Field theory insight from the AdS/CFT correspondence

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ABSTRACT: A survey of ideas, techniques and results from d=5 supergravity for the conformal and mass-perturbed phases of d=4 $\mathcal{N}$=4 Super-Yang-Mills theory.

1. Introduction

The AdS/CFT correspondence [1, 20, 11, 12] allows one to calculate quantities of interest in certain d=4 supersymmetric gauge theories using 5 and 10-dimensional supergravity. Miraculously one gets information on a strong coupling limit of the gauge theory– information not otherwise available– from classical supergravity in which calculations are feasible.

The prime example of AdS/CFT is the duality between $\mathcal{N}$=4 SYM theory and D=10 Type IIB supergravity. The field theory has the very special property that it is ultraviolet finite and thus conformal invariant. Many years of elegant work on 2-dimensional $CFT's$ has taught us that it is useful to consider both the conformal theory and its deformation by relevant operators which changes the long distance behavior and generates a renormalization group flow of the couplings. Analogously, in d=4, one can consider
a) the conformal phase of $\mathcal{N}$=4 SYM
b) the same theory deformed by adding mass terms to its Lagrangian
c) the Coulomb/Higgs phase.
Conformal symmetry is broken in the last two cases. There is now considerable evidence that the strong coupling behavior of all 3 phases can be described quantitatively by classical supergravity. In the conformal phase, the $AdS_5 \times S_5$ "ground state" of D=10 supergravity is relevant, while the phases with RG-flow are described by solitons or domain wall solutions.

The result of work by many theorists over the last few years is a quantitative picture of the strong coupling limit of the theory which is a remarkable advance on previous knowledge. In this lecture we will survey some of the ideas, techniques, and results on the conformal and massive phases of the theory. We attempt to reach non-specialists and are minimally technical.

2. The $\mathcal{N}$=4 SYM theory

The $\mathcal{N}$=4 SUSY Yang-Mills theory in d=4 with $SU(N)$ gauge group can be obtained by dimensional reduction of d=10 SYM. The fields consist of a gauge field A_μ, 4 Weyl fermions λ_a (in $\underline{4}$ of the R-symmetry group SU(4)) and 6 real scalars X^i (in $\underline{6}$ of SU(4)). Each of these fields can be taken as an $N \times N$ traceless Hermitian matrix of the adjoint of $SU(N)$. The R-symmetry or flavor symmetry group will play a more important role in our discussion than the gauge group. Explicit calculations have shown that the β function of the theory is zero up to three-loop order, and arguments for ultraviolet finiteness to all orders have been given [21, 22, 23]. Finiteness implies conformal symmetry. The conformal group of 4-dimensional Minkowski space is $SO(4,2){\sim}SU(2,2)$. This combines with the R-symmetry $SO(6){\sim}SU(4)$ and $\mathcal{N}$=4 SUSY to give

the superalgebra $SU(2,2|4)$ which is the over-arching invariance of the theory. It contains $4\mathcal{N}=16$ supercharges Q_α^a associated with Poincaré SUSY and $4\mathcal{N}=16$ additional conformal supercharges.

The observables of the theory are the correlation functions of gauge invariant operators which are composites of the elementary fields. These operators are classified in irreducible representations (irreps) of $SU(2,2|4)$. There are many such operators, but those of chief interest for AdS/CFT belong to short representations. The scale dimension Δ of these is fixed at integer or 1/2-integer values and correlated with the $SU(4)$ irrep. One basic reference on the irreps of $SU(2,2|4)$ is [25, 24] and there is considerable information in the current literature, e.g. see [2]

Chiral primary operators correspond to lowest weight states of short irreps. The most important ones are

$$TrX^k = TrX^{(i_1}X^{i_2}...X^{i_k)} \qquad (2.1)$$

where the parentheses indicate a symmetric traceless tensor in the $SO(6)$ indices. The rank k chiral primary has dimension Δ=k, and the Dynkin designation of its R-symmetry irrep is $(0,k,0)$. The dimensions of these irreps are 20' for k=2, 50 for k=3, 105 for k=4.... By applying Q_α^a to these operators, we obtain descendant operators in the same $SU(2,2|4)$ irrep. For example, the descendants of TrX^2 include the $SU(4)$ flavor currents J_μ^I and the stress tensor $T_{\mu\nu}$.

The dynamics of $\mathcal{N}=4$ SYM has been much explored through the years. We now mention a few aspects of this dynamics which will be illuminated later, through AdS/CFT.
1. Ward identities, anomalies, and $\mathcal{N}=1$ Seiberg dynamics can be combined [26] to show that 2-point functions of flavor currents and stress tensor are not renormalized. This means that all radiative corrections vanish and the exact correlation functions are given by the free-field approximation. Using $\mathcal{N}=1$ conformal superspace [4], this result can be extended to 2- and 3-point functions of the lowest chiral primary TrX^2 and all descendents. Similar results can be derived using extended superspace [3]. It is also known that 4-point correlation functions do receive radiative corrections, so the theory is not secretly a free theory.

