

Neutrinos: mass spectrum and mixing

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ABSTRACT: The program of reconstruction of the neutrino mass and flavor spectrum is outlined and the present status of research is summarized. We describe the role of future solar and atmospheric neutrino experiments, detection of the Galactic supernovae and double beta decay searches in accomplishing this program. The LSND result and four neutrino mass spectra are considered in connection with recent searches for the sterile components in the solar and atmospheric neutrino fluxes.

1. Introduction

1.1 Two remarks

There is a hope that *detailed* information on the neutrino mass spectrum and lepton mixing may eventually shed the light on

- the origin of the neutrino mass,
 - quark-lepton symmetry, unification of quarks and leptons, Grand Unification,
 - fermion mass problem,
 - physics beyond the standard model in general.
- “Detailed information” are the key words: just knowledge that masses are small is not enough to clarify the points.

Results on atmospheric neutrinos [1] show that the simplest possibility – hierarchical mass spectrum with small flavor mixing has not been realized. The guideline from the quark sector is essentially lost. In this connection we should consider without prejudice all possible mass and mixing spectra which do not contradict experiment.

1.2 The present status.

There are three leptonic flavors: ν_α , $\alpha = e, \mu, \tau$ and at least three neutrino mass eigenstates ν_i with eigenvalues m_i ($i = 1, 2, 3$). The program of reconstruction of the spectrum consists of the determination of

- number of mass eigenstates,

- masses m_i ,
- distribution of the flavor in the mass eigenstates described by the mixing matrix $U_{\alpha i}$ (admixture of the flavor α in the eigenstate ν_i),
- complex phases of $U_{\alpha i}$ and m_i .

An appearance of more than three mass eigenstates implies an existence of sterile neutrinos.

What is the present status?

The atmospheric neutrino data provide us with the most reliable information. With high confidence level we can say that the data imply the ν_μ oscillations with maximal or near maximal depth. Moreover, the oscillations are driven by non-zero Δm^2 . From this interpretation we can infer that

- (i) There is at least one mass eigenstate with

$$m_a \geq \sqrt{|\Delta m_{atm}^2|} \sim (4 - 6) \cdot 10^{-2} \text{ eV}. \quad (1.1)$$

Further implications depend on assumptions about the number of mass eigenstates and the type of mass hierarchy. In the case of 3ν -spectrum with *normal* mass hierarchy (fig.1, left panel), $m_3 \gg m_2, m_1$, the heaviest state ν_3 has the mass (1.1). If the spectrum has the *inverted* mass hierarchy, ν_3 is the lightest state, $m_3 \ll m_a$, and ν_1, ν_2 form a system of degenerate neutrinos with $m_2 \approx m_1 \approx m_a$ (fig.1, right panel). In the case of completely degenerate spectrum one has $m_3 \approx m_2 \approx m_1 \gg m_a$.

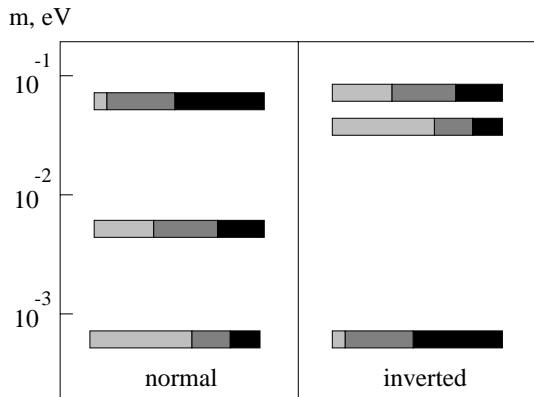


Figure 1: The 3 ν mass spectra with normal and inverted mass hierarchy. Boxes show admixtures of different flavors in the mass eigenstates: electron (light grey), muon (grey) and tau (black).

(ii) The admixture of the ν_μ flavor in the ν_3 mass eigenstate is

$$|U_{\mu 3}|^2 = 0.3 - 0.7 \quad (90\% \text{ CL}). \quad (1.2)$$

(iii) The admixture of the electron neutrino in the third state is zero or small [2]:

$$|U_{e 3}|^2 \leq 0.015 - 0.05. \quad (1.3)$$

Thus, ν_μ is mixed almost maximally with ν_τ or/and ν_s .

(iv) The $\nu_\mu - \nu_\tau$ channel gives better description of the data than $\nu_\mu - \nu_s$ one: the latter is disfavored at 3 σ level [3]. Substantial contribution of the sterile channel is however possible already at 2 σ level.

(v) We assume that ν_1 and ν_2 are responsible for the solution of the solar neutrino problem. The best fit values of the oscillation parameters from all solution regions (LMA, SMA, LOW, VO) satisfy inequality

$$\Delta m_{\odot}^2 \ll \Delta m_{atm}^2.$$

That is, the hierarchy of the Δm^2 exists.

(vi) The distribution of the electron flavor depends on the solution of the ν_{\odot} -problem.

