

# Horizontal $U(1)_H$ symmetry: a non-anomalous model

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ABSTRACT: Spontaneously broken Abelian gauge symmetries can explain the fermion mass hierarchies of the minimal supersymmetric standard model. In most cases it is assumed that the  $U(1)_H$  symmetry is anomalous. However, non-anomalous models are also viable and yield an interesting phenomenology. Cancellation of the gauge anomalies implies the following results: unification of leptons and down-type quarks Yukawa couplings is allowed at most for two generations. The  $\mu$  term is necessarily somewhat below the supersymmetry breaking scale. The superpotential has accidental  $B$  and  $L$  symmetries, and R-parity is automatically conserved in the supersymmetric limit. Anomaly cancellation also implies that the determinant of the quark mass matrix must vanish, which is possible only if  $m_{\text{up}} = 0$ . This solves the strong  $CP$  problem and provides an unambiguous low energy test of the model.

One of the most successful ideas in modern particle physics is that of local gauge symmetries. A huge amount of data is beautifully explained in terms of the standard model (SM) gauge group  $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ . Identifying this symmetry required a lot of experimental and theoretical efforts, since  $SU(2)_L \times U(1)_Y$  is hidden and color is confined. Today we understand particle interactions but we do not have any deep clue in understanding other elementary particle properties, like fermion masses and mixing angles. The SM can only accommodate but not explain these data. Another puzzle is why  $CP$  is preserved by strong interactions to an accuracy  $< 10^{-9}$ . One solution is to postulate that one quark is massless, but within the SM there are no good justifications for this. Adding supersymmetry does not provide us with any better understanding of these issues. In contrast, it adds new problems. A bilinear coupling for the down-type and up-type Higgs superfields  $\mu\phi_d\phi_u$  is allowed both by supersymmetry and by the gauge symmetry. However, phenomenology requires that  $\mu$  should be close to the scale where these symmetries are broken. With supersym-

metry, several operators that violate baryon ( $B$ ) and lepton ( $L$ ) numbers can appear. However, none of the effects expected from these operators has ever been observed. Since a few of them can induce fast proton decay, they must be very suppressed or absent.

In [1] we attempted to see if by insisting on the gauge principle we could gain some insight into these problems. We extended minimally  $G_{SM}$  with a *non anomalous* horizontal Abelian  $U(1)_H$  factor, and imposed the consistency conditions for cancellation of the gauge anomalies. An unambiguous prediction of the non anomalous  $U(1)_H$  is a massless up-quark. This represents a crucial low energy test for our framework. Shall future lattice computations rule out  $m_{\text{up}} = 0$  [2] the model will be disproved.

The fermion mass pattern is accounted for by means of the approach originally suggested by Froggatt and Nielsen (FG) [3]. The  $U(1)_H$  symmetry forbids most of the fermion Yukawa couplings. The symmetry is spontaneously broken by the vacuum expectation value (vev) of a SM singlet field  $S$ , giving rise to a set of effective operators that couple the SM fermions to the electroweak Higgs field. The hierarchy of fermion masses results from the dimensional

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hierarchy among the various higher order operators. This idea was recently reconsidered by several groups, both in the context of supersymmetry [4] and with an anomalous local  $U(1)_H$  [5, 6, 7, 8, 9]. Here we study the non-anomalous case. Our theoretical framework is defined by the following assumptions: 1) Supersymmetry and the gauge group  $G_{SM} \times U(1)_H$ . 2)  $U(1)_H$  is broken only by the vev of a field  $S$  with horizontal charge  $-1$ .<sup>1</sup>  $S$  is a SM singlet and is chiral under  $U(1)_H$ . 3) The ratio between the vev  $\langle S \rangle$  and the mass scale  $M$  of the FN fields is of the order of the Cabibbo angle  $\lambda \simeq \langle S \rangle / M \sim 0.2$ . 4) The only fields chiral under  $U(1)_H$  and charged under  $G_{SM}$  are the minimal supersymmetric SM supermultiplets. 5) We also assume  $\det M^\ell \leq \det M^d$ , as is strongly suggested by the measured values of the eigenvalues of the lepton ( $M^\ell$ ) and down-type quark ( $M^d$ ) mass matrices.