2. The scalar potential of the theory is a positive quartic in X^i of the form

$$V(X) = bg_{YM}^2 Tr([X^i,X^j])^2 \qquad (2.2)$$

There is thus a moduli space of supersymmetric minima, ie $V(\langle X \rangle)=0$, in which the vacuum expectation values $\langle X^i \rangle$ are traceless diagonal matrices. Generically the preserved gauge symmetry is $U(1)^{N-1}$, and the particle content at a generic point of moduli space is $N-1$ massless photons, N^2-N massive gauge bosons and superpartners. This gives a representation of $\mathcal{N}=4$ supersymmetry with central charges.
3. We will also be interested in supersymmetric mass deformations of the theory. It is then convenient to describe the deformed theory using $\mathcal{N}=1$ chiral superfields $\Phi_i (i = 1,2,3)$ whose lowest components are complex superpositions of the 6 X^i. The deformed theory has the superpotential

$$W = g_{YM}Tr\Phi_3[\Phi_1,\Phi_2] + \frac{1}{2}\sum_{i=1}^{3}m_iTr(\Phi_i)^2 \qquad (2.3)$$

The dynamics then turns out to depend very dramatically on the pattern of the mass parameters m_i.
For $m_3 \neq 0$ and $m_1=m_2=0$, the methods of Seiberg dynamics have been used [50] to show that conformal symmetry is broken at intermediate scales, but the theory flows to a non trivial conformal fixed point in the infrared limit.
For $m_1=m_2=m_3=m$, the supersymmetric vacua obey $[\phi^1,\phi^2]=\frac{m}{g_{YM}}\phi_3$ and are in one-to-one correspondence with representations of $SU(2)$ [5]. The long distance physics depends on which vacuum is chosen, and gauge symmetry can be realized in both confined and Higgsed phases.
We have time and space here to discuss only the holographic dual of the first mass deformation [6], despite extremely interesting recent work on the second [31, 41, 51].

3. D=10 Type IIB Supergravity

The bosonic fields of the D=10 IIB supergravity consist of a metric g, a scalar dilaton ϕ and axion C, two three-forms and a self-dual five-form F_5.

The classical equations admit as exact background $AdS_5 \times S^5$ with $F_5 = N vol(S^5)$. N comes from flux quantization. The fields ϕ and C are constant and the other fields vanishes.

The complete Kaluza-Klein mass spectrum on $AdS_5 \times S^5$ was obtained in [8]. It is organized into super-multiplets whose component fields are in representations of the $SO(6)$ isometry group of S^5. The lowest multiplet contains the graviton and its super-partners:

($g_{\mu\nu}, \psi_\mu^\alpha$ in the $\underline{4} + \underline{4}^*$, A_μ in the $\underline{15}$, $B_{\mu\nu}$ in the $\underline{6}_c$, λ in the $\underline{4} + \underline{10} + \underline{4}^* + \underline{10}^*$, the scalars φ^i in the $\underline{1}_c + \underline{10} + \underline{10}^* + \underline{20}$)

Each of these fields is the lowest state of a Kaluza-Klein tower of fields. For example a scalar of the D=10 theory can be expanded as

$$\varphi(z, y) = \sum_{\Delta=1}^\infty \varphi_\Delta(z) Y^\Delta(y) \qquad (3.1)$$

Here z^μ and y^i are coordinates of AdS_5 and S^5, respectively, and $Y^\Delta(y)$ is a rank Δ spherical harmonic on S^5. The masses of the 5-dimensional scalars $\varphi_\Delta(z)$ are eigenvalues of the $SO(6)$ Casimir operator, and masses are related to the rank Δ by $m^2 = \Delta(\Delta - 4)$.

One can now begin to see the duality between d=4, $\mathcal{N}=4$ SYM and the dimensionally reduced D=10 supergravity theory. Type IIB supergravity has 32 supercharges preserved by the $AdS_5 \times S^5$ vacuum. The isometry group is $SO(4, 2) \times SO(6)$ and there are 4 gravitini, which indicates that the superalgebra is indeed $SU(2, 2|4)$. One can look in more detail and find that the Kaluza-Klein spectrum of supergravity contains D=5 fields in exactly the same short representations as those of the chiral primaries $Tr X^k$ of d=4, $\mathcal{N}=4$ SYM. So there is 1:1 correspondence of fields in the D=5 and d=4 theories with a perfect match of $SO(6)$ irreps and scale dimensions. For example, there are scalar fields $\varphi_\Delta(z)$ with Δ=k for each of operators $Tr X^k$. The 15 gauge vectors $A_\mu^I(z)$ of supergravity are dual to the $SO(6)$ flavor currents J_μ^I, of the field theory and fluctuations of the metric $g_{\mu\nu}(z)$ are dual to the stress tensor $T_{\mu\nu}$.

In principle the equations of motion of the D=10 completely determine the interactions in the $AdS_5 \times S^5$ background. However, the dimensional reduction process is already very complicated at the linear level, since the independent (i.e. uncoupled) 5-dimensional fields are actually mixtures of those in the 10-dimensional theory [8]. The nonlinear interactions are even more complicated, and they have been worked out in only a few sectors of the theory [18].

On the other hand there is a known complete nonlinear 5-dimensional supergravity theory, the gauged $\mathcal{N}=8$ theory of [10, 9] which contains the fields of the graviton multiplet above and is invariant under the same superalgebra. This D=5 $\mathcal{N}=8$ theory is believed to be a consistent truncation of the Type IIB theory on $AdS_5 \times S^5$. This means that any classical solution of the D=5 theory can be lifted to an exact solution of the D=10. The Kaluza-Klein "ansatze" required to establish consistent truncation are quite complicated. Thus no complete proof for the D=10 IIB theory has been given. However there has been recent progress in finding the lifts of several solutions [30, 31], and the truncation property has been proven for other theories [32, 52].