Clearly, with this information we are just in the beginning of realization of the program. In what follows I will consider the next steps.

1.3 Further questions

The neutrino mass and flavor spectrum can be described by a set of the *oscillation parameters* (Δm^2 , $|U_{\alpha i}|^2$), δ_{CP}) which can in principle be measured in the oscillation experiments, and the *non-oscillation parameters* like the absolute scale of neutrino mass m_1 and relative complex phases of the mass eigenvalues.

Further questions to be answered can be divided in to two categories:

As far as the oscillation parameters are concerned we should clarify:

1. Channel of oscillations of the atmospheric neutrinos: $\nu_\mu - \nu_\tau$, $\nu_\mu - \nu_s$ or mixed case $\nu_\mu - \nu_\tau/\nu_s$. In general, the answer should be formulated as an upper bound on the admixture of the sterile neutrino in the third state: $|U_{s3}|^2$
2. Admixture of the ν_e flavor in the ν_3 $|U_{e3}|^2$.
3. Sign of Δm_{atm}^2 which is equivalent to identification of the type of hierarchy.
4. Number of the mass eigenstates involved in the atmospheric neutrino studies (number of Δm^2) this includes searches for the effects of Δm_{\odot}^2 responsible for the solar neutrino oscillations and Δm_{LSND}^2 which can explain the LSND result.
5. Distribution of the ν_e flavor in ν_1 and ν_2 states, that is, measurements of $|U_{e1}|^2$ and $|U_{e2}|^2$.
6. Mass splitting between ν_2 and ν_1 : Δm_{21}^2 .
7. Channel of the solar neutrino oscillations $\nu_e - \nu_\mu/\nu_\tau$ or/and $\nu_e - \nu_s$. Results can be formulated as bounds on admixtures of the sterile neutrino in the ν_1 and ν_2 components.
8. Number of Δm^2 relevant for conversion of solar neutrinos.
9. CP-violating phase in the neutrino oscillations δ_{CP} .

As far as non-oscillation parameters are concerned, we need to know

10. The absolute mass scale m_1 .
11. Level of degeneracy of the spectrum.
12. Phases of the mass eigenstates.
13. Violation of the lepton number: that is, establishing that neutrinos are the majorana particles.

In what follows I will comment on perspectives to answer these questions.

2. Distribution of the electron flavor, Hierarchy and Degeneracy

2.1 Where are ν_1 and ν_2 mass eigenstates?

Measurements of Δm_{\odot}^2 will be possible in

1. Future studies of the solar neutrinos. The data on the Day-Night asymmetry as well as zenith angle distribution of events are particularly sensitive to Δm_{\odot}^2 in the LMA and LOW regions.

2. In the LMA solution region the KAMLAND (measurements of the spectrum distortion of the energy spectrum of events) will be able to determine Δm_{\odot}^2 with rather high accuracy.

3. Observations of the excess of the e-like events in the sub-GeV sample of the atmospheric neutrinos will testify for the high values of the Δm_{\odot}^2 in the LMA region.

2.2 Distribution of the electron flavor

The electron flavor should be mainly distributed in the ν_1 and ν_2 mass eigenstates and this can be determined by solving the solar neutrino problem.

According to SMA solution the ν_e flavor is mainly in the lightest mass eigenstate: $|U_{e1}|^2 \approx 1$ with small admixture in the second state: $|U_{e2}|^2 \approx \sin^2 2\theta_{\odot}$.

All other solutions require large mixing: for LMA $|U_{e2}|^2 \approx 0.2 - 0.5$ at 99.7 % CL. For LOW solutions mixing is closer to maximal: $|U_{e2}|^2 \approx 0.4 - 0.6$, that is, the data do not exclude maximal mixing and mixing from the “dark side” when ν_e has larger admixture in the heaviest state: $|U_{e2}|^2 > |U_{e1}|^2$. Vacuum oscillation solutions imply also close to maximal mixing and symmetric regions in the normal and the dark sides.

2.3 U_{e3}

The future long-baseline experiments MINOS [4], CERN-GS [5] will be able to mildly improve present CHOOZ bound. An estimated sensitivity is at most $|U_{e3}|^2 \approx 5 \cdot 10^{-3}$ at $\Delta m_{atm}^2 = 3 \cdot 10^{-3} \text{ eV}^2$.

Further improvements of the reactor bound are rather difficult (see [6]). Signatures of non-zero $|U_{e3}|^2$ exist in the atmospheric neutrinos. In fact, best fit value of $|U_{e3}|^2$ from the atmospheric neutrinos differs from zero, although the deviation is statistically insignificant. However, it is difficult to improve the situation with present experiments and the possibilities of future atmospheric neutrino detectors deserve special study.

Registration of the neutrino bursts from the Galactic supernova by existing detectors SK, SNO (several thousands events) will give information about $|U_{e3}|^2$ down to $10^{-5} - 10^{-4}$ [7].

