Superstring theory on AdS_3 times a coset manifold*

Riccardo Argurio, Amit Giveon and Assaf Shomer
Racah Institute of Physics, The Hebrew University of Jerusalem 91904, Israel
E-mail: argurio@cc.huji.ac.il, giveon@vms.huji.ac.il, shomer@cc.huji.ac.il

ABSTRACT: Superstrings propagating on backgrounds of the form $AdS_3 \times G/H$ are studied using the coset CFT approach. We focus on seven dimensional cosets which have a semiclassical limit, and which give rise to $N = 3$ superconformal symmetry in the dual CFT. This is realized for the two cases $AdS_3 \times \mathrm{SU}(3)/\mathrm{U}(1)$ and $AdS_3 \times \mathrm{SO}(5)/\mathrm{SO}(3)$, for which we present an explicit construction. The spectrum of the two coset models is analyzed and compared. We also briefly comment on the geometrical interpretation of our results.

1. Introduction

String propagation on curved backgrounds with an AdS_3 factor has been of recent interest. One motivation is the fact that $AdS_3 \simeq \mathrm{SL}(2)$ is an exact background which can be treated in string pertubation theory, and thus allows to consider the AdS/CFT correspondence [1] beyond the supergravity limit. Some specific examples that were studied in this context include superstrings propagating on $AdS_3 \times \mathcal{N}$ where $\mathcal{N}$ was a group manifold [2, 3], or an orbifold of a group manifold [4, 5]. In this contribution we study cases in which $\mathcal{N}$ is a coset manifold [6]. This is an interesting generalization of the AdS/CFT correspondence which has been considered in the higher-dimensional cases of type-IIB string theory on $AdS_5 \times \mathcal{N}^5$ [7] and of M-theory on $AdS_4 \times \mathcal{N}^7$ [8, 9], where $\mathcal{N}^5$ and $\mathcal{N}^7$ are Einstein manifolds (generically coset manifolds) preserving a fraction of supersymmetry. This type of construction allows one to consider dual supersymmetric CFTs which are not "orbifolds" of the maximally supersymmetric one. The $AdS_3 \times \mathcal{N}$ case is somewhat different since here we have the possibility of studying $\mathcal{N}$ in the context of coset CFTs.

We choose to study coset CFTs which have a large radius (or large level k) semiclassical limit, corresponding to superstrings propagating on seven-dimensional coset manifolds. Moreover, we focus on cases in which the dual two-dimensional theory (also referred to as the spacetime CFT) has an extended superconformal symmetry. Coset models leading to $N = 2$ can be easily realized as particular cases of the general construction of [10], where $\mathcal{N}$ decomposes as a U(1) factor times a Kazama-Suzuki model [11]. On the other hand, there are no seven-dimensional coset manifolds leading to $N = 4$ supersymmetry in spacetime (except of course the cases [2, 3] in which the cosets are actually group manifolds). Therefore, we shall be interested in the cases where the spacetime CFT has $N = 3$ supersymmetry.

2. Spacetime $N = 3$ superconformal algebra

Extended superconformal algebras in two dimensions also include an affine R-symmetry algebra, which generally leads to a quantization of the central charge in unitary theories. Specifically, the $N = 3$ superconformal algebra has an affine SU(2) subalgebra. The central charge is related to the level $\tilde{k}$ of this affine SU(2), which is an integer, by $\tilde{c} = \frac{3}{2}\tilde{k}$ [12]. Therefore, a necessary condition for string theory on a background of the form $AdS_3 \times \mathcal{N}$ to have spacetime $N = 3$ superconformal symmetry is the existence of an affine SU(2) in spacetime. This is obtained when the worldsheet CFT on $\mathcal{N}$ has an affine SU(2) symmetry as well [2]. If the respective world-

*Talk given by R. Argurio.

sheet levels of SL(2) and of SU(2) are k and k', the analysis of [2, 13] shows that in the spacetime theory we have $\tilde{c} = 6kp$ and $\tilde{k} = k'p$, where p is the integer number introduced in [2], related to the maximal number of "long strings" [13, 14]. A further condition is thus $k' = 4k$ (recall that k is not forced to be an integer).

