions in the $\rho$-$\eta$ plane can be inferred. Obviously, CP violation in $B$ physics becomes a crucial test for this MSSM case with very light $\tilde{t}$ and $\chi^+$. Interestingly enough, such low values of SUSY masses are at the border of the detectability region at LEP II.

In next Section, we will move to the case where, still keeping the minimality of the model, we switch on the new CP violating phases. Later on we will give up also the strict minimality related to the absence of new flavor structure in the SUSY breaking sector and we will see that, in those more general contexts, we can expect SUSY to significantly depart from the SM predictions in CP violating phenomena.

### 2. Flavor Blind SUSY Breaking and CP Violation

We have seen in the previous section that in any MSSM there are additional phases which can cause deviations from the predictions of the SM in CP violation experiments. In fact, in the CMSSM, there are already two new phases present, Eq. (1.2), and for most of the MSSM parameter space, the experimental bounds on the electric dipole moments (EDM) of the electron and neutron constrain these phases to be at most $O(10^{-2})$. However, in the last few years, the possibility of having non-zero SUSY phases has again attracted a great deal of attention. Several new mechanisms have been proposed to suppress supersymmetric contributions to EDMs below the experimental bounds while allowing SUSY phases $O(1)$. Methods of suppressing the EDMs consist of cancellation of various SUSY contributions among themselves \cite{Masiero}, non-universality of the soft breaking
parameters at the unification scale $\frac{1}{5} M_{P}$ and approximately degenerate heavy sfermions for the first two generations $[12]$. In the presence of one of these mechanisms, large supersymmetric phases are naturally expected and EDMs should be generally close to the experimental bounds.  

In this section we will study the effects of these phases in CP violation observables as $\varepsilon_K$, $\varepsilon'/\varepsilon$ and $B^0$ CP asymmetries. Following our work of ref. $[13]$ that the presence of large susy phases is not enough to produce sizeable supersymmetric contributions to these observables. In fact, in the absence of the CKM phase, a general MSSM with all possible phases in the soft-breaking terms, but no new flavor structure beyond the usual Yukawa matrices, can never give a sizeable contribution to $\varepsilon_K$, $\varepsilon'/\varepsilon$ or hadronic $B^0$ CP asymmetries. However, we will see in the next section, that as soon as one introduces some new flavor structure in the soft Susy-breaking sector, even if the CP violating phases are flavor independent, it is indeed possible to get sizeable CP contribution for large Susy phases and $\delta_{CKM} = 0$. Then, we can rephrase our sentence above in a different way: A new result in hadronic $B^0$ CP asymmetries in the framework of supersymmetry would be a direct proof of the existence of a completely new flavor structure in the soft-breaking terms. This means that $B$--factories will probe the flavor structure of the supersymmetry soft-breaking terms even before the direct discovery of the supersymmetric partners $[^4]$. 

3. CP Violation in the presence of new Flavor Structures

In section $^{21}$ we have shown that CP violation effects are always small in models with flavor blind soft--breaking terms. However, as soon as one introduces some new flavor structure in the soft breaking sector, it is indeed possible to get sizeable CP contribution for large Susy phases and $\delta_{CKM} = 0$ $\frac{1}{5} M_{P}$ $\frac{1}{5} M_{P}$ $\frac{1}{5} M_{P}$. To show this, we will mainly concentrate in new supersymmetric contributions to $\varepsilon'/\varepsilon$.

In the CMSSM, the SUSY contribution to $\varepsilon'/\varepsilon$ is small $\frac{1}{5} M_{P}$ $\frac{1}{5} M_{P}$ $\frac{1}{5} M_{P}$. However in a MSSM with a more general framework of flavor structure it is relatively easy to obtain larger SUSY effects to $\varepsilon'/\varepsilon$. In ref. $[^{26}]$ it was shown that such large SUSY contributions arise once one assumes that: i) hierarchical quark Yukawa matrices are protected by flavor symmetry, ii) a generic dependence of Yukawa matrices on Polonyi/moduli fields is present (as expected in many supergravity/superstring theories), iii) the Cabibbo rotation originates from the down–sector and iv) the phases are of order unity. In fact, in $[^{26}]$, it was illustrated how the observed $\varepsilon'/\varepsilon$ could be mostly or entirely due to the SUSY contribution.