The study of the dynamical implications of the AdS/CFT correspondence is somewhat limited by the absence of a complete dimensionally reduced action which includes interactions of all Kaluza-Klein modes. For perturbative questions (and some others) one can work at the level of Type IIB on $AdS_5 \times S_5$. One cannot always do this when exact solutions are needed, and one works instead with the D=5 $\mathcal{N}=8$ theory.

4. AdS/CFT correspondence

4.1 General setting

Maldacena [1] conjectured an exact duality between d=4, $\mathcal{N}=4$ SYM and Type IIB string theory on $AdS_5 \times S^5$. String theory on this spacetime is not yet well defined, so Maldacena argued further that duality holds at the level of classical Type *IIB* supergravity on $AdS_5 \times S^5$ provided two conditions hold:
1) the AdS length scale L and string scale α' must satisfy $L^2 \equiv \alpha'(g_{YM}^2 N)^{\frac{1}{2}} >> \alpha'$ so that stringy corrections to supergravity are small. This requires $\lambda \equiv g_{YM}^2 N$ large, i.e. strong coupling in SYM.

2) $N \to \infty$ so that loop corrections in SG or in string theory can be neglected.

Maldacena's conjectured duality was given dynamical content and predictive power by Gubser, Klebanov and Polyakov [11] and by Witten [12]. For our present purposes the duality means that observables of the field theory such as the correlation functions can be calculated from classical supergravity if the two conditions $N \to \infty$, $\lambda >> 1$ hold.

We will explore the dynamics of the AdS/CFT correspondence in a toy model of gravity and a scalar field in 5 bulk dimensions. The action is

$$S = \int d^5 x \sqrt{g}\left(-\frac{1}{4}R + \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) \quad (4.1)$$

where we work in units in which $\kappa_5^2 = 8\pi G_5 = 2$. We assume that the potential $V(\phi)$ has one or more critical points $\bar{\phi}$ at which $V(\bar{\phi}) < 0$. The classical equations of motion are

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) + V'(\phi) = 0 \qquad (4.2)$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 2\partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}[(\partial\phi)^2 - 2V(\phi)] \qquad (4.3)$$

At each critical point there is a solution with constant $\phi(z) = \bar{\phi}$ and an AdS geometry satisfying

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 2V(\bar{\phi})g_{\mu\nu} \qquad (4.4)$$

The cosmological constant Λ and $AdS_{(d+1)}$ scale are related to $V(\bar{\phi})$ by $\Lambda = 2V(\bar{\phi}) = -\frac{d(d-1)}{L^2}$. There are several common coordinate systems in which the $AdS_{(d+1)}$ metric can be presented – each making different features of the geometry evident. For the purposes of this lecture the most useful form is

$$g = e^{2A(r)}\eta_{ij}dx^i dx^j - dr^2 \qquad (4.5)$$

with the scale factor $e^{2A(r)} = e^{\frac{2r}{L}}$ and Minkowski$_d$ metric $\eta_{ij} = (+ - - \cdots -)$. These coordinates are not global; they cover only the Poincaré patch of the full space-time. The Poincaré patch is natural for the AdS/CFT correspondence because it makes the relevant symmetries evident (although interesting issues beyond the scope of this lecture do arise from the non-global property). The continuous symmetries of this metric include

1) an obvious d-dimensional Poincaré symmetry group with $\frac{d(d+1)}{2}$ parameters.

2) the scale tranformation $r \to r + a$, $x^i \to e^{\frac{-a}{L}}x^i$.

3) an additional d parameters of special conformal transformations with usual infinitesimal form $\delta x^i = (x)^2 c^i - 2(c.x)x^i$. Readers are invited to find the corresponding transformation of r.

The complete continuous isometry group is $SO(d, 2)$. Thus the isometry group of AdS_5 is the same as the conformal group of Minkowski$_4$.

It is also important to realize that the surface $r \to \infty$ is technically a boundary. A null geodesic gets there in finite time, and boundary conditions must be supplied to obtain unique solutions of wave equations in the AdS geometry. The boundary is conformal to d=4 Minkowshi space. We are interested in stable solutions and classical stability is determined by the M^2 of scalar fluctuations about the background, $h(z) \equiv \phi(z) - \bar{\phi}$. We express the mass in units of the AdS scale, ie. $M^2 = V''(\bar{\phi}) \equiv m^2/L^2$. An AdS solution can be stable even for negative m^2 provided that the stability bound [13, 14] $m^2 \geq -4$ is satisfied. In more realistic models there are several scalars ϕ^i and masses are eigenvalues m_I^2 of the hessian matrix $L^2 \frac{\partial^2 V}{\partial\phi_i\partial\phi_j}$. Stability requires $m_I^2 \geq -4$.

In realistic models there is a CFT_4 associated with each stable critical point of $V(\phi^i)$ with a map between bulk fields ϕ^i and field operators $\mathcal{O}_{\Delta_i}$ whose scale dimension is $\Delta_i = 2 + \sqrt{4 + m_i^2}$. In the AdS/CFT prescription for correlation functions of the $\mathcal{O}_{\Delta_i}$, their bulk duals act as sources in a generating functional. In more detail the prescription [12] is:

1. Solve the classical equations of motion for the fluctuations,

$$\frac{\delta S}{\delta\phi^i} = \Box\phi^i + \frac{\partial V}{\partial\phi^i} = 0 \qquad (4.6)$$

subject to boundary conditions as $r \to \infty$

$$\phi^i(\vec{x}, r) \to e^{\frac{(\Delta_i - 4)r}{L}}\bar{\phi}^i(\vec{x}) \qquad (4.7)$$

This is a Dirichlet problem modified to accomodate the exponential scaling rate found from standard Frobenius analysis of the equation.

