LONG RANGE FORCES FROM QUANTUM FIELD THEORY AT ZERO AND FINITE TEMPERATURE

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ABSTRACT: We discuss the derivation of Newtonian potentials in the framework of quantum field theory. We focus on two particular points: on long range forces i.e. forces which fall off as $1/r^n$ being mediated by light quanta like neutrinos or Goldstone bosons and on possible temperature dependence of such forces arising in situations when the exchanged quanta are in a thermal heat bath. Examples of the latter are cosmic relic photons and relic neutrinos. Among other things, we will show that the existence of cosmic relic neutrinos modifies the long tail of the two-neutrino exchange Feinberg-Sucher force drastically. Results concerning the potential mediated by two Goldstone bosons are also presented.

1. Overview

One of the most important key concepts in theoretical physics, is the concept of a force introduced by Newton some three hundred years ago. Without any doubts, this concept continues to play a fruitful role in physics, despite the fact that classical mechanics has been superseded by the more general quantum theory. Indeed, modern theories of interactions use the tools of quantum field theory (QFT) as a general framework. It bears therefore a certain charm when we can span a bridge between classical mechanics and QFT by deriving new forces, especially the long range forces, within the framework of the latter. However, this is not the only reason which makes the subject worthwhile as the following rough classification of long range forces demonstrates.

1. Quantum corrections to classical results. The QFT can, of course, reproduce the classical long range forces of electromagnetism and gravity. In addition QFT predicts also quantum mechanical corrections to these classical results. So, for instance the Coulomb potential receives corrections of the following type [1]

$$V_{em}(r) = \frac{e^2}{r}\left[1 + \delta V_{em}^{QM}(r)\right]$$

$$\delta V_{em}^{QM}(r) = \frac{2\alpha}{3\pi}\left(\ln(1/m_e r) - C - \frac{5}{6}\right) - \frac{2\alpha^2 e^2}{225\pi}\frac{1}{(m_e r)^4} \qquad (1.1)$$

where $C = 0.577$ and m_e is the electron mass. The first correction ($\sim \alpha$) is due to vacuum polarization and valid for $m_e r \ll 1$. The second correction ($\sim \alpha^2$) has its root in the Heisenberg-Euler Lagrangian ($\gamma\gamma$ scattering). Similarly, for gravity using low energy effective field theory techniques, one derives quantum corrections of the form [2]

$$V_{gravity}(r) = -\frac{G_N M_1 M_2}{r}\left[1 + \delta V_{gravity}^{QM}(r)\right]$$

$$\delta V_{gravity}^{QM}(r) = -\frac{G_N(M_1 + M_2)}{r} + \frac{127 G_N}{30\pi^2 r^2} \qquad (1.2)$$

2. New forces from old quanta. All forces in the QFT arise from the exchange of quanta, massless or very light in the case of long range forces. Apart from the corrections to the classical results given above, the next logical step is to search for possible long range forces mediated by some light particles in the experimentally established particle spectrum. Neutrinos, being either very light or massless, are the natural candidates. This was

suspected by Feynman [3] and demonstrated in detail by Feinberg and Sucher [4]. We quote below the more general results for massive neutrinos distinguishing between Dirac and Majorana type [5]

$$
\begin{aligned}
V_{Dirac}(r) &= \frac{G_F^2 m_\nu^3 g_V g_V'}{4\pi^3 r^2} K_3(2m_\nu r) \\
V_{Majorana}(r) &= \frac{G_F^2 m_\nu^2 g_V g_V'}{2\pi^3 r^3} K_2(2m_\nu r)
\end{aligned}
\tag{1.3}
$$

where g_V and g_V' are vector coupling constants and K_n are modified Bessel functions. For $m_\nu = 0$ both results reduce to the formula obtained originally by Feinberg and Sucher, namely

$$
V_{FS}(r) = \frac{G_F^2 g_V g_V'}{4\pi^3 r^5}. \tag{1.4}
$$

Another famous example is the Casimir-Polder force mediated by two photons between polarizable particles [6]. Its analytical form reads

$$
V_{CP}(r) = -\frac{23(\alpha_E^2 + \alpha_B^2) - 14(\alpha_E \alpha_B)}{(4\pi)^3 r^7} \tag{1.5}
$$

where α_E and α_B are electric and magnetic polarizabilities of the external particle. It is worthwhile mentioning that it took some fifty years to verify this force experimentally [7]. This also shows that a technologically difficult task, not possible at the moment, could still become feasible in the future. We emphasize this, because all forces we are discussing here are feeble and difficult to detect experimentally.