The universality of the breaking is a strong assumption and is known not to be true in many supergravity and string inspired models $[^{21}]$. In these models, we expect at least some non–universality in the squark mass matrices or trilinear terms at the supersymmetry breaking scale. Hence, sizeable flavor--off-diagonal entries will appear in the squark mass matrices. In this regard, gluino contributions to $\varepsilon'/\varepsilon$ are especially sensitive to $(\delta_{12})_{LR}$; even $|\text{Im}(\delta_{12})_{LR}| \sim 10^{-5}$ gives a significant contribution to $\varepsilon'/\varepsilon$ while keeping the contributions from this MI to $\Delta m_K$ and $\varepsilon_K$ well below the phenomenological bounds. The situation is the opposite for $L$--$L$ and $R$--$R$ mass insertions; the stringent bounds on $(\delta_{12})_{LL}$ and $(\delta_{12})_{RR}$ from $\Delta m_K$ and $\varepsilon_K$ prevent them to contribute significantly to $\varepsilon'/\varepsilon$.

The LR squark mass matrix has the same flavor structure as the fermion Yukawa matrix and both, in fact, originate from the superpotential couplings. It may be appealing to invoke the presence of an underlying flavor symmetry restricting the form of the Yukawa matrices to explain their hierarchical forms. Then, the LR mass matrix is expected to have a very similar form as the Yukawa matrix. Indeed, we expect the components of the LR mass matrix to be roughly the SUSY breaking scale (e.g., the gravitino mass) times the corresponding component of the quark mass matrix. However, there is no reason for them to be simultaneously diagonalizable based on this general argument. To make an order of magnitude estimate, we take the down quark mass matrix for the first and second gen-

\[1\] In a more general (and maybe more natural) MSSM there are many other CP violating phases $[^{21}]$ that contribute to CP violating observables.
erations to be (following our assumption iii),
\[ Y^{d_{U1}} \simeq \begin{pmatrix} m_d & m_s V_{us} \\ m_s & m_s \end{pmatrix}, \]
where the (2,1) element is unknown due to our lack of knowledge on the mixings among right-handed quarks (if we neglect small terms \( m_d V_{cd} \)). Based on the general considerations on the LR mass matrix above, we expect
\[ m^2_{LR} \simeq m_{3/2} \begin{pmatrix} a m_d & b m_s V_{us} \\ c m_s & m_s \end{pmatrix}, \]
where \( a, b, c \) are constants of order unity. Unless \( a = b = c \) exactly, \( M_d \) and \( m^2_{LR} \) are not simultaneously diagonalizable and we find
\[ (\delta^2_{12}) \simeq \frac{m_{3A} m_{V_{us}}}{m_q^2} = 2 \times 10^{-5} \left( \frac{m_s(M_{Pl})}{50 \text{ MeV}} \right) \times \left( \frac{m_{3/2}}{m_q} \right) \left( \frac{500 \text{ GeV}}{m_q} \right). \]
It turns out that, following the simplest implementation along the lines of the above described idea, the amount of flavor changing LR mass insertion in the s and d–squark propagator results to roughly saturate the bound from \( \varepsilon'/\varepsilon \) if a SUSY phase of order unity is present.

This line of work has received a great deal of attention in recent times, after the last experimental measurements of \( \varepsilon'/\varepsilon \) in KTeV and NA31 [21],[22]. The effects of non–universal \( A \) terms in CP violation experiments were previously analyzed by Abel and Frere [21] and after this new measurement discussed in many different works [12]. In the following we show a complete realization of the above Masiero–Murayama (MM) mechanism from a Type I string–derived model recently presented by one of the authors [23].

### 3.1 Type I string model and \( \varepsilon'/\varepsilon \)

In first place we explain our starting model, which is based on type I string models. Our purpose is to study explicitly CP violation effects in models with non–universal gaugino masses and \( A \)–terms. Type I models can realize such initial conditions. These models contain nine–branes and three types of five–branes \( (5_a, a = 1, 2, 3) \). Here we assume that the gauge group \( SU(3) \times U(1)_Y \) is on a 9–brane and the gauge group \( SU(2) \) on the 51–brane like in Ref. [24],[25], in order to get non–universal gaugino masses between \( SU(3) \) and \( SU(2) \). We call these branes the \( SU(3) \)–brane and the \( SU(2) \)–brane, respectively.

Chiral matter fields correspond to open strings spanning between branes. Thus, they must be assigned accordingly to their quantum numbers. For example, the chiral field corresponding to the open string between the \( SU(3) \) and \( SU(2) \) branes has non–trivial representations under both \( SU(3) \) and \( SU(2) \), while the chiral field corresponding to the open string, which starts and ends on the \( SU(3) \)–brane, should be an \( SU(2) \)–singlet.