Focusing on coset manifolds $\mathcal{N}$ which have 7 dimensions, so that a large k semi-classical limit is possible, one sees that the only two cases which satisfy the conditions given above are:

$$AdS_3 \times \frac{\mathrm{SU}(3)}{\mathrm{U}(1)}\,, \qquad AdS_3 \times \frac{\mathrm{SO}(5)}{\mathrm{SO}(3)}\,, \tag{2.1}$$

It is straightforward to show that the two models above are critical when the level of the SU(3) and of the SO(5) respectively is $4k$. We now show that these two models indeed possess $N = 3$ superconformal symmetry in spacetime by explicit construction. Since the construction is similar in the two cases, we will focus here on the first case, the second one goes along very similar lines.

We first have to set some notations, starting from the SL(2) WZW part. We mainly follow the formalism of [11] and [2]. For simplicity we only treat the holomorphic sector.

The SL(2) supersymmetric WZW model is constituted of the three currents $J^P(z)$, $P = 1, 2, 3$ of the SL(2) affine algebra at level k, and the three fermions ψ^P implied by the $N = 1$ worldsheet supersymmetry. As usual in supersymmetric WZW models, the currents can be decomposed in two pieces:

$$J^P = \hat{J}^P - \frac{i}{k}\eta^{PQ}\epsilon_{QRS}\,\psi^R\,\psi^S\,, \tag{2.2}$$

where $\eta^{PQ} = (++-)$ and $\epsilon^{123} = 1$. The first piece $\hat{J}^P$ constitutes an affine algebra at level $k+2$, and has regular OPE with the fermions ψ^P. We thus refer to $\hat{J}^P$ as the bosonic currents. The second part constitutes an affine algebra at level -2, and is the fermionic part of the current.

Let us now turn to the SU(3)/U(1) coset CFT. We start from the SU(3) affine superalgebra at level $k' = 4k$ realized as follows:

$$K^A(z)K^B(w) \sim \frac{(k'/2)\delta^{AB}}{(z-w)^2} + \frac{if_{ABC}K^C(w)}{z-w}\,,$$
$$K^A(z)\chi^B(w) \sim \frac{if_{ABC}\chi^C(w)}{z-w}\,,$$
$$\chi^A(z)\chi^B(w) \sim \frac{(k'/2)\delta^{AB}}{z-w}\,. \tag{2.3}$$

Here $A, B, C, D = 1, \ldots, 8$ and the structure constants f_{ABC} are $f_{123} = 1$, $f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = 1/2$ and $f_{458} = f_{678} = \sqrt{3}/2$. As before, we split the currents into their bosonic and fermionic parts:

$$K^A = \hat{K}^A - \frac{i}{k'}f_{ABC}\,\chi^B\chi^C\,. \tag{2.4}$$

The bosonic currents realize an affine algebra at level $k' - 3$.

We now choose to mod out the SU(3) by the U(1) generated by K^8. The SU(2) subgroup generated by K^1, K^2, K^3 is orthogonal to this U(1), and thus survives as an affine algebra in the coset CFT. The stress-energy tensor and the supercurrent of the coset CFT are built as in [11], using the decomposition $T_{\mathrm{SU}(3)} = T_{\mathrm{SU}(3)/\,\mathrm{U}(1)} + T_{\mathrm{U}(1)}$, and similarly for the supercurrent G.

Our goal now is to build the spacetime supercharges. For that we would like to construct spinfields via bosonization following [15]. Note that since we are dealing with a coset and not with a group manifold, the fermions are generically not free. Of course since the SU(2) is preserved as an affine symmetry, the fermions belonging to it are free. Despite the above remark, we bosonize the 10 fermions into 5 scalars. Define:

$$\begin{aligned} \partial H_1 &= \frac{2}{k}\psi^1\psi^2\,, \qquad \partial H_2 = \frac{2}{k'}\chi^1\chi^2\,, \\ i\partial H_3 &= \frac{1}{k}\psi^3\chi^3\,, \qquad \partial H_4 = \frac{2}{k'}\chi^4\chi^5\,, \\ \partial H_5 &= \frac{2}{k'}\chi^6\chi^7\,. \end{aligned} \tag{2.5}$$

The scalars H_I are all canonically normalized: $H_I(z)H_J(w) \sim -\delta_{IJ}\log(z-w)$. Obviously, the scalars H_4 and H_5 are not free in the coset CFT. However, it is also easy to see that there is a linear combination of them which is free. This is what will enable us to build the $N = 3$ spacetime superalgebra.