2. Substitute the solution into the action to obtain the on-shell action $S[\bar{\phi}^i(\vec{x})] \equiv S[\phi^i(\vec{x}, r)]$ and regard this as a functional of the boundary data.

3. The CFT_4 correlators are then given by

$$\langle \mathcal{O}_\Delta(\vec{x}_1) \cdots \mathcal{O}_\Delta(\vec{x}_n) \rangle = \frac{\delta}{\delta\bar{\phi}(\vec{x}_1)} \cdots \frac{\delta}{\delta\bar{\phi}(\vec{x}_n)} e^{iS[\bar{\phi}]}$$
$$(4.8)$$

The functional prescription can be turned into a precise diagrammatic algorithm, and some Witten diagrams for 2-, 3-, and 4-point functions are shown in Fig 1.
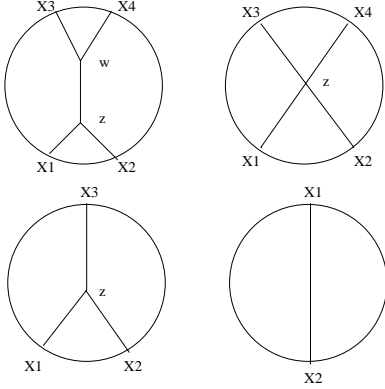

Figure 1: Witten diagrams

a. a Witten diagram (Fig. 1) is much like a Feynman diagram. There are boundary points $\vec{x}$ at which the CFT operators are inserted, while bulk points z, w are integrated over AdS_5

b. There are quite simple bulk-to-boundary propagators

$$K_\Delta(\vec{z} - \vec{x}, z_0) = C_\Delta \left(\frac{z_0}{(\vec{z} - \vec{x})^2 + z_0^2}\right)^\Delta \quad (4.9)$$

They are solutions of the linear wave equation $(\Box + \frac{m^2}{L^2})K_\Delta = 0$ where $\Box$ is the invariant wave operator.

c. For exchange graphs one also needs bulk-to-bulk propagators. They are hypergeometric functions of the variable $u = \frac{(z_0 - x_0)^2 + (\vec{z} - \vec{x})^2}{2z_0 x_0}$. For scalars, they can be obtained from the AdS literature of the 1980's.

d. The Feynman rules for bulk vertices come, as expected, from S_{int} of the supergravity Lagrangian.

In a long program of work the MIT-UCLA collaboration [15, 16] developed systematic techniques to evaluate the AdS integrals in these diagrams and extract physical results for field theory. It was also necessary to derive new bulk-to-bulk

propagators for photons and gravitons [17]. Witten diagrams automatically give conformally covariant amplitudes which contain the precise scaling factors to justify the bulk-boundary relation between scale dimension and mass given above. This is just kinematics, but the quantitative agreement found between $\mathcal{N}=4$ SYM and IIB SG on $AdS_5 \times S^5$ extends far beyond kinematics and symmetry. There is space here only for the briefest summary of the key results.

1. In a tour de force technical calculation, the cubic couplings of the supergravity scalars dual to the operators TrX^k were obtained in [18]. Results were combined with the corresponding AdS integrals in [15] and revealed that the supergravity results for all 3-point correlators

$$\langle TrX^k TrX^l TrX^m \rangle$$

agreed with the free-field Feynman diagrams in the field theory. This suggested that this whole family of correlators was not renormalized, at least in the $N \to \infty$, $\lambda >> 1$ limit in which the correspondence is valid. This surprising result was then investigated at weak coupling in the field theory and it was shown that order g^2 radiative corrections to all 2- and 3- point correlators of the TrX^k (and order g^4 in a few cases) vanish. General arguments indicating that radiative corrections vanish to all orders in g^2 were developed in [34, 35] and elsewhere. Remarkably, the field theory results hold for all N and for any gauge group [33]. Thus an important, unrecognized property of the field theory was revealed through supegravity.

2. Most 4-point functions obtained from supergravity are not trivial. One can extract an operator product expansion from the amplitudes of exchanged graphs. All singular powers agree with the expected contribution of operator dual to the exchanged field and its conformal descendents. The schematic form of the resulting double OPE is

$$\langle \mathcal{O}_{\Delta_1}(\vec{x1}) \cdots \mathcal{O}_{\Delta_2}(\vec{x4}) \rangle = \sum_\Delta \frac{C_{\Delta_1 \Delta_2 \Delta}}{x_{12}^{\Delta_1 + \Delta_3 - \Delta}} \frac{C_\Delta}{x_{13}^{2\Delta}} \frac{C_{\Delta_3 \Delta_4 \Delta}}{x_{34}^{\Delta_3 + \Delta_4 - \Delta}}$$
$$(4.10)$$

The supergravity amplitudes contain a single power of $log(x_{12}^2 x_{34}^2 / x_{13}^2 x_{24}^2)$. This can be interpreted as an anomalous dimension of double trace opera-

tors : TrX^jTrX^k :. These operators are not directly in the map between supergravity fields and the single trace TrX^k. Instead their effects are found in an appropriate short distance limit of n-point functions with $n \geq 4$. The scale dimensions of most double trace operators are renormalized since they are typically lowest weight operators of long representations. The 4-point calculations yield $\Delta_{jk} = j + k + \gamma_{jk}/N^2$. The $1/N^2$ corrections are strong coupling predictions of AdS/CFT. One curious point is that in a generic bulk supergravity theory, exchange graphs can have log^2 singularities rather than the single power that appears in the case of fields and couplings of the Type IIB theory. Only the single log can be interpreted as the correction to the scale dimension of an operator.