There is only one type of the open string that spans between the 9 and 5–branes, that we denote as the \( C^{5_9}_{1} \). However, there are three types of open strings which start and end on the 9–brane, that is, the \( C^{9}_{i} \) sectors \( (i = 1, 2, 3) \), corresponding to the \( i \)–th complex compact dimension among the three complex dimensions. If we assign the three families to the different \( C^{9}_{i} \) sectors we obtain non–universality in the right–handed sector. Notice that, in this model, we can not derive non–universality for the squark doublets, i.e. the left–handed sector. In particular, we assign the \( C^{9}_{1} \) sector to the third family and the \( C^{9}_{2} \) and \( C^{9}_{3} \), to the first and second families, respectively.

Under the above assignment of the gauge multiplets and the matter fields, soft SUSY breaking terms are obtained, following the formulae in Ref. [22]. The gaugino masses are obtained
\[ M_1 = m_1 = \sqrt{3} m_{3/2} \sin \theta e^{-i \alpha_3}, \]
\[ M_2 = \sqrt{3} m_{3/2} \cos \theta \Omega_1 e^{-i \alpha_2}, \]
While the \( A \)–terms are obtained as
\[ A_{a_1} = - \sqrt{5} m_{3/2} \sin \theta e^{-i \alpha_3} = - M_3, \]
for the coupling including \( C^{9}_{1} \), i.e. the third family,
\[ A_{a_2} = - \sqrt{5} m_{3/2} (\sin \theta e^{-i \alpha_3} + \cos \theta (\Omega_1 e^{-i \alpha_1} - \Omega_2 e^{-i \alpha_2})), \]
for the coupling including \( C^{9}_{2} \), i.e. the second family and
\[ A_{a_3} = - \sqrt{5} m_{3/2} (\sin \theta e^{-i \alpha_3} + \cos \theta (\Omega_1 e^{-i \alpha_1} - \Omega_3 e^{-i \alpha_3})), \]
for the coupling including $C_3^q$, i.e. the first family. Here $m_{3/2}$ is the gravitino mass, $\alpha_S$ and $\alpha_i$ are the CP phases of the F–terms of the dilaton field $S$ and the three moduli fields $T_i$, and $\theta$ and $\Theta_i$ are goldstino angles, and we have the constraint, $\sum \Theta_i^2 = 1$.

Thus, if quark fields correspond to different $C_3^q$ sectors, we have non–universal A–terms. We obtain the following A–matrix for both of the up and down sectors,

$$ A = \begin{pmatrix} A_{C_3^q} & A_{C_2^q} & A_{C_1^q} \\ A_{C_2^q} & A_{C_1^q} & A_{C_0^q} \\ A_{C_1^q} & A_{C_0^q} & A_{C_{-1}^q} \end{pmatrix} \quad (3.9) $$

The trilinear SUSY breaking matrix, $(Y^A)_{ij} = (Y)_{ij}(A)_{ij}$, itself is obtained

$$ Y^A = \begin{pmatrix} Y_{ij} \end{pmatrix} \cdot \begin{pmatrix} A_{C_3^q} & 0 & 0 \\ 0 & A_{C_2^q} & 0 \\ 0 & 0 & A_{C_1^q} \end{pmatrix} \quad (3.10) $$

in matrix notation.

In addition, soft scalar masses for quark doublets and the Higgs fields are obtained,

$$ m_{C_i}^2 = m_{3/2}^2 (1 - \frac{3}{2} \cos^2 \theta (1 - \Theta_i^2)). \quad (3.11) $$

The soft scalar masses for quark singlets are obtained as

$$ m_{C_i}^2 = m_{3/2}^2 (1 - 3 \cos^2 \theta \Theta_i^2), \quad (3.12) $$

if it corresponds to the $C_i^q$ sector.

Now, below the string or SUSY breaking scale, this model is simply a MSSM with non–trivial soft–breaking terms from the point of view of flavor. Scalar mass matrices and tri–linear terms have completely new flavor structures, as opposed to the super–gravity inspired CMSSM or the SM, where the only connection between different generations is provided by the Yukawa matrices.

This model includes, in the quark sector, 7 different structures of flavor, $M_{d}^{0}$, $M_{d}^{1}$, $M_{d}^{2}$, $Y_{d}$, $Y_{u}$, $Y_{d}^{A}$ and $Y_{u}^{A}$. From these matrices, $M_{d}^{0}$, the squark doublet mass matrix, is proportional to the identity matrix, and hence trivial, then we are left with 6 non–trivial flavor matrices. Notice that we have always the freedom to diagonalize the hermitian squark mass matrices (as we have done in the previous section, Eqs. (3.11,3.12)) and fix some general form for the Yukawa and tri–linear matrices. In this case, these four matrices are completely observable, unlike in the SM or CMSSM case.