We thus write:

$$H_\pm = \frac{1}{\sqrt{2}}\big(H_4 \pm H_5\big)\,. \tag{2.6}$$

The expression for the stress-energy tensor T is therefore:

$$T = \frac{1}{k}\big(\hat{J}^1\hat{J}^1 + \hat{J}^2\hat{J}^2 - \hat{J}^3\hat{J}^3\big)$$

$$
\begin{aligned}
&+\frac{1}{k'}(\hat{K}^1\hat{K}^1+\cdots+\hat{K}^7\hat{K}^7)- \\
&-\frac{1}{2}(\partial H_1\partial H_1+\partial H_2\partial H_2 \\
&+\partial H_3\partial H_3+\partial H_-\partial H_-)- \\
&-\frac{1}{2}\left(1-\frac{3}{k'}\right)\partial H_+\partial H_+ + \frac{i\sqrt{6}}{k'}\hat{K}^8\partial H_+\,.
\end{aligned}
\tag{2.7}
$$

We conclude that H_- is the fourth free scalar, namely that ∂H_- is a primary field of weight 1.

We now write the worldsheet $N=1$ supercurrent, which will be used to enforce the BRST condition on the spin fields. The supercurrent for the coset CFT reads:

$$
G_{\rm coset}=\frac{2}{k'}\left(\chi^{\bar a}\hat{K}^{\bar a}-\frac{i}{3k'}f_{\bar a\bar b\bar c}\chi^{\bar a}\chi^{\bar b}\chi^{\bar c}\right), \tag{2.8}
$$

where $\bar a$ are indices in the coset G/H. We get the following expression for $G_{tot}=G_{sl}+G_{\rm coset}$ in our model:

$$
\begin{aligned}
G_{tot} = &\ \frac{2}{k}(\psi^1\hat{J}^1+\cdots-\psi^3\hat{J}^3) \\
&+\frac{2}{k'}(\chi^1\hat{K}^1+\cdots+\chi^7\hat{K}^7)+ \\
&+\frac{i}{\sqrt{k}}\Big\{\partial H_1(e^{iH_3}-e^{-iH_3}) \\
&-\frac{1}{2}\left(\partial H_2+\frac{1}{\sqrt{2}}\partial H_-\right)(e^{iH_3}+e^{-iH_3})\Big\}+ \\
&+\frac{1}{2\sqrt{k}}\left(e^{iH_2-i\sqrt{2}H_-}-e^{-iH_2+i\sqrt{2}H_-}\right).
\end{aligned}
\tag{2.9}
$$

Before going on to the BRST condition for the spin-fields, we write for completeness the expressions for the SU(2) currents. Writing $K^\pm = K^1\pm iK^2$ and similarly for the bosonic currents and the fermions, we get:

$$
\begin{aligned}
K^\pm &= \hat{K}^\pm \mp e^{\mp iH_2}(e^{iH_3}+e^{-iH_3})\pm e^{\mp i\sqrt{2}H_-} \\
K^3 &= \hat{K}^3 - i\left(\partial H_2+\frac{1}{\sqrt{2}}\partial H_-\right).
\end{aligned}
\tag{2.10}
$$

Note that since these currents are primaries of weight 1, this could have been an alternative way of showing that H_- is a free scalar.

In order to construct the spacetime superconformal algebra we need, in particular, to construct physical supercharges which we choose to write in the $-1/2$ picture [15]:

$$
Q\propto\oint e^{-\varphi/2}u^\alpha S_\alpha(z)\,dz\,. \tag{2.11}
$$

Here S_α is a basis of spin-fields, u^α are constants, and φ is the bosonized superconformal ghost. The set of operators $e^{-\varphi/2}u^\alpha S_\alpha(z)$ should be BRST invariant and mutually local. We choose a basis of spin-fields

$$
S_{[\epsilon_1\epsilon_2\epsilon_3\epsilon_-]}=e^{i/2\left(\epsilon_1H_1+\epsilon_2H_2+\epsilon_3H_3+\epsilon_-\sqrt{2}H_-\right)}, \tag{2.12}
$$

where $\epsilon_I=\pm1$. Because H_- is a free scalar, these 16 spin-fields are primaries of weight 5/8 and, therefore, $e^{-\varphi/2}u^\alpha S_\alpha(z)$ are primaries of weight 1, as they should be.

