

# On the $c$ -theorem in more than two dimensions

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ABSTRACT: Several pieces of evidence have been recently brought up in favour of the  $c$ -theorem in four and higher dimensions, but a solid proof is still lacking. We present two basic results which could be useful for this search: *i*) the values of the putative  $c$ -number for free field theories in any even dimension, which illustrate some properties of this number; *ii*) the general form of three-point function of the stress tensor in four dimensions, which shows some physical consequences of the  $c$ -number and of the other trace-anomaly numbers.

## 1. Introduction

### 1.1 The $c$ -theorem in two dimensions

The renormalization-group (RG) flow is defined as the one-parameter motion in the space of (renormalized) coupling constants  $\{g^i, i = 1, 2, \dots\}$ ,

$$\frac{d}{dt} \equiv -\beta^i(g) \frac{\partial}{\partial g^i}, \quad (1.1)$$

with “velocities” given by the beta-functions; the flow corresponds to a change of scale in the field theory which grows towards the infrared.

The Zamolodchikov  $c$ -theorem[1] holds for unitary, renormalizable quantum field theories in two dimensions; it says that there exists a positive-definite real function of the coupling constants  $c(g)$  such that:

*i*) it is monotonically decreasing along the flow,

$$\frac{d}{dt} c \leq 0; \quad (1.2)$$

*ii*) it is stationary at the fixed points  $g^i = (g^*)^i$ ,

$$\beta^i(g^*) = 0 \leftrightarrow \left. \frac{\partial}{\partial g^i} c(g) \right|_{g^*} = 0; \quad (1.3)$$

*iii*) at the fixed points, it equals the Virasoro central charge  $c$  of the corresponding conformal field theory,

$$c(g^*) = c. \quad (1.4)$$

This theorem implies some fundamental properties of the RG flow:

*i*) The flow necessarily ends into fixed points (or fixed surfaces); there cannot exist limit cycles or strange attractors, which are other possible asymptotic behaviours for the solutions of non-linear differential equations (1.1).

*ii*) The fixed points are classified according to the value of their central charge; we can think the space of theories<sup>1</sup> as a mountain landscape, with the fixed points located at the tips, the saddles and the valleys bottoms.

The central charge is a measure of the “number of degrees of freedom” and its decreasing along the RG flow can be viewed as the consequence of “coarse graining”, the integration of high-energy degrees of freedom in the Wilsonian approach to the renormalization group [2]. Note that a theory with an asymptotic limit cycle would have a never-ending infrared flow, with degrees of freedom periodically dying out and coming back; it would be very difficult to make sense of this RG behaviour in a unitary field theory. In conclusion, the  $c$ -theorem confirms our intuitive understanding of the RG flow.

Let us remark some aspects of the Zamolodchikov proof that will be useful for the fore-coming discussion:

<sup>1</sup>Namely, the space of coupling constants.

- i*) The inputs of the proof are just “kinematics”, i.e. general properties of Poincare invariance, unitarity and renormalizability – there are no hypotheses on the dynamics of the theory.
- ii*) The function  $c(g)$  is finite once the coupling constants are renormalized; moreover, its critical value is uniquely defined, since the trace anomaly is both finite and universal (as any other anomaly). It follows that  $c(g)$  is defined globally (i.e. non-perturbatively) on the whole space of theories.

### 1.2 The $c$ -theorem in higher dimension: motivations and overview of the results

The consequences of the  $c$ -theorem on the RG flow are so general that it is natural to expect its extension to higher-dimensional field theories. However, more than ten years have passed since the first attempts to a generalization [3][4][5]. First of all, a straightforward extension of the Zamolodchikov argument is not possible [5]. Secondly, in odd dimension  $d = 3, 5, \dots$ , the  $c(g)$ -function lacks the natural global definition given by the trace anomaly, because the latter is equal to zero (actually, it is very easy to construct functions which are monotonically decreasing along the flow but are discontinuous at fixed points).