Even better sensitivity $|U_{e3}|^2 > 3 \cdot 10^{-5}$ may be achieved at the neutrino factories [8].

Intuitively, it is difficult to expect very small $|U_{e3}|^2$ if mixing between the second and the third generation is almost maximal and the mixing of the electron neutrinos is also maximal or large (unless some special arrangements are done). This has been quantified recently in terms of the neutrino mass matrices which lead to the solutions of the solar and atmospheric neutrino problems [9]. In the assumption that there is no special fine tuning of the matrix elements $m_{e\mu}$ and $m_{e\tau}$, so that $|m_{e\mu} - m_{e\tau}| \sim \max[m_{e\mu}, m_{e\tau}]$, the following relation has been found [9]:

$$U_{e3}^2 \approx \frac{1}{4} \frac{\tan^2 2\theta_{\odot}}{\sqrt{1 + \tan^2 2\theta_{\odot}}} \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2}. \quad (2.1)$$

For parameters from the LMA region we get values

$$|U_{e3}|^2 = 0.003 - 0.02, \quad (2.2)$$

where the upper edge is the present experimental bound.

2.4 Hierarchy and Degeneracy

Phenomenology of schemes with normal and inverted mass hierarchy is different. The hierarchy can be identified by studies of (i) the neutrinoless double beta decay, (ii) Earth matter effect on 1 - 3 mixing in the atmospheric neutrinos and in the long baseline experiments, (iii) neutrino burst from supernovae.

In the scheme with inverted hierarchy the contribution of neutrinos to the energy density of the Universe can be two times larger than

that in the scheme with normal hierarchy: $\Omega_\nu \geq 2\sqrt{\Delta m_{atm}^2} n_\nu$ (n_ν is the concentration of one neutrino species).

The scheme with normal mass hierarchy (or in general, the scheme with ν_3 being the heaviest state) may have partial degeneracy when $m_2 - m_1 \ll m_2$. In this case for a given oscillation pattern both Ω_ν and m_{ee} can be larger than in the hierarchical case.

The scheme with complete degeneracy is not excluded, being however substantially restricted by the neutrinoless double beta decay data.

3. Identifying the solution of the ν_\odot problem

Identification of the solution is one of the major steps in the reconstruction of the spectrum which will significantly determine further strategy of the research. It will allow us to: (i) measure $\Delta m_{21}^2 \equiv \Delta m_\odot^2$ as well as the distribution of the electron flavor: $|U_{e1}|^2, |U_{e2}|^2$; (ii) find or restrict the presence of the sterile neutrinos, (iii) estimate a possibility to measure the CP-violation and to discover the neutrinoless beta decay.

Let us describe some recent results.

3.1 Flux during the night

The zenith angle (θ_Z) distributions of events during the nights differ for different solutions and therefore precise measurements of the distribution can be used to discriminate among the solutions.

The LMA solution predicts rather flat distribution of events with slightly lower rate in the first night bin N1 ($\cos \theta_Z = 0 - 0.2$). The reason is that the oscillation length is small and substantial averaging of oscillations occurs in all the bins [10].

For the LOW solution, the maximal rate is expected in the second night bin N2 ($\cos \theta_Z = 0.2 - 0.4$) [11]. Indeed, for parameters from the LOW region, the oscillation length in matter is determined basically by the refraction length, $l_m \approx l_0$, and it depends weakly on $E/\Delta m^2$ and mixing. No averaging occurs. It turns out that the

average length of the neutrino trajectories in the N2 bin equals half of the refraction length, so that the phase is about π and the oscillation effect is maximal. The length of the trajectory is about l_0 in the N3 bin, where minimum of the rate is expected. The height of the peak in N2 bin decreases with Δm^2 .

In the case of SMA solution maximal rate is expected in the N5 (core) bin [12], where the parametric enhancement of oscillations can take place. The peak decreases with mixing angle and at $\sin^2 2\theta \sim 3 \cdot 10^{-3}$ it transforms to the deep.

No enhancement of the night rate should be seen for VO solution.

Thus, using information on integrated day-night asymmetry and signals in N2 and N5 bins one can identify the solution.

Clearly, just using energy spectra during the day and during the night one loses important information.

Notice that the zenith distribution observed by SK does not fit any of expected distributions: maximal rate is in the N1 bin, and there is no enhancement of rate neither in N2 nor in N5 bins. Clearly statistics is not enough to make any conclusion.

In the SNO the expected zenith angle distributions have similar character, however absolute value of the regeneration effect is larger due to absence of damping related to ν_μ and ν_τ contribution to the SK signal.

3.2 Correlations of observables

Present searches for the “smoking guns” of certain solutions of the ν_\odot -problem give just $(1-2)\sigma$ indications (which some time are in favor of different solutions). To enhance the identification power of the analysis we suggest to study correlations of various observables [13]. Indeed, correlations of observables appear for different solution of the ν_\odot -problem and they can be considered as signatures of corresponding solutions.