At this point, to specify completely the model, we need not only the soft–breaking terms but also the complete Yukawa textures. The only available experimental information is the Cabbibo–Kobayashi–Maskawa (CKM) mixing matrix and the quark masses. Here, we choose our Yukawa texture following two simple assumptions: i) the CKM mixing matrix originates from the down Yukawa couplings (as done in the MM case) and ii) our Yukawa matrices are hermitian [26]. With these two assumptions we fix completely the Yukawa matrices,

$$ Y_u = \frac{1}{v_2} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \quad (3.13) $$

$$ Y_d = \frac{1}{v_1} K \cdot \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}, \quad K (3.14) $$

with $v = v_1/(\cos \beta) = v_2/(\sin \beta) = \sqrt{2} M_W/g$, and $K$ the CKM matrix. We take $\tan \beta = v_2/v_1 = 2$ in the following in all numerical examples. In this basis we can analyze the down tri–linear matrix,

$$ Y_d^{A}(M_{St}) = \frac{1}{v_1} K^{\dagger} \cdot \begin{pmatrix} A_{C_3^q} & 0 & 0 \\ 0 & A_{C_2^q} & 0 \\ 0 & 0 & A_{C_1^q} \end{pmatrix}, \quad (3.15) $$

with $M_d = \text{diag}(m_d, m_s, m_b)$.

Hence, together with the up tri–linear matrix we have our MSSM completely defined. The next step is simply to use the MSSM Renormalization Group Equations to obtain the whole spectrum and couplings at the low scale, $M_W$. The dominant effect in the tri–linear terms renormalization is due to the gluino mass which produces the well–known alignment among A–terms and gaugino phases. However, this renormalization is always proportional to the Yukawa couplings and not to the tri–linear terms, Eq. (3.10). This implies that, in the SCKM basis, the gluino
effects will be diagonalized in excellent approximation, while due to the different flavor structure of the tri-linear terms large off-diagonal elements will remain with phases $O(1)$. To see this more explicitly, we can roughly approximate the RGE effects as,

$$Y_d^A(M_W) = g^*_3 m_3 Y_d + c_A Y_d \cdot \left( \begin{array}{ccc} A_{C_3^3} & 0 & 0 \\ 0 & A_{C_2^2} & 0 \\ 0 & 0 & A_{C_1^1} \end{array} \right) \tilde{K}$$

with $m_3$ the gluino mass and $g^*_3$, $c_A$ coefficients order 1 (typically $g^*_3 \approx 5$ and $c_A \approx 1$). We go to the SCKM basis after diagonalizing all the Yukawa matrices (that is, $K Y_d \tilde{K}^\dagger = \hat{M}_d/v_1$). In this basis, we obtain the tri-linear couplings as,

$$v_1 \ Y_d^A(M_W) = g^*_3 m_3 \ M_d + c_A M_d \cdot \tilde{K} \left( \begin{array}{ccc} A_{C_3^3} & 0 & 0 \\ 0 & A_{C_2^2} & 0 \\ 0 & 0 & A_{C_1^1} \end{array} \right) \tilde{K}^\dagger \ (3.16)$$

From this equation we can get the $L-R$ down squark mass matrix

$$m_{LR}^{(d)} = v_1 \ Y_d^{A^*} - \mu e^{i\varphi_v} \tan \beta \ M_d \ (3.18)$$

And finally using unitarity of $K$ we obtain for the $L-R$ Mass Insertions,

$$(\delta_{LR}^{(d)})_{ij} = \frac{m_i}{m_q^2} \left( \begin{array}{ccc} \delta_{ij} (c_A A_{C_3^3}^* + c^*_3 m_3 - \mu e^{i\varphi_v} \tan \beta) \\ + K_{i3} K_{j2}^* c_A (A_{C_2^2}^* - A_{C_1^1}^*) \\ + K_{i3} K_{j3}^* c_A (A_{C_1^1}^* - A_{C_2^2}^*) \end{array} \right) \ (3.19)$$

where $m_q^2$ is an average squark mass and $m_i$ the quark mass. The same rotation must be applied to the $L-L$ and $R-R$ squark mass matrices,

$$M_{LL}^{(d)}(M_W) = K \cdot M_Q^{21}(M_W) \cdot \tilde{K}$$
$$M_{RR}^{(d)}(M_W) = K \cdot M_D^{21}(M_W) \cdot \tilde{K} \ (3.20)$$

However, the off-diagonal MI in these matrices are sufficiently small in this case thanks to the universal and dominant contribution from gluino to the squark mass matrices in the RGE. At this point, with the explicit expressions for $(\delta_{LR}^{(d)})_{ij}$, we can study the gluino mediated contributions to EDMs and $\varepsilon'/\varepsilon$. In this non-universal scenario, it is relatively easy to maintain the SUSY contributions to the EDM of the electron and the neutron below the experimental bounds while having large SUSY phases that contribute to $\varepsilon'/\varepsilon$.  