It seems that the extension of the  $c$ -theorem to higher dimension requires a new ingredient, possibly involving the field-theory dynamics. In this respect, we believe that the eventual proof of the theorem could teach us new properties of field theories and of their interaction with gravity (through their stress tensor). Therefore, the interest of the  $c$ -theorem extends beyond the proof of mandatory properties of the RG flow.

Several works have recently discussed the  $c$ -theorem in four dimensions, by providing new arguments for the proof [6][7] [8] and by analysing examples of RG flows for the trace anomaly coefficients  $a$ ,  $c$  and  $a'$  [9][10]. These are defined by the following expression<sup>2</sup> [11]:

$$\langle T_{\mu}^{\mu} \rangle = \lambda (a E - 3c W + a' D^2 \mathcal{R}) , \quad (1.5)$$

where  $\int \sqrt{g} E = \chi$  is the Euler characteristics,  $W$  is the square of the Weyl tensor and  $\mathcal{R}$  is the curvature scalar.

<sup>2</sup>The coefficient  $\lambda = -1/(2880 \cdot 4\pi^2)$  is included to normalize the values of  $a$  and  $c$  to one for the free conformal-invariant scalar field.

#### $a$ -theorem:

$a_{UV} > a_{IR}$  has been exactly proven [9] for the non-trivial RG flows among  $N = 1$  supersymmetric gauge theories found by Seiberg, most notably those in the “conformal window” [12].

#### $c$ -theorem:

$c_{UV} > c_{IR}$  cannot be true in general, because counterexamples are known [5][9]; however, it holds for the field theories which have a gravity dual theory according to the AdS/CFT correspondence [13][10], such as the  $N = 4$  supersymmetric gauge theories. In these theories, the ratio  $c/a$  is an overall fixed constant, thus these results also support the  $a$ -theorem. The AdS/CFT correspondence has provided a lot of evidence for the irreversibility of the RG flow and a proof of the theorem has been found in this context [10] (see also Ref.[8]).

#### $a'$ -theorem:

the decreasing of  $a'$  along the flow can be easily proven, but this does not imply the theorem, because the function  $a'(g)$  cannot be globally defined in the space of theories [14]. Actually,  $a'$  is not well defined at fixed points, because it corresponds to a scheme-dependent term in the trace anomaly (1.5):  $D^2 \mathcal{R}$  is the Weyl variation of the local term  $\int \sqrt{g} \mathcal{R}^2$  in the effective action [15]. This problem can be cured by assuming a proportionality between  $a'$  and  $a$ , as proposed in Ref.[7]; however, this amounts to a strong dynamical hypothesis on the effective action.

In conclusion, the most promising formulation of the theorem in four dimensions involves the coefficient  $a$  of the Euler term in the trace anomaly (as first suggested in Ref. [3]).

## 2. The anomaly $a$ as a measure of degrees of freedom

In this Section, we suppose as a working hypothesis that the  $a$ -theorem is true in any even dimension  $d \geq 4$  (the Euler term is always present in the trace anomaly [16]); we want to understand how the  $a$ -number is actually measuring the number of degrees of freedom in field theory. To this extent, it is interesting to compute the value of  $a$  in several free theories and study its dependence on spin and dimension [17].

We consider the free conformal invariant theories of the scalar ( $S$ ), Dirac fermion ( $F$ ) and antisymmetric tensor ( $AT$ ) fields<sup>3</sup> and compute their trace anomalies on the  $d$ -dimensional sphere  $S^d$ . We use the well-known zeta-function regularization of the Euclidean partition function, given by the determinant of the Laplacian  $\Delta$  (the Hodge-de Rham operator [17]) acting on the respective fields. Under a scale transformation of the metric,  $g_{\mu\nu} \rightarrow \exp(2\alpha) g_{\mu\nu}$ , the variation of the partition function is:

$$\frac{d}{d\alpha} \log Z [S^d] = \zeta_{\Delta}(s=0) = 2 \int_{S^d} \langle T_{\mu}^{\mu} \rangle, \quad (2.1)$$

where

$$\zeta_{\Delta}(s) = \sum_n \frac{1}{\lambda_n^s}, \quad (2.2)$$

is the zeta function associated to the Laplacian, whose eigenvalues are denoted by  $\lambda_n$ .