In the following we will use the same symbol to denote a field and its horizontal charge. Upon  $U(1)_H$  breaking, the Yukawa couplings  $Y^u$ ,  $Y^d$  and  $Y^\ell$  of the up-type and down-type quarks and of the leptons are generated. They satisfy the following relations:

$$Y_{ij}^u = \begin{cases} A_{ij}^u \lambda^{Q_i+u_j+\phi_u} & \text{if } Q_i + u_j + \phi_u \geq 0, \\ 0 & \text{if } Q_i + u_j + \phi_u < 0, \end{cases}$$

and similar ones for  $Y^d$  and  $Y^\ell$ . The zero entries arise from holomorphy, while  $A_{ij}^u$  are numerical coefficients of order  $\lambda^0$  that we will often leave understood. Let us introduce the following combinations of charges:

$$\begin{aligned} n_u &= \sum_i (Q_i + u_i), & n_d &= \sum_i (Q_i + d_i), \\ n_Q &= \sum_i Q_i, & n_L &= \sum_i L_i, \\ n_\ell &= \sum_i (L_i + \ell_i), & n_\phi &= \phi_u + \phi_d. \end{aligned} \quad (1)$$

After electroweak symmetry breaking, the Yukawa couplings  $Y_{ij}^{u,d,\ell}$  give rise to the fermion mass matrices  $M^u$ ,  $M^d$  and  $M^\ell$ . In the absence of vanishing eigenvalues their determinants read

$$\det M^u = \langle \phi_u \rangle^3 \lambda^{n_u+3\phi_u} \det A^u, \quad (2)$$

$$\det M^d = \langle \phi_d \rangle^3 \lambda^{n_d+3\phi_d} \det A^d, \quad (3)$$

$$\det M^\ell = \langle \phi_d \rangle^3 \lambda^{n_\ell+3\phi_d} \det A^\ell. \quad (4)$$

<sup>1</sup>We assume that a tree level Fayet-Iliopoulos  $D$ -term triggers the breaking of  $U(1)_H$  while preserving supersymmetry.

Since all the entries in  $A^{u,d,\ell}$  are of order  $\lambda^0$ ,  $\det A^{u,d,\ell}$  is of order 1. Then the size of the determinants (2)-(4) is fixed by the horizontal charges and by the ratio of the Higgs doublets vevs  $\tan \beta = \langle \phi_u \rangle / \langle \phi_d \rangle$ .

The SM Yukawa operators are invariant under a set of global  $U(1)$  symmetries:  $B$ ,  $L$ , hypercharge ( $Y$ ) and a symmetry  $X$  with charges  $X(d) = X(\ell) = -X(\phi_d)$  and  $X = 0$  for all the other fields. Therefore, shifts of the horizontal charges proportional to  $L$ ,  $B$ ,  $Y$  and  $X$  do not affect the fermion mass matrices. In the following, we will denote as *equivalent* two sets of charges that can be transformed one into the other by means of shifts of this kind. Note that the superpotential term  $\mu \phi_u \phi_d$  (the  $\mu$ -term) is not invariant under  $X$ , and hence it can be different for two equivalent sets. Experimental evidences for non-vanishing neutrino mixings [10] imply that shifts proportional to individual lepton flavor numbers  $L_a$  ( $a = e, \mu, \tau$ ) transform between phenomenologically *non equivalent* set of charges. In fact, while these shifts do not affect the charged lepton masses, they still produce different patterns of neutrino mixings. In our analysis we will work with the following linear combinations of generators:  $X$ ,  $B$ ,  $B-L$ ,  $L_\tau-L_\mu$ ,  $L_\mu-L_e$ , and  $Y$ .