3. The explicit structure of supergravity amplitudes suggested that "extremal" 4-point (and $n \geq 4$) correlators are also not renormalized [36]. An extremal 4-point function is

$$< TrX^{k_1}TrX^{k_2}TrX^{k_3}TrX^{k_4} >$$

in the case $k_1 = k_2 + k_3 + k_4$. This is another very curious prediction of AdS/CFT which was subsequently confirmed by order g^2 and instanton calculations [37]. General arguments based on harmonic superspace appeared soon after [39]. There are further results of this type for "subextremal" correlators including a prediction from field theory [39] later confirmed in supergravity [38].

The situation may be summarized by saying that an interplay of work by both supergravity and field theory methods has given much new information about the conformal phase of the $\mathcal{N}$=4 SYM theory. It confirms that the AdS/CFT correspondence has quantitative predictive power, so we can go ahead and apply it in non-conformal settings.

5. Basics of holographic RG flows

A conformal field theory can be perturbed by adding a relevant perturbation $A_\Delta \int d^4\vec{x}\mathcal{O}_\Delta(\vec{x})$ to the action, where the operators $\mathcal{O}_\Delta$ have scale dimension $\Delta < 4$. One reason to restrict to relevant deformations is to avoid uncontrollable ultraviolet divergences, but relevant deformations

are also more interesting physically since they change the low energy behavior of the theory. As relevant deformations of $\mathcal{N}$=4 SYM we shall be interested in the operators TrX^2, $Tr\lambda^2$, and TrX^3, that is mass terms and cubic scalar couplings.

In this section we will explore the ideas involved in finding the holographic duals of such perturbed CFT_4's. Eventually we will be led to the gauged $\mathcal{N}$=8 D=5 supergravity which contains all the relevant operators of the parent Type IIB theory. However it is useful to begin the discussion in terms of the gravity/scalar toy model of Sec III. We shall describe the basic ideas [49, 46] of holograhic RG flows in this model and then gradually move toward the more realistic case.

Symmetry considerations are a useful starting point. In the perturbed theory $SO(4,2)$ symmetry is valid only at short distances, but the true spacetime symmetry is reduced to that of the Poincaré group in d=4. Since symmetries of the bulk and boundary theories must match, we look for classical solutions of the D=5 bulk theory with Poincaré symmetry. The most general Poincaré$_4$ metric and scalar configuration with this symmetry takes the form

$$ds^2 = e^{2A(r)}[(dt)^2 - (d\vec{x})^2] - dr^2 \qquad (5.1)$$

$$\phi = \phi(r) \qquad (5.2)$$

(Other equivalent forms are found in the literature and differ by change of the radial coordinate r.) If ϕ=const and $A(r)=\frac{r}{L}$, we recover AdS_5 with enhanced $SO(4,2)$ symmetry, but we will now be more general. With this ansatz, the equations of motion become

$$\phi''(r) + 4A'(r)\phi'(r) = \frac{\partial V}{\partial \phi} \qquad (5.3)$$

$$A'(r)^2 = -\frac{1}{3}V(\phi) + \frac{1}{6}(\phi')^2 \qquad (5.4)$$

These coupled non-linear equations are usually difficult to solve. One thing which can be done quite generally is to linearize about the pure AdS_5 solution with scale L associated with a fixed point $\bar{\phi}$ near which

$$V(\bar{\phi} + h) \simeq \frac{1}{L^2}(-3 + \frac{1}{2}m^2 h^2) \qquad (5.5)$$

The fixed point is approached by exact solution as $r \rightarrow \pm\infty$. Moreover, letting $m^2 = \Delta(\Delta - 4)$, a standard Frobenius analysis gives the scaling rates

$$\lim_{r \to -\infty} \phi = A e^{(\Delta - 4)\frac{r}{L}} + B e^{-\Delta \frac{r}{L}} \qquad (5.6)$$

The AdS/CFT correspondence gives the following physical interpretation of the solution approaching the boundary region $r \rightarrow +\infty$:

1. a generic solution with $A \neq 0$ corresponds to addition of the operator dual to ϕ to the CFT_4 Lagrangian, i.e. $\Delta \mathcal{L} = A \mathcal{O}_\Delta(\vec{x})$.

2. Special solutions with $A = 0$, $B \neq 0$ correspond to a deformation of CFT by the vev $\langle \mathcal{O}_\Delta \rangle_{CFT} \sim B$ [40].