The trace anomaly in any even dimension contains the Euler term we are interested in, plus a number of terms which are Weyl-covariant polynomials of the Weyl tensor and its derivatives [16]. These additional terms vanish on the geometry of the sphere, which is related to Euclidean space by a Weyl transformation; therefore, the trace anomaly (2.1) is completely given by the Euler term.

Thus, we can write the following equation for the trace anomaly on any conformally-flat space  $\mathcal{M}$ :

$$\frac{d}{d\alpha} \log Z [\mathcal{M}] = 2\lambda a \chi(\mathcal{M}) = \frac{\zeta_{\Delta}(0)}{2} \chi(\mathcal{M}), \quad (2.3)$$

where  $\chi$  is the Euler characteristic in  $d$  dimensions ( $\chi(S^d) = 2$ ),  $\lambda$  is the normalization constant for  $a$  and  $\zeta_{\Delta}(0)$  is computed on  $S^d$  [17].

Equation (2.3) determines the values of  $a$  once the normalization  $\lambda$  is chosen. We first consider the normalization  $\lambda = 1$  (call  $a = \hat{a}$  in this case); this is a rather natural choice, because  $\hat{a}$  becomes the proportionality constant between two universal pure numbers which are  $d$  and scale independent: a topological number on the r.h.s. of (2.3) and the regularized number of modes of the Laplacian on the l.h.s. (which can also be

<sup>3</sup>This is the  $p$ -form field, with  $p = (d-2)/2$  for conformal invariance at the classical level [17].

thought of as the number of “effective zero modes” [17]).

The anomaly number  $\hat{a}(\sigma)$  divided by the number of field components  $n(\sigma)$ ,  $\sigma = S, F, AT$ , is found to decrease with the dimension and to vanish in the limit  $d = \infty$  [17]:

$$\frac{\hat{a}(\sigma)}{n(\sigma)} \rightarrow 0, \quad \text{for } d \rightarrow \infty, \quad (2.4)$$

with

$$\begin{aligned} n(S) &= 1, & n(F) &= 2^{d/2}, \\ n(AT) &= \frac{(d-2)!}{[(\frac{d}{2}-1)!]^2}. \end{aligned} \quad (2.5)$$

The behaviour of  $\hat{a}$  is consistent with the known fact that these free theories become semiclassical in the limit of large dimensionality<sup>4</sup>: the anomaly is a quantum effect and should go to zero (once properly normalized).

We now discuss the use of  $a$  as a measure of degrees of freedom in the spirit of the  $c$ -theorem; we should use another normalization  $\lambda = \lambda(d)$  in (2.3), such that the scalar field is counted the same value in any dimension, say:

$$a(B) \equiv 1, \quad \text{any } d. \quad (2.6)$$

(This determines the value of  $\lambda = \lambda(4)$  in (1.5)). The values of  $a$  in this normalization are reported in Table 1 together with the ratios per field component,

$$r(\sigma) \equiv \frac{a(\sigma)}{n(\sigma)}. \quad (2.7)$$

We find that the ratios do not approach 1 for large  $d$ , but actually grow like  $O(d)$  and  $O(d^3)$  for the fermion and antisymmetric tensor fields, respectively. The measure of degrees of freedom given by  $a$  is very different from the classical value  $n(\sigma)$ , even when the theories become semiclassical; the higher-spin fields are weighted much more than the lower-spin ones, as is already apparent in  $d = 4$ . This is the main result of the work [17]. The same qualitative enhancement is found [17] for the other coefficient  $c$  in the trace anomaly (1.5) and for the gravitational chiral anomaly [18].

<sup>4</sup>This can be seen by putting the theories on a space-time lattice.