The observables (denote them by X, Y) include rates of events at different detectors, characteristics of spectrum distortion (e.g., shift of the first moment of the spectrum) and parameters of the time variations of signals (day-night asymmetry, seasonal asymmetry, etc.). To find

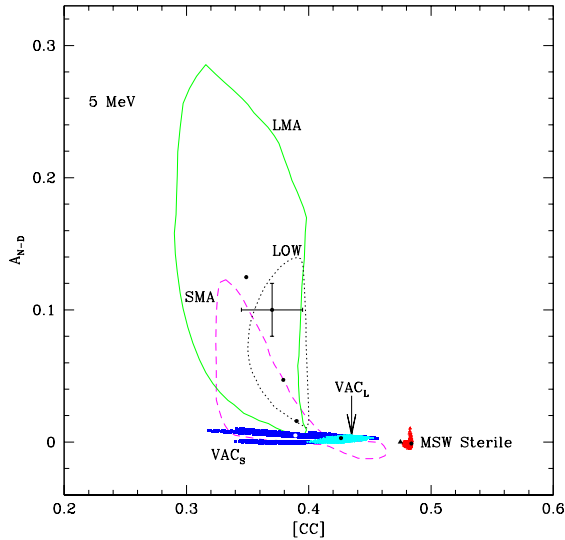


Figure 2: The allowed regions for the day-night asymmetry versus reduced rate (CC events at SNO above 5 MeV). The best fit points for each solution are indicated by black circles and no-oscillation case is shown by a triangle. The cross is a simulated measurement with 1σ error bars. (From [13]).

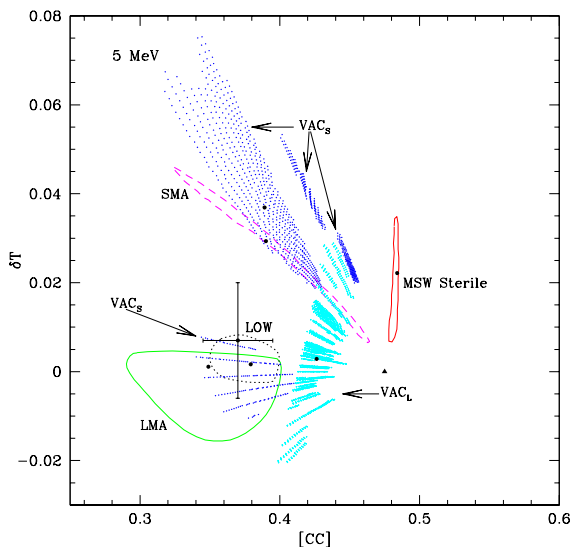


Figure 3: The allowed regions for the first moment of the recoil energy spectrum versus reduced rate (CC events at SNO above 5 MeV). The best fit points for each solution are indicated by black circles and no-oscillation case is shown by a triangle. The cross is a simulated measurement with 1σ error bars. (From [13]).

the correlations we have performed the mapping of the solution regions in the $\Delta m^2 - \sin^2 2\theta$ plane onto the plane of observables X and Y . If $\Delta m^2 -$

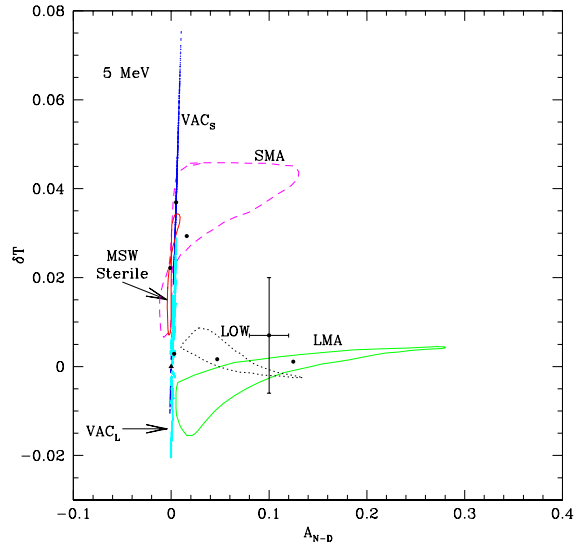


Figure 4: The allowed regions for the day-night asymmetry versus first moment of the energy spectrum (CC events at SNO above 5 MeV). The best fit points for each solution are indicated by black circles and no-oscillation case is shown by a triangle. The cross is a simulated measurement with 1σ error bars. (From [13]).

$\sin^2 2\theta$ region projects onto the line in the $X - Y$ the correlations is very strong. In general, the criteria for strong correlation is that the area of the projected region, S_{XY} , is much smaller than the product $\Delta X \times \Delta Y$, where ΔX and ΔY are allowed intervals of X and Y when they are treated independently.

In fig. 2 - 4 we show mapping of the $\Delta m^2 - \sin^2 2\theta$ regions of solutions (taken at 99% C.L.) onto the planes of various SNO observables which will be measured in the charged current interactions above 5 MeV (electron energy).