Since  $G_{SM} \times U(1)_H$  is a local symmetry, it is mandatory to study the consistency conditions for cancellation of the gauge anomalies. The mixed  $SU(n)^2 \times U(1)_H$  anomalies, quadratic in  $SU(n) = SU(3)_C, SU(2)_L, U(1)_Y$  and linear in the horizontal charges, can be expressed in terms of the coefficients

$$\begin{aligned} C_3 &= n_u + n_d, \\ C_2 &= n_\phi + (3n_Q + n_L), \\ C_1 &= n_\phi + \frac{8}{3}n_u + \frac{2}{3}n_d + 2n_\ell - (3n_Q + n_L). \end{aligned} \quad (5)$$

The coefficient of the mixed  $U(1)_Y \times U(1)_H^2$  anomaly quadratic in the horizontal charges reads

$$C^{(2)} = \phi_u^2 - \phi_d^2 + \sum_i [Q_i^2 - 2u_i^2 + d_i^2 - L_i^2 + \ell_i^2].$$

The pure  $U(1)_H^3$  and the mixed gravitational anomalies can always be canceled by adding SM singlet fields with suitable charges, and we assume they vanish. If the  $C_n$ 's in (5) do not vanish, the Green-Schwarz (GS) mechanism [11] can be

invoked to remove the anomalies by means of a  $U(1)_H$  gauge shift of an axion field  $\eta(x) \rightarrow \eta(x) - \xi(x) \delta_{GS}$ . The consistency conditions for this cancellation read [12]

$$C_1/k_1 = C_2 = C_3 = \delta_{GS}, \quad (6)$$

where the Kac-Moody levels of the  $SU(2)_L$  and  $SU(3)_C$  gauge groups have been assumed to be unity and  $k_1$  is the  $U(1)_Y$  (arbitrary) normalization factor. Then the weak mixing angle (at some large scale  $\Lambda$ ) is given by  $\tan^2 \theta_W = g'^2/g^2 = 1/k_1$ . Using (5), conditions (6) translate into

$$2(n_\phi - n_d + n_\ell) = (k_1 - \frac{5}{3}) \delta_{GS}. \quad (7)$$

Now, one can assume that the gauge couplings unify for the canonical value  $\tan^2 \theta_W = 3/5$  [8]. Then  $n_\phi = n_d - n_\ell$  is obtained. Alternatively, one can assume that for some reasons the l.h.s. in (7) vanishes, and thus predict canonical gauge couplings unification [6]. However, in the absence of a GUT symmetry the value  $k_1 = 5/3$  is not compelling. Other values of  $k_1$  can be in reasonable agreement with unification at scales  $\Lambda \neq \Lambda_{GUT}$  [12] so that  $n_\phi$  and  $n_d - n_\ell$  are not necessarily related in any simple way. If  $U(1)_H$  is non-anomalous (6) and (7) still hold with  $\delta_{GS} = 0$ , so that the interplay with gauge couplings unification is lost. However,  $n_\phi = n_d - n_\ell$  now follows as an unavoidable consistency condition, giving a first constraint on the permitted horizontal charges.

Let us now study the symmetry properties of the coefficients (5). Since for each  $SU(2)_L$  multiplet  $\text{Tr}[T_3 Y H] = Y H \text{Tr}[T_3] = 0$ , the mixed electromagnetic- $U(1)_H$  anomaly can be expressed in terms of  $C_1$  and  $C_2$  as  $C_Q = \frac{1}{2}(C_1 + C_2)$ . Being  $SU(3)_C \times U(1)_Q$  vectorlike, it is free of  $B$  and  $L$  anomalies, and then  $C_3$  and  $C_Q$  must be invariant under shifts of the horizontal charges proportional to  $B$  and  $L$ . The same is not true for  $C_1$  and  $C_2$  separately. However, the SM is free of  $B$ - $L$  anomalies and thus  $C_1$  and  $C_2$  are invariant under the corresponding shift. Also  $L_\tau$ - $L_\mu$  and  $L_\mu$ - $L_e$  have vanishing anomalies with  $G_{SM}$ , so they identify two more possible shifts that leave invariant the  $C_n$ 's. In the following we state the consistency conditions for cancellation of the  $G_{SM} \times U(1)_H$  gauge anomalies.