3. New forces from new quanta. While going beyond the Standard Model we can, in principle, encounter many other light quanta, mostly light scalars, pseudo-scalars or true Goldstone bosons. Famous examples are Axions [8], Majorons [9] and scalars and pseudo-scalars in no-scale supergravity [10] and others [11]. A tower of massive gravitons is also possible by compactification of extra higher space dimensions [12]. Hence a search for long range forces mediated by such exotic particles could be a harbinger of new physics. For more details, especially regarding the experimental aspect of such a search, we refer the reader to [13].

4. New effects: temperature dependent forces. From the point of view of QFT at finite temperature, an exchange of quanta which are in a thermal bath at a temperature T leads, of course, to temperature dependent amplitudes and therefore also to temperature dependent forces. This is indeed a curious prediction of QFT at finite temperature. Physical examples of quanta in a thermal heat bath are cosmic relic photons (microwave background radiation) and relic neutrinos (the latter not yet experimentally verified). In the real time approach to finite temperature field theory the full propagator is a matrix out of which we need for the actual calculations of potentials only the 1-1 component given by

$$
\begin{aligned}
&S_T^{fermion}(k) = (\not{k} + m)[(k^2 - m^2 + i\epsilon)^{-1} \\
&+2\pi i\delta(k^2 - m^2)(\theta(k^0)n_+(T) + \theta(-k^0)n_-(T))] \\
&S_T^{boson}(k) = (k^2 - m^2 + i\epsilon)^{-1} \\
&-2\pi i\delta(k^2 - m^2)(\theta(k^0)n_+(T) + \theta(-k^0)n_-(T))
\end{aligned}
\tag{1.6}
$$

where n_+ and n_- are distribution functions for particle and antiparticle, respectively. Temperature corrections to various long range forces have been calculated in [14, 15, 16, 17, 18, 19].

5. Forces not derivable from QFT. We mention here for completeness that an example of such a force would be the Newtonian limit of Einstein's gravity with a cosmological constant Λ. For a spherical object or point-like particle, the gravitational potential reads

$$
\Phi_\Lambda(r) = -\frac{G_N M}{r} - \frac{1}{6}\Lambda r^2 \tag{1.7}
$$

The second part, proportional to Λ, cannot be derived from QFT. If certain recent experimental indications of a non-zero cosmological constant should be confirmed, the Λ-force in the Newtonian approximation would be "longest" out of the long range forces in nature. For peculiarities of the Newtonian limit in the presence of non-zero Λ, see [20].

Before discussing concrete examples, a few comments about the actual method to calculate a potential from an amplitude $\mathcal{M}$ are in order. There are essentially 2 equivalent methods. The more standard one is to take the Fourier transform of a matrix element in the static limit i.e. approximating the four momentum transfer q by

$q \simeq (0, \mathbf{Q})$ $(Q = |\mathbf{Q}|)$.

$$V(r) = \int \frac{d^3Q}{(2\pi)^2} \exp(i\mathbf{Qr})\mathcal{M}(\mathbf{Q}) = \frac{1}{2\pi^2 r}\int_0^\infty dQQ\mathcal{M}(Q)\sin Qr \quad (1.8)$$

The other, more elaborate, method uses dispersion techniques and defines [21]

$$V(r) = \frac{-i}{8\pi^2 r}\int_{4m^2}^\infty dt[\mathcal{M}]_t \exp(-\sqrt{t}r) \quad (1.9)$$

where the integration variable t equals the four–momentum transfer squared, q^2. Here, $[\mathcal{M}]_t$ denotes the discontinuity of the Feynman amplitude across the cut in the real t axis.

In the next sections we will focus on two particular examples with slightly different emphasis. The first example will be the two-neutrino exchange force (Feinberg-Sucher force). We will examine here the aforementioned temperature dependence taking different thermal distributions $n_\pm(T)$. The second example deals with the two-boson exchange force and emphasizes the difference between light pseudoscalar and Goldstone bosons.