---

**Figure 1:** Allowed values for $\alpha_2 - \alpha_S$ (open blue circles) and $\alpha_3 - \alpha_S$ (red stars)

**Figure 2:** $(\delta_{LR}^{(d)})_{21}$ versus $m_{3/2}$ for experimentally allowed regions of the SUSY parameter space
This is due to the fact the EDM are mainly controlled by flavor-diagonal MI, while gluino contributions to $e'/\epsilon$ are controlled by $(\delta_{LR}^{(d)})_{12}$ and $(\delta_{LR}^{(u)})_{21}$. Here, we can have a very small phase for $(\delta_{LR}^{(d)})_{11}$ and phases $O(1)$ for the off-diagonal elements without any fine-tuning $\Delta_{\phi_{\mu}}$. It is important to remember that the observable phase is always the relative phase between these mass insertions and the relevant gaugino mass involved. In Eq. (3.19) we can see that the diagonal elements tend to align with the gluino phase, hence to have a small EDM, it is enough to have the phases of the gauginos and the $\mu$ term approximately equal, $\alpha_S = \alpha_1 = -\varphi_\mu$. However $\alpha_2$ and $\alpha_3$ can still contribute to off-diagonal elements. In figure 2 we show the allowed values for $\alpha_2, \alpha_3$ assuming $\alpha_1 = \varphi_\mu = 0$. We impose the EDM, $\epsilon_K$ and $b \rightarrow s\gamma$ bounds separately for gluino and chargino contributions together with the usual bounds on SUSY masses. We can see that, similarly to the CMSSM situation, $\varphi_\mu$ is constrained to be very close to the gluino and chargino phases (in the plot $\alpha_S \simeq 0, \pi$), but $\alpha_2$ and $\alpha_3$ are completely unconstrained.

Finally, in figure 3, we show the effects of these phases in the $(\delta_{LR}^{(d)})_{21}$ MI as a function of the gravitino mass. All the points in this plot satisfy all CP-conserving constraints besides EDM and $\epsilon_K$ constraints. We must remember that a value of $|\text{Im}(\delta_{LR}^{(d)})_{12}| \sim 10^{-5}$ gives a significant contribution to $e'/\epsilon$. In this plot, we can see a large percentage of points above or close to $1 \times 10^{-5}$. Hence, we can conclude that, in the presence of new flavor structures in the SUSY soft-breaking terms, it is not difficult to obtain sizeable SUSY contributions to CP violation observables and specially to $e'/\epsilon$ $\Delta_{\phi_{\mu}}$.2

4. Conclusions and Outlook

Here we summarize the main points of this talk:

- Flavor and CP problems constrain low-energy SUSY, but, at the same time, provide new tools to search for SUSY indirectly.
- In all generality, we expect new CP violating phases in the SUSY sector. However, these new phases are not going to produce sizeable effects as long as the SUSY model we consider does not exhibit a new flavor structure in addition to the SM Yukawa matrices.
- In the presence of a new flavor structure in SUSY, we showed that large contributions to CP violating observables are indeed possible.

In summary, given the fact that LEP searches for SUSY particles are close to their conclusion and that for Tevatron it may be rather challenging to find a SUSY evidence, we consider CP violation a potentially precious ground for SUSY searches before the advent of the “SUSY machine”, LHC.

Acknowledgments

We thank D. Demir, T. Kobayashi, S. Khalil and H. Murayama as co-authors of some recent works reported in these lectures. We are grateful to S. Bertolini, L. Silvestrini and F.J. Botella for enlightening conversations. A.M. thanks the organizers for the stimulating setting in which the school took place. The work of A.M. was partly supported by the TMR project “Beyond the Standard Model” contract number ERBFMRX CT96 0090; O.V. acknowledges financial support from a Marie Curie EC grant (TMR-ERBFMB1 CT98 3087).

References


T. Ibrahim and P. Nath, Santa Barbara U.
preprint no. NSF-ITP-99-129, October 1999,