A set of horizontal charges  $\{H\}$  is equivalent to a second set  $\{H''\}$  for which the coefficients  $C_n''$  of the mixed linear anomalies vanish, if and only if the mixed  $U(1)_Q^2$ - $U(1)_H$  and  $SU(3)_C^2$ - $U(1)_H$  anomaly coefficients are equal:

$$C_Q - C_3 = 0 \iff C_1'' = C_2'' = C_3'' = 0. \quad (8)$$

Moreover, if for  $\{H''\}$  the charge of the  $\mu$  term  $n_\phi''$  is different from zero, the coefficient of the quadratic anomaly  $\tilde{C}^{(2)}$  can always be set to zero:

$$n_\phi'' \neq 0 \implies \tilde{C}^{(2)} = 0. \quad (9)$$

As it stands, this condition is sufficient but not necessary. However, if all the neutrinos are mixed at a measurable level, condition (9) turns out to be also necessary [13]. In the following we take  $n_\phi \neq 0$  in the strong sense.

To prove (8) and (9), let us start by assuming that for the initial set  $\{H\}$   $C_n \neq 0$ . Then we can shift the charges proportionally to the  $X$  quantum numbers.  $H \rightarrow H + \frac{a}{3} X$  yields:

$$C_n \rightarrow C'_n = C_n + \alpha_n a, \quad (10)$$

with  $\alpha_3 = 1$ ,  $\alpha_2 = -1/3$  and  $\alpha_1 = +7/3$ . We fix  $a = -C_3/\alpha_3$  so that  $C'_3 = 0$ . Note that the combination  $(C_1 + C_2)/(\alpha_1 + \alpha_2) - C_3/\alpha_3 = C_Q - C_3$  besides being  $B$  and  $L$  invariant, is also  $X$  invariant by construction. Now a shift proportional to  $B$  can be used to set  $C''_2 = 0$ . Since  $C_3$  is  $B$  invariant,  $C''_3 = C'_3 = 0$ . The sum  $C'_1 + C'_2$  is also  $B$  invariant and thus  $C''_1 = C'_1 + C'_2 = 2C'_Q$ . However, by assumption  $C'_Q = C'_3 (= 0)$  and then the set  $\{H''\}$  has vanishing mixed linear anomalies. Now, in order to cancel the quadratic anomaly while keeping vanishing  $C''_n$ , we can use any of the SM anomaly free symmetries  $B$ - $L$ ,  $L_\tau$ - $L_\mu$ ,  $L_\mu$ - $L_e$ . Since  $L_\tau$ - $L_\mu$  and  $L_\mu$ - $L_e$  transform between non equivalent set of charges, we keep this freedom to account for two neutrino mixings (the third one results as a prediction) and we use  $B$ - $L$ . Under the charge redefinition  $H \rightarrow H + \beta(B-L)$

$$\begin{aligned} C^{(2)''} \rightarrow \tilde{C}^{(2)} &= C^{(2)''} + \beta \left[ \frac{4}{3} n_u'' - \frac{2}{3} n_d'' + 2 n_\ell'' \right] \\ &= C^{(2)''} - 2\beta n_\phi'', \end{aligned} \quad (11)$$

where in the last step we have used the identity  $\frac{4}{3} n_u - \frac{2}{3} n_d + 2 n_\ell = C_1 + C_2 - \frac{4}{3} C_3 - 2 n_\phi$  and

the vanishing of the  $C_n''$ . If, as we have assumed,  $n_\phi'' \neq 0$ , we can always set  $\tilde{C}^{(2)} = 0$  by choosing  $\beta = C^{(2)''}/(2n_\phi'')$ . It is also useful to note that a set of horizontal charges  $\{H\}$  for which  $C_n = C^{(2)} = 0$  identifies a one parameter family of anomaly free charges generated by shifts proportional to hypercharge:  $H \rightarrow H + yY$ . For the  $C_n$ 's this is trivial due to the vanishing of the SM anomalies  $\text{Tr}[SU(n)^2 Y] = 0$ . For  $C^{(2)}$  we have  $\text{Tr}[Y H^2] \rightarrow \text{Tr}[Y (H + Y)^2] = 2C_1 = 0$ .