In fig. 2 we show mapping of the solution regions on to the $[CC] - A_{D-N}$ plane, where $[CC] \equiv N_{obs}/N_{SSM}$ is the reduced rate of the charged current events and $A_{D-N} \equiv 2(N-D)/(N+D)$ is the day-night asymmetry of the charged current events. The region of solution based on the $\nu_e \rightarrow \nu_s$ - conversion is clearly separated from all other regions (basically by $[CC]$). There is a significant overlap of the SMA, LOW and LMA regions.

Fig. 3 shows mapping onto the $[CC] - \delta T$ - plane, where δT is the shift of the first moment of the recoil electron spectrum due to oscillations. One can see strong correlation of the observables

for the SMA solution as well as for VAC_S solution.

The mapping of the solutions onto the $A_{D-N} - \delta T$ is shown in the fig. 4. Some correlations are seen for the LMA, LOW and SMA solutions.

Notice that in all the figures there are non-trivial avoidance zones where no points are expected. We show by crosses simulated results of measurements with 1σ error bars which correspond to 5000 CC events and include all known systematic errors. As follows from the figures, SNO may or may not give strong discrimination among solutions depending on real measured values of observables.

3.3 Large or Maximal?

The three among five solutions of the solar neutrino problem require large mixing angle of the electron neutrino. Moreover, the LMA solution gives the best global fit of the data. The best fit of the atmospheric neutrino data corresponds to maximal mixing. Is large (or maximal) mixing the generic property of leptons? What is a deviation from maximal mixing?

These questions are important for theory [14]: The deviation from maximal mixing can be related to small parameter $\lambda \sim 0.22$ which characterizes the fermion mass hierarchy and appears in the theories with flavor symmetry. We describe the deviation by $\epsilon \equiv \cos 2\theta$, so that $\sin^2 \theta = (1 - \epsilon)/2$ ($\epsilon = 0$ at the maximal mixing). Depending on model one can get $\epsilon = \lambda^n$, where usually $n = 1$ or 2 , or $\epsilon \sim \sqrt{m_e/m_\mu} = 0.07$, or $\epsilon \sim m_e/m_\mu = 0.005$, etc..

In contrast with theory, maximal mixing is not a special point for phenomenology. Nothing dramatic happens when ϵ changes the sign: no divergencies or discontinuities appear, all observables depend on ϵ rather smoothly [15, 11].

Performing a global fit of all available solar neutrino data we find [11] that maximal mixing is allowed in the LMA region at 99.9 % CL, and in LOW region at 99 % CL. For $\epsilon = 0.07$ the interval $\Delta m^2 = (2 - 30) \cdot 10^{-5} \text{ eV}^2$ (LMA) is accepted at 99 % CL, etc..

Future perspectives of measurement of the deviation depend significantly on the range of Δm^2 .

Observables depend linearly on ϵ in the range of the MSW conversion (LMA, LOW). In particular, the survival probability is proportional to $(1 - \epsilon)$, the day-night asymmetry $\propto (1 + \epsilon)$, the distortion of spectrum $\propto \epsilon$. Clearly, the dependences of observables are asymmetric under interchange ($\epsilon \rightarrow -\epsilon$). In contrast, in the VO regions the dependence of observables on ϵ is quadratic: $\propto (1 + \epsilon^2)$ in the average oscillation case and $\propto (1 - \epsilon^2)$ in the non-averaged case. The observables as functions of ϵ are symmetric under ($\epsilon \rightarrow -\epsilon$).

Thus, for small ϵ the sensitivity of measurements of the deviation is much higher in the MSW regions of Δm^2 . The most precise determination of ϵ will be possible with the SNO results. Simultaneous measurements of the double ratio $[NC]/[CC]$ (of the neutral to charged current reduced rates) and the day-night asymmetry of the CC events will allow to determine ϵ with accuracy $\Delta\epsilon \sim 0.07$ (1σ) [11].

4. Double beta decay, supernova neutrinos etc.

4.1 Double beta decay and test equalities

The effective Majorana mass of the electron neutrino m_{ee} which determines the rate of the neutrinoless double beta decay: $\Gamma \propto m_{ee}^2$, depends both on oscillation and on non-oscillation parameters (m_1, ϕ_i – absolute mass scale, phases of the mass eigenstates), so that in general no predictions for m_{ee} can be made on the basis of oscillation data without additional assumptions.

The most important assumptions concern with the type of the mass hierarchy and the level of degeneracy of the spectrum. Definite predictions can be given if dominant contribution to m_{ee} comes from one mass eigenstate so that the phases ϕ_i are irrelevant.

Remarks:

For any oscillation pattern (values of Δm^2 and $|U_{\alpha i}|^2$) the effective Majorana mass of the electron neutrino relevant for the $\beta\beta_{0\nu}$ decay, m_{ee} , can take any value from zero to experimental upper bound for the normal mass hierarchy

and $|m_{ee}|$ has non-zero minimum value for the inverted hierarchy provided that $|U_{e1}|^2 > 1/2$ [16].