Suppose now that $V(\varphi)$ has two fixed points at values $\varphi_{UV(IR)}$. The non-linear equations will then have a solution $\varphi(r)$ which approaches the constants $\varphi_{UV(IR)}$ and $A(r)$ which is asymptotic to $\frac{r}{L_{UV(IR)}}$ as r goes to $\pm\infty$. This is just a domain wall which interpolates between the boundary region of one AdS_5 geometry with scale L_{UV} and the deep interior region of another AdS_5 geometry with scale L_{IR}. The scalar field profile will typically have dominant scale behavior $A_{UV} \neq 0$ as $r \rightarrow \infty$. In order to approach $\bar{\varphi}_{IR}$ as $r \rightarrow -\infty$, it must behave as $h(r) \rightarrow e^{(\Delta_{IR} - 4)r}$ with $\Delta_{IR} > 4$. This is the gravity dual of a field theory which flows from a CFT_{UV} perturbed by the relevant operator $\mathcal{O}_\Delta$, toward a different CFT_{IR} at long distance along an RG trajectory corresponding to perturbation of the latter theory by an irrelevant operator.

5.1 A c-theorem

When a CFT_4 is coupled to a curved external metric $g_{ij}(\vec{x})$, the expected invariance under the Weyl transformation $\delta g_{ij}(\vec{x}) = 2\delta\sigma(\vec{x})g_{ij}(\vec{x})$ is broken due to ultraviolet divergences. An anomalous contribution to the vacuum expectation value $\langle T_i^i \rangle$ of the form

$$\langle T_i^i \rangle = \frac{c}{16\pi^2} W_{ijkl}^2 - \frac{a}{16\pi^2} \tilde{R}_{ijkl}^2 \qquad (5.7)$$

is then generated. The first term is the square of the Weyl tensor and the second is the topological Euler density. The Weyl anomaly coefficients c and a, also known as conformal central charges,

are important data of a CFT_4. They can also be obtained from the long and short distance limits of correlation functions of the stress tensor. In $\mathcal{N} = 4$ SYM theory the anomaly coefficients obey non-renormalization theorems [26] and can be calculated using only the free-field content of the theory. The result is $c = a = \frac{N^2 - 1}{4}$.

For CFT_2 there is the fundamental Zamolodchikov c-theorem which uses the 2-point function of the stress tensor to construct a positive monotonic function interpolating between the central charges of the UV and IR CFT's in any RG flow. Thus one always has $c_{UV} > c_{IR}$ for the unique central charge in 2-dimensions. There is no generally accepted proof of a similar theorem in 4-dimensions [27], but there is a great deal of evidence from model examples [28, 26] that the Euler anomaly coefficient does satisfy $a_{UV} > a_{IR}$ (while the same inequality for c is violated in many cases).

In a very elegant paper [19], Henningson and Skenderis have shown how to obtain the conformal anomaly holographically. Formally the on-shell action which is the generating function for field theory correlators is invariant under 5-dimensional diffeomorphisms. However, the diffeomorphism group contains a subgroup which induces Weyl transformations of the boundary metric $g_{ij}(\vec{x})$, so one really should expect an anomaly. Indeed the on-shell action is actually divergent. It must be cutoff at a large finite value of r and counter terms added to cancel the divergence. The counter terms are local functions of the boundary metric whose variation generates the trace anomaly in the form (5.7). The specific counter terms that appear guarantee that $c = a$ in any field theory which has a holographic dual in this framework. Further the holographic value agrees with the field theory result for $\mathcal{N} = 4$ SYM.

It is very curious that for holographic RG flows, one can prove the c-theorem [49] that remains elusive in field theory. To do this one defines the (dimensionless) function

$$a(r) = \frac{1}{G_5 A'(r)^3} = \frac{N^2}{4L^3 A'(r)^3}$$

whose boundary limit agrees with the Henningson-Skenderis calculation of the central charge. It is easy to manipulate the domain wall equations

(5.3), (5.4) to show that $A''(r)=-2\phi'(r)^2/3 < 0$ so that the function $a(r)$ decreases monotonically as one moves from the boundary into the interior of the space-time. This result is actually independent of the specific dynamics of the bulk theory, because it simply follows from the Einstein equations for a metric of the domain wall form that $A''=-8\frac{\pi G_5}{3}(T_t^t - T_r^r) < 0$. The inequality $(T_t^t - T_r^r) > 0$ is one of the standard positive energy conditions in general relativity. A direct consequence of monotonicity is the c-theorem in the form $a_{UV} > a_{IR}$.
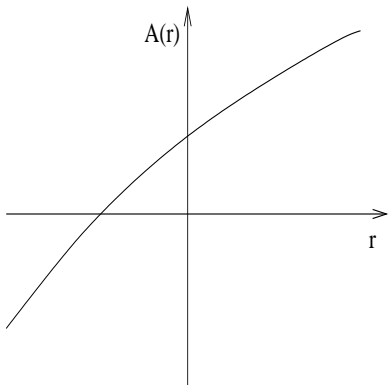

Figure 2: Profile of the scale factor $A(r)$

The result $A''(r) < 0$ also implies that the profile of the scale factor $A(r)$ is always concave downward, as shown in (Fig. 5.1). Since $A'(\pm\infty)=\frac{1}{L_{(UV,IR)}}$ and $V(\bar{\phi})=-\frac{3}{L^2}$ at a critical point, it is also the case that the IR endpoint of the flow occurs at a deeper critical point of the potential than the UV endpoint.
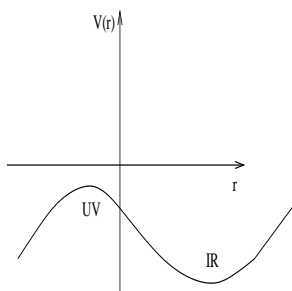

Figure 3: Potential $V(\phi)$

Let us suppose that the potential $V(\phi)$ is as shown in (Fig. 3). The critical point labelled UV is a maximum because we are speaking of a relevant deformation, while that labelled IR

is a minimum since it describes an irrelevant direction in the CFT_{IR}. We have been discussing the classical solution which interpolates between these two extrema.