In summary, cancellation of the  $G_{SM} \times U(1)_H$  gauge anomalies implies the following constraints on the fields charges

$$n_\phi \neq 0, \quad n_\phi = n_d - n_\ell \simeq \ln_\lambda \frac{\det M^d}{\det M^\ell}, \quad (12)$$

where the last relation follows from (3) and (4). Since  $n_d \neq n_\ell$  we conclude that Yukawa coupling unification is permitted at most for two families. Together with assumption 5), we also obtain  $n_\phi < 0$ , so that the superpotential  $\mu$  term is forbidden by holomorphy and vanishes in the supersymmetric limit. Let us confront these results with phenomenology. To a good approximation the mass ratios  $m_e/m_\mu \sim \lambda^3$ ,  $m_\mu/m_\tau \sim \lambda^2$ ,  $m_d/m_s \sim \lambda^2$  and  $m_s/m_b \sim \lambda^2$  are renormalization group invariant. Then, since Yukawa coupling unification works remarkably well for the third family,  $\det M^\ell/\det M^d \sim \lambda$  and the preferred value of  $n_\phi$  is  $-1$ . Then a  $\mu$  term arising from the (non-holomorphic) Kähler potential [14] will have a value somewhat below the supersymmetry breaking scale  $m_{3/2}$ :

$$\mu \sim \lambda^{|n_\phi|} m_{3/2} \quad \text{with} \quad n_\phi = -1. \quad (13)$$

As we have explicitly shown, the anomaly cancellation condition  $C_Q - C_3 = 0$  (8) is  $Y$ ,  $B$ ,  $L$  and  $X$  invariant and hence it shares the same invariance of the Yukawa couplings. Therefore, any product of the determinants (2)-(4) for which the overall horizontal charge can be recasted just in terms of the  $C_n$ 's must depend precisely on this combination. Such a relation was first found in [8]. Given that  $C_Q - C_3 = n_\ell - \frac{2}{3}n_d + \frac{1}{3}n_u + n_\phi$  we can write it down at once:

$$\left(\frac{\det M^\ell}{\langle \phi_d \rangle^3}\right) \left(\frac{\det M^d}{\langle \phi_d \rangle^3}\right)^{-\frac{2}{3}} \left(\frac{\det M^u}{\langle \phi_u \rangle^3}\right)^{\frac{1}{3}} \simeq \lambda^{C_Q - C_3}. \quad (14)$$

Let us confront this relation with phenomenology. Anomaly cancellation implies that the r.h.s.

is unity, while the l.h.s. is bounded by the upper limit  $[(\det M^d/\langle \phi_d \rangle^3) (\det M^u/\langle \phi_u \rangle^3)]^{1/3} \ll 1$ . This inconsistency (or similar ones) led several authors to conclude that  $U(1)_H$  must be anomalous [5, 6, 7, 8]. However, (14) is meaningful only under the assumption that none of the determinants vanishes, and since low energy phenomenology is still compatible with a massless up quark [2, 15] (see however [16]) this might not be the case. In the following we show that insisting on the vanishing of the gauge anomalies yields  $m_{\text{up}} = 0$  as a prediction.

We start by noticing that if the determinant of the matrix  $U_{ij} \sim \lambda^{Q_i + u_j + \phi_u}$  has an overall negative charge  $\eta^U \equiv n_u + 3\phi_u \sim \log_\lambda \det U < 0$ , then  $M^u$  has vanishing eigenvalues. In fact  $\det U$  consists of the sum of six terms of the form  $\lambda^{n_1} \cdot \lambda^{n_2} \cdot \lambda^{n_3}$  where  $n_1 + n_2 + n_3 = \eta^U < 0$ . Then at least one of the  $n_i$  must be negative, corresponding to a holomorphic zero in the mass matrix. Hence each one of the six terms vanishes.