If the neutrinoless double beta decay will be discovered and the rate will give m_{ee} , then under assumption that the Majorana masses are the only source of the decay, we can say that at least one mass should satisfy inequality $m_j > m_{ee}/n$, where n is the number of mass eigenstates [16].

Definite predictions for m_{ee} can be given in the context of certain neutrino mass spectra (see [17, 16] and references therein): m_{ee} can be related (especially in the cases when dominant contribution comes from one mass eigenstate) with the oscillation parameters. Therefore coincidence of the measured value m_{ee} with some combination of the oscillation parameters will testify for certain neutrino mass spectrum. We can call these relations the *test equalities*. (In some cases one can get certain inequalities for m_{ee} which will allow to discriminate corresponding scheme.) Notice also that due to uncertainties in the nuclear matrix elements it will be difficult to check the equalities with precision better than say 50 %.)

Let us give examples of the test equalities:

1). In the case of normal mass hierarchy, and SMA, LOW or VO solutions of the solar neutrino problem the dominant contribution comes from ν_3 :

$$m_{ee} \approx \sqrt{\Delta m_{atm}^2} |U_{\mu 3}|^2.$$

2). For normal mass hierarchy, LMA solution and small $|U_{e3}|^2$ one gets

$$m_{ee} \approx \sin^2 \theta_{\odot} \sqrt{\Delta m_{\odot}^2} \leq 5 \cdot 10^{-3} \text{eV} \quad (4.1)$$

which can be tested by the 10 ton version of GENIUS experiment.

3). In the case of inverted mass hierarchy, SMA or LMA solutions with equal phases of the mass eigenvalues m_1 and m_2 , one gets [17, 16]

$$m_{ee} \approx \sqrt{\Delta m_{atm}^2} = (5 - 7) \cdot 10^{-2} \text{eV} \quad (4.2)$$

which can be achieved already in the next generation of the double beta decay experiments.

4). For inverted mass hierarchy and large mixing solutions with relative phase of the degenerate states $\phi_1 - \phi_2 = \pi$ (which holds for the

pseudo Dirac system) we find

$$m_{ee} \approx \cos 2\theta_{\odot} \sqrt{\Delta m_{atm}^2}.$$

As follows from the above analysis, the double beta decay searches may help to determine the type of mass hierarchy: the bound $m_{ee} < 3 \cdot 10^{-3} \text{eV}$ will testify for the normal hierarchy, whereas values $m_{ee} > 10^{-2} \text{eV}$ are the signature of the inverted mass hierarchy provided that $\sin^2 2\theta_{\odot} \leq 0.9$. Predictions overlap in the case of partial or complete degeneracy of spectra.

4.2 Supernova neutrinos

In the three neutrino scheme there are two relevant resonances: high (density) resonance at $\rho_h \sim 10^4 \text{g/cm}^3$ the low resonance at $\rho_l \sim 10 - 100 \text{g/cm}^3$ related to Δm_{atm}^2 and Δm_{\odot}^2 correspondingly. Since at the production point $\rho \gg \rho_h, \rho_l$, the supernova neutrinos probe whole neutrino mass spectrum. Moreover, mixings associated with both Δm^2 can be matter enhanced.

In general, the neutrino conversion effects are influenced by uncertainties in the density profile and in characteristics of the original spectra. However, in spite of uncertainties related to the density profile and, especially, to parameters of the original spectra, some observables are largely *supernova model independent* which opens the possibility to get reliable information on neutrino mass spectrum. In particular, inequalities of the average energies:

$$E(\nu_e) < E(\bar{\nu}_e) < E(\nu_{\mu}) \quad (4.3)$$

are SN model independent. Violation of these inequalities will testify for the neutrino conversion. It is expected that spectra emitted during short time intervals $\Delta t \ll 10 \text{s}$ are “pinched”. Observation of wide spectra will testify for its compositeness which appears as a result of conversion.

Another SN model independent possibility is to study the Earth matter effects on the neutrino fluxes from supernovae. The oscillations of the SN neutrinos in the matter of the Earth can induce irregular structures in the otherwise smooth energy spectra. These oscillations will

lead also to different signals at different detectors (for which neutrino trajectories in the Earth are different).

The observable effects are the result of the interplay of neutrino conversion inside the star and oscillations inside the Earth. The level crossing schemes are different for normal and inverted mass hierarchies. The high resonance is in the neutrino channel if the hierarchy is normal and it is in the antineutrino channel for the inverted mass hierarchy. This can lead to completely different patterns of conversion.

If $|U_{e3}|^2 > 10^{-3}$, the conversion in the high resonance is completely adiabatic. Taking into account also that original ν_μ and ν_τ fluxes are practically identical one gets that in the normal hierarchy case: (i) ν_e converts completely to ν_μ/ν_τ , (ii) at the Earth ν_e should have hard spectrum of the original ν_μ and (iii) the Earth matter effect does not influence this flux. In contrast, the oscillation effect in the matter of the Earth can be observed in the $\bar{\nu}_e$ -spectrum.