There is another classical solution which departs from the UV critical point and moves to the left as r decreases into the interior. If there is no other critical point in this direction the geometry eventually develops a curvature singularity, sometimes at finite and sometimes at infinite geodesic distance from an interior point. Such singularities are a major problem for the AdS/CFT correspondence, since infinite curvature means that that the supergravity approximation to string theory is invalid. In some cases stringy mechanisms to resolve the singularity have been studied [42, 41]. In other cases one attempts a physical interpretation in spite of the singularity by requiring that fluctuations about the background are regular at the singularity. Gubser [43] has formulated a thermodynamic criterion for acceptable singularities.

Let us now discuss an interesting approach to the problem of solutions of the equations (5.3, 5.4) for a domain wall background. The approach arose from the study of supersymmetric domain walls in both D=4 [29] and D=5 [6], but was shown to apply more broadly [45]. Let us define an auxiliary function of φ, the superpotential $W(\varphi)$ as follows:

$$\frac{1}{8}(\frac{dW}{d\varphi})^2 - \frac{1}{3}(W)^2 = V(\varphi) \qquad (5.8)$$

and suppose that one can solve this as an ODE for $W(\varphi)$ given $V(\varphi)$. Then consider the following set of ODE's

$$\frac{d\varphi}{dr} = \frac{1}{2}W'(\varphi) \qquad (5.9)$$

$$A'(r) = -\frac{1}{3}W(\varphi(r)) \qquad (5.10)$$

which can be easily solved by successive quadrature. One can then easily show that the solution $\varphi(r)$, $A(r)$ is also a solution of the original 2nd order gravity-scalar equations (5.3), (5.4). This turns out to be equivalent [44] to applying Hamilton-Jacobi theory to the original dynamical system with $W(\varphi)$ as the Hamilton-Jacobi function. Unfortunately, it is usually hard to find

analytic solutions for $W(\varphi)$, especially in realistic case of several φ^i when the H-J equation becomes a PDE. It is worth pointing out that the first order scalar equation then generalizes to the gradient flow equations

$$\frac{d\varphi^i}{dr} = \frac{1}{2}\frac{dW(\varphi^j)}{d\varphi^i} \qquad (5.11)$$

5.2 SUSY Flows

This leads to the miracle of SUSY RG flows: the transformation rules of a d=5 supergravity theory generate the superpotential $W(\varphi)$ in an indirect and rather surprising way. Given $W(\varphi)$, one can either solve the first order flow equations exactly or gain insight or accurate numerical solutions from the well developed theory of gradient flow equations. There is another important reason to restrict to $SUSY$ flows. It is only for supersymmetric deformations of $\mathcal{N}$=4 SYM, that methods of Seiberg dynamics give sufficient control of the IR behavior of the boundary field theory.

To see how all this happens, consider a d=5 supergravity theory with a five-bein e^m_μ, several scalars φ^i, other bosonic fields A^I_μ, $B^\alpha_{\mu\nu}$ and fermionic fields χ^A and ψ^a_μ. The other fields are not "turned on" in a domain wall background since that would violate Lorentz invariance, but these fields are important as fluctuations dual to operators in the field theory.

The fermionic transformation rules have the form:

$$\delta\psi^a_\mu = D_\mu\epsilon^a - \frac{1}{6}W^a_b\gamma_\mu\epsilon^b \qquad (5.12)$$

$$\delta\chi^A = (\gamma^\mu P_\mu(\varphi)^A_a - Q^A_a(\varphi))\epsilon^a \qquad (5.13)$$

The matrices W^a_b, $P^A_{\mu a}$ and Q^A_a are functions of scalars φ^i which are part of the specification of the classical supergravity theory. Killing spinors $\epsilon^a(\vec{x}, r)$ are spinor configurations which satisfy $\delta\psi^a_\mu$=0 and $\delta\chi^A$=0. When they exist they contain $4n$ arbitrary real parameters, and the background is said to preserve $4n$ supercharges. This symmetry is then matched in the boundary field theory which posesses $\mathcal{N}$=n 4-dimensional Poincaré supersymmetry.

Let us examine the $\delta\psi^a_\mu$=0 condition and outline how it leads to the flow equation (5.9), (5.10). In the obvious diagonal local frame for the domain wall metric (5.1), the spin connection is

$$\omega_{\mu ab}\sigma^{ab} = \begin{cases} -A'(r)\gamma_j\gamma_5 & \mu = j \\ 0 & \mu = 5 \end{cases}$$

The condition (5.12) can be written in detail as

$$\delta\psi^a_j = \partial_j\epsilon - \frac{1}{2}A'(r)\gamma_j\gamma_5\epsilon^a - \frac{1}{6}W^a_b\gamma_j\epsilon^b = 0 \qquad (5.14)$$

We can drop the first term because the Killing spinor must be translation invariant. What remains is a purely algebraic condition, and we can see that the flow equation (5.10) for the scale factor directly emerges with superpotential $W(\phi)$ identified as one of the eigenvalues of the tensor W^a_b. In detail one actually has a symplectic eigenvalue problem, with 4 generically distinct W's as solutions. Each of these is a candidate superpotential. One must then examine the 48 conditions $\delta\chi^A = 0$ to see if SUSY is supported on one of the eigenspaces, and this leads to the gradient flow equation (5.11). Success is not guaranteed and generically occurs on one of the four (symplectic) eigenspaces, giving $\mathcal{N}$=1 SUSY. Extended $\mathcal{N} > 1$ SUSY requires further degeneracy of the eigenvalues. This process can be implemented reasonably efficiently using Mathematica programs (if you have the right collaborators).