$d$	4	6	8	10	12	14		$2k$
$a(S)$	1	1	1	1	1	1		
$a(F)$	11	$\frac{191}{5}$	$\frac{2497}{23}$	$\frac{73985}{263}$	$\frac{92427157}{133787}$	$\frac{257184319}{157009}$	...	
$a(AT)$	62	$\frac{3978}{5}$	$\frac{161020}{23}$	$\frac{13396610}{263}$	$\frac{44166621324}{133787}$	$\frac{310708060404}{157009}$	...	
$r(S)$	1	1	1	1	1	1		1
$r(F)$	2.75	4.77	6.79	8.79	10.79	12.80	...	$\simeq 2k$
$r(AT)$	31	132.6	350.0	727.7	1310.	2142.	...	$\simeq (2k)^3$

**Table 1:** Values of  $a$  and of the weight per field component  $r$  in various even dimensions  $d$ , with asymptotic behaviours for  $d \rightarrow \infty$ .

This result for the  $a$ -counting is rather counter-intuitive but does not directly imply an obstruction for the  $a$ -theorem in higher dimensions: it does not lead to contradictions in the RG flows checked so far. It is a peculiar behaviour that one should keep in mind for further investigations of the  $a$ -theorem.

### 3. The three stress-tensor correlator in four dimensions.

The previous discussions have shown the important role of the trace anomaly in the various attempts to extend the  $c$ -theorem above two dimensions. In this respect, a better understanding of the physical consequences of the trace anomaly is very useful. Since the  $a$  and  $c$  coefficients of the  $d = 4$  trace anomaly (1.5) are scheme-independent quantities, it is possible to relate them to finite, scheme-independent amplitudes of the stress-tensor correlators and thus to physical quantities in flat space [19].

#### 3.1 Two-dimensional preliminaries

Let us first review the relation of the trace anomaly to two-dimensional correlators and its key role for the dispersive proof of the  $c$ -theorem [5]. In two dimensions, the two-point correlator of the stress tensor can be written in momentum space as follows:

$$\begin{aligned} & \langle T_{\mu\nu}(p) T_{\rho\sigma}(-p) \rangle \\ &= \frac{\pi}{3} A(p^2) (p_\mu p_\nu - \delta_{\mu\nu}) (p_\rho p_\sigma - \delta_{\rho\sigma}), \end{aligned} \quad (3.1)$$

where the form of the tensor structure is required by conservation, i.e. by Diffeomorphism invariance. The dimensional analysis shows that the

scalar amplitude  $A(p^2)$  has dimension  $(-2)$  and therefore is finite in perturbation theory and scheme independent. At fixed points, it becomes:

$$A(p^2) = \frac{c}{p^2}, \quad \text{fixed points.} \quad (3.2)$$

This Equation gives the desired relation of the anomaly coefficient  $c$  with the scheme-independent correlator, which plays an important role in the conformal field theory [20] ( $\langle T(z)T(0) \rangle = c/2z^4$  in coordinate space).

Off criticality, the amplitude satisfies the dispersion relation:

$$\begin{aligned} A(p^2) &= \int ds \frac{\rho(s)}{s + p^2}, \\ \rho(s) &= \frac{1}{\pi} \text{Im} A(p^2 = -s). \end{aligned} \quad (3.3)$$

In this Equation,  $\rho(s)ds$  is a positive-definite dimensionless spectral measure [5], whose critical limit is:

$$\rho(s) \rightarrow c \delta(s), \quad \text{fixed points.} \quad (3.4)$$

Using this measure, one can obtain another proof of the Zamolodchikov theorem, as follows [5]: off-criticality, the measure contains a delta term plus a smooth positive function peaked at  $s = m^2$ , where  $m$  is the typical mass scale of the theory:

$$\rho(s) = c_0 \delta(s) + \rho_{\text{smooth}}(s/m^2). \quad (3.5)$$

In the infrared limit  $m \rightarrow \infty$ , the peak will move to infinity and the smooth function will go to zero in a weak sense (i.e. as a distribution); therefore, the coefficient of the remaining delta-function is identified with the central charge of the infrared theory:  $c_0 = c_{IR}$ . On the other hand, in the ultraviolet limit  $m \rightarrow 0$ , the smooth function

should go to a delta function which adds up to the first term of (3.5), such that the total integral gives the central charge of the ultra-violet theory:  $\int_0^\infty \rho(s) ds = c_{UV}$ .