In the case of inverted mass hierarchy ν_e and $\bar{\nu}_e$ interchange the roles: (i) $\bar{\nu}_e$ transforms in the star into $\bar{\nu}_\mu/\bar{\nu}_\tau$, (ii) at the surface of the Earth $\bar{\nu}_e$ -flux will have a hard spectrum, (iii) the Earth matter effect should not be seen in the $\bar{\nu}_e$ signal but it can be observed in the ν_e -signal.

Thus, the fact of observation of the Earth matter effect in the $\bar{\nu}_e$ flux, but not in ν_e , will testify for the normal hierarchy. An opposite situation: the Earth matter effect in ν_e channel and an absence of the effect in $\bar{\nu}_e$ will be an evidence of the inverted mass hierarchy. Substantial matter effect is possible for the LMA parameters only.

If $|U_{e3}|^2 \ll 10^{-3}$ the high resonance is inefficient and significant matter effect can be observed both in $\bar{\nu}_e$ and ν_e spectra.

5. More than three neutrinos

It is widely accepted that simultaneous explanation of the solar, atmospheric and LSND [18] results in terms of oscillations requires an existence of the sterile neutrino (see e.g. [19]). (Three strongly different values Δm^2 implied by

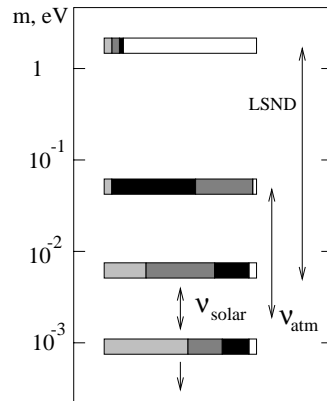


Figure 5: The $(3 + 1)$ spectrum of the neutrino mass. Boxes show the admixtures of the flavors in the mass eigenstates: electron (light grey) muon (grey) tau (black) and sterile (white).

these results can not be obtained with three mass eigenstates.)

Less appreciated fact is that the explanation requires the sterile neutrino to be a dominant component in oscillations of solar or atmospheric neutrinos. That is, either $\nu_e \rightarrow \nu_s$ is the dominant channel of the solar neutrino conversion, or $\nu_\mu \rightarrow \nu_s$ is the dominant oscillation mode for the atmospheric neutrinos. The extreme situation is when sterile channels contribute 1/2 both in the solar and the atmospheric neutrino transformations.

This statement holds for the so called $(2 + 2)$ scheme of the neutrino mass in which two pairs of the mass eigenstates with mass splitting Δm_{\odot}^2 and Δm_{atm}^2 are separated by the mass gap related to Δm_{LSND}^2 . In these schemes one easily gets the depth of $\bar{\nu}_\mu - \bar{\nu}_e$ oscillations required by the LSND. It is claimed [19] that the $(2 + 2)$ scheme is the only possibility and the alternative schemes give too small mixing for the LSND. Situation, however, may change:

1. Recent atmospheric neutrino data disfavor $\nu_\mu \rightarrow \nu_s$ as a dominant mode of oscillations. Although up to 0.5 contribution of the sterile channel still gives a good fit [20].

2. The mode $\nu_e \rightarrow \nu_s$ although accepted, does not give the best fit of the solar neutrino data. The $\nu_e \rightarrow \nu_s$ solution can be identified soon by (i) equality of the reduced charged current rate at SNO and electron scattering rate at SK:

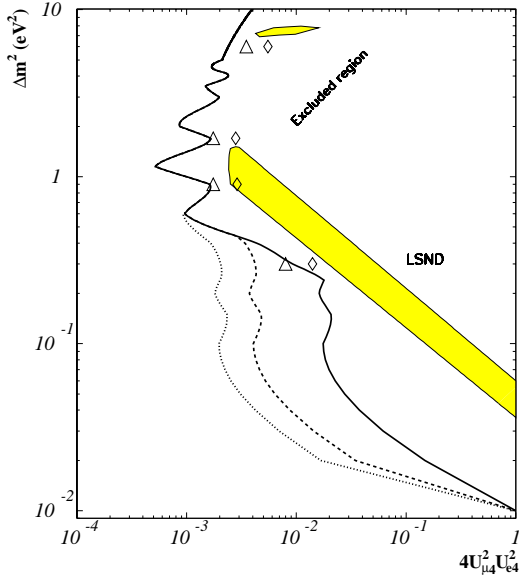


Figure 6: The bounds on the effective mixing parameter $\sin^2 2\theta_{LSND}$ for the LSND experiment. Shown are: the product of the 90 % CL bound from BUGEY and CDHS experiments (solid line) Solid, dashed and dotted lines below $\Delta m^2 \sim 0.2 - 0.3$ correspond to the bound from the atmospheric neutrinos for $\sin^2 2\theta_{23} = 1.0, 0.9, 0.8$ correspondingly. Rombs and triangles show 99 % and 95% CL bounds obtained from the BUGEY and CDHS results [22].