We now outline the specific application in [6] to the first of the several known RG flows found within the gauged D=5 $\mathcal{N}$=8 supergravity theory.

1. There are 42 scalars in the bulk theory, and no human being or computer can handle the eigenvalue problem analytically in such a large setting. One uses generalized symmetry arguments to truncate to possible flows involving a small number of fields. In the specific case we describe the final truncation involved 2 scalars and preserved an $SU(2) \times U(1)$ flavor symmetry.

2. The Killing spinor conditions were found to be satisfied on a subspace of 2 canonically normalized scalars called ϕ_1 and ϕ_3 (with $\rho = exp(\phi_3/\sqrt{6})$). The superpotential found was

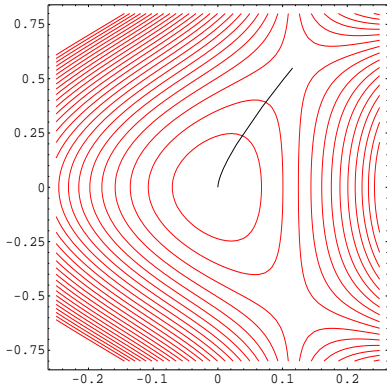$$W(\phi_3, \phi_1) = \frac{1}{\rho^2}[ch(2\phi_1)(\rho^6 - 2) - 3\rho^6 - 2] \quad (5.15)$$

Figure 4: Contour plot of $W(\phi_3, \phi_1)$

3. A contour plot of $W(\phi_3, \phi_1)$ is shown in (Fig. 4) and reveals three critical points.

a. there is a local maximum at the origin which has the full $SU(4)$ symmetry and anomalies of the unperturbed $\mathcal{N} = 4$ SYM which is the UV endpoint of the flow.

b. another pair of saddle points at $\phi_3 = ln(2)/\sqrt{6}$, $\phi_1 = \pm ln(3)/2$. There is a Z_2 symmetry under reflection of ϕ_1, so we can restrict to the upper half-plane.

4. The fields ϕ_3, ϕ_1 are $SU(2) \times U(1)$ singlets so all gradient flow trajectories have this symmetry. We are most interested in the critical trajectory which terminates at the saddle point b. According to our general discussion the associated geometry approaches the deep interior of an AdS_5 with scale L_{IR}. This determines an anomaly coefficient $a_{IR}=c_{IR}= 27N^2/128$. As pointed out in [53], the anomaly and symmetries, including $\mathcal{N}=1$ SUSY, exactly match those of the $\mathcal{N}=4$ SYM deformed by the Leigh-Strassler mass term $\Delta\mathcal{L} = mTr\Phi_3^2/2$. This is evidence that the critical trajectory describes the holographic dual of the RG flow in that theory.

5. The gradient flow equations for $W(\phi_3, \phi_1)$ are easy to write down, but they cannot be solved analytically. An accurate numerical determination of the critical trajectory is not difficult, but an analytic solution would be very useful[1]

6. One can check the holographic description

of the dynamics of the field theory by computing the mass eigenvalues of all fields in the theory, namely all fields in the graviton multiplet listed in section 3, at the IR critical point. Scale dimensions are then assigned using the formula $\Delta = 2 + \sqrt{4 + m^2}$ for scalars and its generalizations to other spins.

7. The next step is to assemble component fields into multiplets of the $SU(2,2|1)$ superalgebra. There are several short multiplets, including chiral multiplets for which the formula $\Delta=3R/2$ relating $U(1)_R$ charge to scale dimension is a helpful constraint. Results on the 8 short multiplets agree perfectly with the field theory description. This constitutes a non-trivial confirmation of the supergravity description of the dynamics of the field theory. There are additional results on 3 previously unrocognized semi-short mulitplets, headed by the 3 supercurrents of the $\mathcal{N}=4$ theory which are broken by the $\mathcal{N}=1$ perturbation, and one long multiplet. Their scale dimensions are non-perturbative predictions of the holographic description.

8. There is an infinite number of other gradient flow trajectories emerging from the $\mathcal{N} = 4$ critical point. Generically the associated geometries, obtained from the flow equation for $A(r)$ have curvature singularities. There is a reasonable physical interpretation of the trajectories with $\phi_1(r)=0$ as the supergravity dual of a Coulomb branch deformation of $\mathcal{N}=4$ SYM [7, 47]. Since there is only one scalar active in the flow, the domain wall profile can be found analytically [2]. Some 2-point correlation functions can be calculated and exhibit a mass gap and other features which are not fully understood from the viewpoint of the strongly coupled field theory.

There is much more to be said about the active subject of holographic RG flows, and many interesting papers, including [48, 41, 44] and more, that deserve study by interested theorists.

Acknowledgments

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[1]One of the authors (DZF) offers a gourmet dinner to the first person to find analytic solutions. See http://www.ClayMath.org/ for problems with more substantial rewards.

[2]no dinner for this one

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