2. Two-neutrino exchange force

Given (1.8), the potential follows once we have calculated the matrix element $\mathcal{M}$ using (1.6). Let us start with a simple example of classical Boltzmann-distribution.

a.Boltzmann-distribution: $n_\pm = e^{[(\pm\mu - |k^0|)/T]}$.
With this distribution, the integrations involved in the calculation of potentials can be easily done by conveniently choosing the order in which they are performed. The results can be expressed again in terms of Bessel functions and read [15]:

$$V_T^{Dirac}(r) = -\frac{G_F^2 m_\nu^4 g_V g_V'}{\pi^3 r}\cosh(\mu/T) \times \left[\frac{K_1(\rho)}{\rho} + \frac{4K_2(\rho)}{\rho^2}\right] \quad (2.1)$$

and

$$V_T^{Majorana}(r) = -\frac{4G_F^2 m_\nu^4 g_V g_V'}{\pi^3 r}\frac{K_2(\rho)}{\rho^2} \quad (2.2)$$

where we have defined

$$\rho \equiv \frac{m_\nu}{T}\sqrt{1 + (2rT)^2}. \quad (2.3)$$

For massless neutrinos (and $\mu = 0$) both potentials collapse to

$$V_T(r) = -\frac{8G_F^2 m^4 g_V g_V'}{\pi^3 r}\frac{1}{\rho^4} \quad (2.4)$$

which is the result given in reference [14]. We see that for distances much larger than T^{-1} the potential reads

$$V_T(r) \simeq -\frac{G_F^2 g_V g_V'}{2\pi^3 r^5}. \quad (2.5)$$

When added to the vacuum result (1.4), the total potential is

$$V_{tot}(r) \simeq -\frac{G_F^2 g_V g_V'}{4\pi^3 r^5} \quad (2.6)$$

that is, in the presence of a thermal neutrino background, distributed according to the Boltzmann distribution, the original Feinberg-Sucher force switches sign, i.e. a repulsive force turns into an attractive one. On the other hand, for $(rT \ll 1)$, the temperature dependent potential (16) behaves as follows

$$V_T(r) \simeq -\frac{8G_F^2 g_V g_V' T^4}{\pi^3 r} \quad (2.7)$$

which is negligible compared to the vacuum contribution in equation (1.4).

b.Cold degenerate neutrinos: $n_+ = \theta(\mu - k^0)$.
The main interest in such distributions is the physics of supernova. Here we find for the potential (assuming $m_\nu = 0$)

$$V_T(r) \simeq -2V_{FS}(r)[1 - \cos 2\mu r - \mu r \sin 2\mu r] \quad (2.8)$$

which agrees with the result given in [14] and in [22, 16].

c.Fermi-Dirac: $n_\pm = (e^{(k^0 \mp \mu)/T} + 1)^{-1}$
The result for $m_\nu = 0$ can be written in the form [16]

$$V_T(r) = -\frac{G_F^2 g_V g_V'}{4\pi^3 r^4}\left[1 - r\frac{d}{dr}\right] I_T(r;\mu) \quad (2.9)$$

with the final result being expressible in terms of the hypergeometric function $F(a, b; c; z)$. Indeed, we have

$$\begin{aligned} I_T(r;\mu) = \frac{1}{4r}[&F(1, -2irT; 1 - 2irT; -e^{-\mu/T}) \\ &+ F(1, -2irT; 1 - 2irT; -e^{\mu/T}) \\ &+ F(1, 2irT; 1 + 2irT; -e^{-\mu/T}) \\ &+ F(1, 2irT; 1 + 2irT; -e^{\mu/T}) \\ &- 8\pi rT \cos 2r\mu \,\mathrm{csch}\, 2\pi rT], \end{aligned} \quad (2.10)$$

Let us take nondegenerate neutrinos ($\mu = 0$). After some algebra we obtain $I_T(r;\mu=0)$ in the form

$$I_T(r;\mu=0) = \frac{1}{2r}[1 - 2\pi rT \,\mathrm{csch}\, 2\pi rT] \quad (2.11)$$

such that the temperature dependent potential for nondegenerate relic neutrinos is:

$$V_T(r) = -V_{FS}(r) \times \\ [1 - \pi rT \,\mathrm{csch}\, 2\pi rT(1 + 2\pi rT \,\coth 2\pi rT)] \quad (2.12)$$

where $V_{FS}(r)$ is the Feinberg-Sucher potential. At large distances (i.e. $rT \gg 1$) the temperature dependent effect exactly cancels the vacuum component,

$$V_T(r) \approx -V_{FS}(r) \quad (2.13)$$

This is, indeed, a drastic effect of relic cosmic neutrinos. It makes the long tail of the Feinberg-Sucher force effectively non-operative. Note that the new scale set by the temperature is $T^{-1} \simeq 1mm$. In a supra-millimeter range a future experiment searching for the Feinberg-Sucher force should give a zero result due to cosmic relic neutrinos!