[CC] $\approx R_{SK}$; (ii) small Day-Night asymmetry ($< 2\%$) in SK and SNO; (iii) unchanged double ratio $[NC]/[CC] \approx 1$ (see [13]).

If it will be proven that both in the solar and atmospheric neutrinos the contribution of the sterile component is smaller than 1/2, the (2 + 2) scheme should be rejected, and the oscillation interpretation of the LSND will be questioned.

In this connection we have reconsidered the (3 + 1) scheme (see fig. 5) in which three mass eigenstates with splittings Δm_{atm}^2 and Δm_{\odot}^2 form the flavor block with small admixtures of sterile neutrino and the fourth state (predominantly sterile) is isolated from the flavor block by the mass gap Δm_{LSND}^2 [22]. Both solar and atmospheric neutrinos transform into active ones.

The effective mixing parameter for $\bar{\nu}_{\mu} - \bar{\nu}_e$ oscillations driven by Δm_{LSND}^2 equals

$$\sin^2 2\theta_{LSND} = 4U_{e4}^2 U_{\mu 4}^2, \quad (5.1)$$

where U_{e4}^2 and $U_{\mu 4}^2$ are the admixtures of the ν_e and ν_{μ} in the fourth mass eigenstate. U_{e4}^2 and

$U_{\mu 4}^2$ determine the ν_e - and ν_{μ} - disappearance in oscillations driven by Δm_{LSND}^2 and are restricted by the results of BUGEY [23] and CDHS [24] experiments correspondingly. For low values of Δm^2 better limit on $U_{\mu 4}^2$ follows from the atmospheric neutrinos [19]. In fig.4 we reproduce the bound on $\sin^2 2\theta_{LSND}$ obtained in [19] using Eq. (5.1) and the 90 % CL bounds from BUGEY and CDHS. The bound excludes whole allowed LSND region which led to conclusion that (3 + 1) scheme can not reproduce the LSND result. However, the question is: which confidence level should be prescribed to this bound?

For several values of Δm^2 we have found the 95 % CL and 99 % CL bounds on $\sin^2 2\theta_{LSND}$ in assumption that distributions of the $U_{\alpha 4}^2$ ($\alpha = e, \mu$) implied by the experiments are Gaussian (see fig. 6). We used central values of $U_{\alpha 4}^2$ and 90 % CL bounds published in the papers [23, 24] to restore parameters of the Gaussian distributions.

As follows from the fig. 6, in the range $\Delta m^2 \sim 1$ eV², the product of the 90% CL bounds corresponds to $\sim 95\%$ CL. The CL decreases with increase of Δm^2 . At 99% CL the LSND region at $\Delta m^2 \sim 1$ eV² becomes acceptable. Moreover, new analysis [25] shifts the allowed LSND region to smaller $\sin^2 2\theta$, so that now some part of the region is acceptable even at 95% CL.

The (3 + 1) scheme leads to a number of the phenomenological consequences which can be checked in the forthcoming experiments. It has also interesting astrophysical and cosmological consequences [22].

6. Conclusions

What are perspectives of the reconstruction of the neutrino mass and flavor spectrum?

1. Identification of the dominant mode of the atmospheric neutrino oscillations has a good chance with further studies at SK and LBL experiments. The bound on the presence of sterile neutrinos will be better than $|U_{s3}|^2 < 1/2$, which has important implications for theory.

2. Distribution of the electron flavor ($|U_{e1}|^2$, $|U_{e2}|^2$) as well as Δm_{\odot}^2 will be determined together with identification of the solution of the

ν_{\odot} -problem. Sooner or later (depending on our luck) this will be done by future measurement at SK, SNO, GNO, SAGE, BOREXINO. Studies of correlations of observables will allow us to enhance an identification power of analysis. Ironically, the solution of the ν_{\odot} problem can be found without solar neutrinos – in KAMLAND experiment. Important bound will be obtained on presence of ν_s .

There is some chance to measure $|U_{e3}|^2$ in the forthcoming LBL experiments.

3. Determination of the type of mass hierarchy, the level of spectrum degeneracy, the CP-violating phase and the absolute scale on the neutrino mass will require much more serious efforts. Progress will be related to the oscillation experiments with very long base lines and probably with direct measurements of the neutrino mass. Identification of the type of mass hierarchy and important bounds on value $|U_{e3}|^2$ can be obtained by the detection of the neutrino burst from the Galactic supernova. Observation of the $\beta\beta_{0\nu}$ -decay means the discovery of the lepton number violation and the Majorana nature of neutrinos. Measurements of the effective mass m_{ee} will allow to check “test equalities” which relate m_{ee} and oscillation parameters in the context of certain schemes of neutrino mass. In this way we will be probably able to identify the scheme and to get information on the absolute scale of neutrino mass.

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