Now, if  $U(1)_H$  is anomaly free and assumption 5) holds, it is easy to see that the determinant of the six quark mass matrix  $\mathcal{M}_q$  vanishes:

$$\left. \begin{array}{l} n_\ell \geq n_d \\ C_n = C^{(2)} = 0 \end{array} \right\} \implies \det \mathcal{M}_q = 0. \quad (15)$$

Adding and subtracting  $3n_\phi$  to  $C_3 = 0$  yields

$$\eta^U + \eta^D = 3n_\phi < 0. \quad (16)$$

Then at least one of the two  $\eta$  must be negative, and the corresponding determinant vanishes. Of course, on phenomenological grounds, a massless up quark is the only viable possibility [2, 15]. Using the  $d$ -quark mass ratios given above, and taking  $m_b/m_t \sim \lambda^3$  and  $n_\phi = -1$ , we obtain

$$\eta^U \simeq -9 - 3 \log_\lambda \left( \frac{m_b}{m_t} \tan \beta \right), \quad (17)$$

that ranges between  $-9$  and  $-18$  for  $\tan \beta$  between  $m_t/m_b$  and  $1$ . Because of the constraints from holomorphy,  $\eta^U < 0$  results in an accidental  $U(1)_u$  symmetry acting on the  $SU(2)_L$  singlet up-quark:  $u_1 \rightarrow e^{i\alpha} u_1$ . By means of this chiral transformation the QCD  $CP$  violating parameter  $\bar{\theta} \equiv \theta + \arg \det \mathcal{M}_q$  can be rotated away, and is no more physical. However, holomorphy plays a crucial role to obtain this result, and we

must check if it is maintained after supersymmetry breaking. While Yukawa couplings redefinition needed to bring the kinetic terms into canonical form cannot lift zero eigenvalues [1], general soft supersymmetry breaking terms do not respect the  $U(1)_u$  symmetry, and can induce  $m_{\text{up}} \neq 0$  radiatively. A conservative estimate gives [1]  $m_{\text{up}} \lesssim (\alpha_s/\pi) \lambda^{|\eta^U-4|} \langle \phi_u \rangle \lesssim 10^{-6}, (10) \text{ eV}$  [for  $\tan \beta \sim 1, (m_t/m_b)$ ]. The resulting contribution to the neutron electric dipole moment is [17]:  $d_n/e \lesssim 10^{-28} \bar{\theta}, (10^{-22} \bar{\theta}) \text{ cm}$ . Therefore, for moderate values of  $\tan \beta$ , the neutron dipole moment remains safely below the experimental limit  $d_n/e < 6.3 \times 10^{-26} \text{ cm}$  [18] even for  $\bar{\theta} \sim 1$ .

Gauge symmetry and supersymmetry, together with constraints from fermion charges relations, also imply that the superpotential has accidental  $B$  and  $L$  symmetries. This result is deeply related to the solutions of the  $\mu$  and of the strong CP problems ( $n_\phi < 0, \eta^U < 0$ ). The proof requires certain phenomenological inputs, like fermion mass ratios and CKM mixings, and the assumption that neutrinos mixings are sizeable. Since it is somewhat lengthy, it will be presented elsewhere [13]. An intuitive argument goes as follows: given a set of minimal charges that fit well the fermion masses and mixings,  $\eta^U < 0$  (17) also implies that  $C^{(2)}$  is negative. To cancel  $C^{(2)}$  the shift  $H \rightarrow H + \beta \cdot (B-L)$  is required, where  $\beta = C^{(2)}/2n_\phi$  is positive. All the R-parity violating operators  $\mu_L L \phi_u, \lambda L L \ell, \lambda' L Q d$  and  $\lambda'' u d d$  have  $B-L = -1$ , so that under this shift their charges are driven to negative values implying that they cannot appear in the superpotential. Of course, dimension five see-saw operators for neutrino masses are also forbidden. However, the same mechanism that generates  $\mu$  will generate (with larger suppressions) also  $\mu_L L \phi_u$  terms, that induce s-neutrinos vevs. Canonical diagonalization of  $L-\phi_d$  mixed kinetic terms will produce tiny  $\lambda$  and  $\lambda'$  from the Yukawa couplings. Both these effects can result in small neutrino masses [9]. However, since none of the  $\lambda''$  can be generated in this way, proton stability is not in jeopardy. Finally, we stress that except for  $\eta^U < 0$  the condition  $C_Q - C_3 = 0$  does not imply other serious constraints on charge assignments. Within our framework the mass matrices of popular models [4, 7] can be easily reproduced

and, apart from  $m_{\text{up}} = 0$ , also the same phenomenology [13].

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