3. Two-boson exchange forces

In the following we will need two generic interactions: one of heavy Higgses (called in the following H) with fermions and of two light or massless pseudoscalars a with the heavy scalars H. We assume that the pseudoscalars do not have tree level coupling to the fermions. Contracting the heavy Higgs propagator, the Feynman diagram looks formally the same like the diagram responsible for the Feinberg-Sucher force, of course with the internal fermions exchanged by bosons [17]. For two-boson forces arising from yet different Feynman diagrams see [23].

a.Light pseudoscalar. Consider the case of some generic non-derivative interaction terms of the form

$$\mathcal{L}_{int} = g_{Hff}\bar{f}fH, \quad \mathcal{L}'_{int} = g_{Haa}aaH \quad (3.1)$$

where f are standard fermions, H is the heavy Higgs with mass m_H and a is the very light pseudoscalar with mass m_a. It is convenient to define global coupling constants as

$$G(q^2) \equiv \frac{g_{Hff}g_{Haa}}{q^2 - m_H^2}, \quad G'(q^2) \equiv \frac{g_{Hf'f'}g_{Haa}}{q^2 - m_H^2} \quad (3.2)$$

To compute the potential we now use equation (1.9) and obtain for the discontinuity

$$[\Gamma]_t = \int \frac{d^4k}{(2\pi)^6}\delta(k^2 - m_a^2)\delta(\bar{k}^2 - m_a^2)\theta(k^0)\theta(\bar{k}^0) \\ = \frac{1}{8\pi}\sqrt{1 - \frac{4m_a^2}{t}}. \quad (3.3)$$

with $[\mathcal{M}]_t = -i2G(0)G'(0)[\Gamma]_t$ which has to be inserted into (1.9) to compute the final expression [17].

$$V(r) = -\frac{G(0)G'(0)}{4\pi^2 r}\int_{4m_a^2}^{\infty} dt[\Gamma]_t \exp(-\sqrt{t}r) \\ = -\frac{G(0)G'(0)m_a}{8\pi^3 r^2}K_1(2m_a r) \\ \simeq -\frac{G(0)G'(0)}{16\pi^3 r^3} \quad (3.4)$$

where the last expression is valid for $rm_a \ll 1$.

b.The case of Goldstone bosons. It is now convenient to use the following derivative interaction

$$\mathcal{L}''_{int} = \tilde{g}_{Haa}H(\partial^\mu a)(\partial_\mu a). \quad (3.5)$$

We define also over-all coupling constants $\tilde{G}(q^2)$ and $\tilde{G}'(q^2)$ in analogy to (3.2). As in the preceding case we start with the dispersion theoretical definition of the potential i.e. eq. (1.9) where we denote now the matrix element by $\tilde{\mathcal{M}}$ given by

$$\tilde{\mathcal{M}} \simeq -2i\tilde{G}(0)\tilde{G}'(0)\cdot\tilde{\Gamma} \\ \tilde{\Gamma} = \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2}\frac{i}{\bar{k}^2}(k\cdot\bar{k})^2 \quad (3.6)$$

where as before $\bar{k} = q - k$. For the discontinuity we obtain

$$\left[\tilde{\Gamma}\right]_t = \frac{q^\mu q^\nu}{(2\pi)^2}\int d^4k\,\delta(k^2)\delta(\bar{k}^2)\,k_\mu k_\nu \\ = \frac{q^\mu q^\nu}{(2\pi)^2}\frac{\pi}{2}\left[\frac{1}{3}\left(q_\mu q_\nu - \frac{1}{4}g_{\mu\nu}q^2\right)\right] \\ = \frac{t^2}{32\pi} \quad (3.7)$$

with $q^2 = t$ as usual. It remains to calculate the integral transform of this discontinuity. To distinguish the potential from the results in the preceding section we will call the potential due to two pseudoscalar exchange arising from the interaction (3.5), $\tilde{V}$. For the latter we get [17]

$$\begin{aligned}\tilde{V}(r) &= -\frac{\tilde{G}(0)\tilde{G}'(0)}{128\pi^3 r}\int_0^\infty dt \exp(-\sqrt{t}r)t^2 \\ &= -\frac{15\tilde{G}(0)\tilde{G}'(0)}{8\pi^3 r^7}. \qquad (3.8)\end{aligned}$$

Had we used a non-derivative coupling scheme for the Goldstone boson interaction with heavy Higgses we would get in the zeroth order $(GG')(q^2 = 0) = 0$ and only in the next order $(GG')(q^2) \propto q^4$. This actually means that the calculations with the two different coupling schemes yield the same result which is also a consequence of a general theorem. The latter ensures independence of physical results on the parameterization of the fields [24].

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