These properties of the spectral measure imply the following sum rule, which is an equivalent form of the  $c$ -theorem:

$$\begin{aligned} c_{UV} - c_{IR} &= \int_\epsilon^\infty ds \rho(s) \\ &= \frac{3}{4\pi} \int_{|x|>\epsilon} d^2x x^2 \langle T_\mu^\mu(x) T_\nu^\nu(0) \rangle \\ &> 0 . \end{aligned} \quad (3.6)$$

In this Equation, we also wrote the expression of the sum rule in coordinate space [21].

The previous analysis can be extended to four dimensions [5][19] starting from:

$$\begin{aligned} \langle T_{\mu\nu}(p) T_{\rho\sigma}(-p) \rangle \\ = A_0(p^2) \mathcal{P}_{\mu\nu,\rho\sigma}^{(0)} + A_2(p^2) \mathcal{P}_{\mu\nu,\rho\sigma}^{(2)} , \end{aligned} \quad (3.7)$$

where  $\mathcal{P}^{(0)}$  is the polynomial in (3.1) and  $\mathcal{P}^{(2)}$  is an analogous expression projecting on spin-two intermediate states. The two amplitudes ( $A_0$ ,  $A_2$ ) now have zero dimension, thus they are superficially divergent and scheme dependent; their critical limits are:

$$\begin{aligned} A_0(p^2) &\rightarrow \lambda a' , & \text{fixed points,} \\ A_2(p^2) &\rightarrow \lambda \frac{c}{4} \log\left(\frac{p^2}{\mu^2}\right) , \end{aligned} \quad (3.8)$$

where  $a'$  is the coefficient of the scheme-dependent term in the trace anomaly (1.5) and  $\mu$  is the renormalization scale. These expressions explicitly show the scheme dependences:  $A_i(p^2) \rightarrow A_i(p^2) + \text{const.}$ ; it follows that the two-point function, although positive definite, cannot be used for proving the  $c$ -theorem in four dimensions [5][22].

### 3.2 The three-point function

The three-point function has the following structure [19]:

$$\begin{aligned} \langle T_{\mu_3\nu_3}(-k_1 - k_2) T_{\mu_1\nu_1}(k_1) T_{\mu_2\nu_2}(k_2) \rangle \\ = \sum_{i=1}^{20} A_i(q^2, k^2) \mathcal{P}_{\mu_3\nu_3, \mu_1\nu_1, \mu_2\nu_2}^{(i)} , \\ k^2 \equiv k_1^2 = k_2^2 , \quad q^\mu \equiv -k_1^\mu - k_2^\mu , \end{aligned} \quad (3.9)$$

where the tensors  $\mathcal{P}^{(i)}$  have dimensions greater or equal to four. This results has been obtained by solving the Ward identities for Diffeomorphism invariance starting from a general expansion involving 137 basic polynomials; the involved tensor algebra can be overcome by using algebraic programs.

In Equation (3.9), 16 amplitudes are proper of the three-point function, while 4 are linked to the two-point function. Two among the 16 amplitudes match the Euler and Weyl anomalies at criticality:

$$\begin{aligned} A_E(q^2, k^2) &\rightarrow \lambda \frac{a}{q^2} , & \text{fixed points,} \\ A_W(q^2, k^2 = 0) &\rightarrow -\lambda \frac{3c}{q^2} , \end{aligned} \quad (3.10)$$

These limits are obtained by solving the Ward identity for the Weyl symmetry of the critical theory, which is anomalous according to (1.5). The amplitudes in (3.10) have dimension  $(-2)$  because the corresponding tensors are six-dimensional, and thus are scheme independent. Equations (3.10) give the expected relation between the anomaly coefficients and the scheme-independent correlations in four dimensions; the corresponding two-dimensional relation is given by Eq.(3.2).

Other scheme-independent amplitudes are non-vanishing at criticality (see also the analyses of the Refs.[23]); two further amplitudes of zero dimension account for the scheme dependence of the three-point function, including that pertaining to  $a'$  [19].

Each amplitude in the expansion (3.9) can be singled out by projecting the three-point function with the help of the dual tensor basis defined by:

$$\left( \mathcal{P}^{*(i)} \mid \mathcal{P}^{(j)} \right) = \delta^{ij} , \quad (3.11)$$

where the non-degenerate scalar product is obtained by contracting the six indices.

### 3.3 Results and Conclusions

Let us now discuss some consequences of the general expression (3.9) of the three-point function: *i)* It disentangles the kinematic properties of field theory, such as Poincaré, Weyl and Bose symmetry, from the dynamics encoded in the scalar amplitudes.

ii) The imaginary part of any scheme-independent amplitude describes a physical quantity such as a scattering or a decay process.

iii) The results (3.9, 3.10) amount to a re-derivation of the trace anomaly within the dispersive renormalization, in close analogy to the well-know analysis of the chiral triangle  $\langle AVV \rangle$  of Ref.[24]; incidentally, the relations (3.9, 3.10) can be practically useful for deriving the trace anomaly by Feynman diagram calculations.

iv) We can write sum rules for the RG flows of the  $a$  and  $c$  coefficients in close analogy with the two-dimensional case described by Eqs.(3.3-3.6). For the  $A_E$  amplitude, we write (similar expressions can be written for  $A_W$  at  $k^2 = 0$ ):

$$A_E(q^2, k^2) = \int ds \frac{\rho_E(s, k^2)}{s + q^2}, \quad (3.12)$$

where the measure  $\rho_E(s, k^2)ds$  reduces at criticality to (cf. (3.10)):

$$\rho_E(s, k^2) ds \rightarrow \lambda a \delta(s) ds, \quad \text{fixed points.} \quad (3.13)$$

The properties of this measure are very similar to that of its two-dimensional counterpart (3.3): it is a finite dimensionless function of the renormalized coupling constants (i.e. of the mass scale off-criticality  $m$ ), which satisfies an homogeneous RG equation; note, however, the dependence on two variables rather than one.

Following the same steps as in Section 3.1, we arrive to the sum rule:

$$\begin{aligned} a_{UV} - a_{IR} &= \frac{1}{\lambda} \int_{\epsilon}^{\infty} ds \rho_E(s, k^2 = 0) \\ &= \frac{1}{\lambda} \int_{\epsilon}^{\infty} ds \text{Im} \langle TTT \rangle|_{\mathcal{P}_E} \end{aligned} \quad (3.14)$$

In this Equation, we have set the second momentum  $k^2$  to zero, in order to let the measure to depend on the ratio  $s/m^2$  only.

Note that the sum rule (3.14) is not enough to prove  $a_{UV} > a_{IR}$ , because the  $\rho_E$  measure is not manifestly positive definite. A positivity condition for the three-point function has been proposed in the first of the Refs.[23], following from the (quantum) weak-energy condition, but its consequences on  $\rho_E$  and  $\rho_W$  remain to be explored. Manifestly positive amplitudes occur in the four-point function, which could also be analysed using the same tools.

In conclusion, we hope that the general expansion (3.9) and its dispersive analysis will be useful for further investigations.

## Acknowledgments

The authors thank the organizers of this Conference for the opportunity to present this work; we also thank all our partners of this European network for the many occasions of informal discussion and collaboration. This work is supported in part by the European Community Network grant FMRX-CT96-0012.

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