The super BRST condition on $e^{-\varphi/2}u^\alpha S_\alpha$ further requires that there will be no $(z-w)^{-3/2}$ singular terms in the OPE of $u^\alpha S_\alpha$ with the supercurrent G_{tot} (note that the only dangerous terms in G_{tot} are the ones trilinear in the fermions, i.e. the last three lines in (2.9)). This leaves 8 combinations $u^\alpha S_\alpha$ out of the 16 spin-fields (2.12). The GSO condition, i.e. mutual locality, further leads to one of two choices of chirality: $\epsilon_1\epsilon_2\epsilon_3=-1$ or $\epsilon_1\epsilon_2\epsilon_3=1$, under which 6 or 2 of the combinations $u^\alpha S_\alpha$ survive, respectively.

For spacetime chirality $\epsilon_1\epsilon_2\epsilon_3=-1$, the 6 physical spin-fields carry quantum numbers of the global SL(2) and SU(2) symmetries, in the $(\mathbf{2},\mathbf{3})$ representation, as can be checked by taking the OPEs with the respective currents. It can be shown [6] that this model reproduces the full $N=3$ superconformal algebra, in the NS sector.

For the other spacetime chirality $\epsilon_1\epsilon_2\epsilon_3=1$, the 2 physical spin-fields have regular OPEs with the SU(2) currents. This leads to only $N=1$ in spacetime.

3. General conditions for obtaining $N=3$

We now present general conditions for the appearance of the $N=3$ superconformal algebra in the context of string theory on $AdS_3\times\mathcal{N}$. Such a background leads to $N=3$ superconformal algebra in spacetime provided that:

(i) $\mathcal{N}$ has an affine SU(2) current algebra at level $k'=4k$, where k is the level of SL(2).

(ii) $\mathcal{N}/\,$U(1) has $N=2$ worldsheet supersymmetry, where U(1) is the Cartan subalgebra of the above SU(2). This condition alone allows one to construct an $N = 2$ superconformal algebra in spacetime (for a definite GSO projection).

(iii) This spacetime $N = 2$ algebra is enhanced to $N = 3$ if the scalar H_0 constructed as in [10] can be decomposed as $\sqrt{3}H_0 = H_2 + \sqrt{2}\tilde{H}_0$, where H_2 derives from the bosonization of the two remaining charged fermions of the SU(2), and $\tilde{H}_0$ is orthogonal to it.

Interestingly, these conditions imply as a by-product that for the opposite GSO projection we also get supersymmetry in spacetime, namely $N = 1$.

Let us present the proof by constructing the $N = 3$ superalgebra generators given the above conditions. Recall that besides the scalar $i\sqrt{3}\partial H_0 = J_R^{\mathcal{N}/U(1)} - \frac{4}{k'}K^3$ defined in [10], we define also the scalars $\partial H_1 = (2/k)\psi^1\psi^2$ and $i\partial H_3 = (1/k)\psi^3\chi^3$. The existence of the affine SU(2) allows us to define also $\partial H_2 = (2/k')\chi^1\chi^2$. Consider now the currents K^3 and $K^\pm$. Since they form an SU(2) supersymmetric WZW model (embedded inside the CFT on $\mathcal{N}$), they can be split into orthogonal pieces:

$$K^3 = \tilde{K}^3 - i\,\partial H_2, \qquad K^\pm = \tilde{K}^\pm \mp \frac{2}{\sqrt{k'}}\, e^{\mp i H_2}\chi^3\,. \tag{3.1}$$

We start now by noting that condition (iii) implies the following:

$$i\sqrt{2}\,\partial\tilde{H}_0(z)K^3(w) \sim -\frac{1}{(z-w)^2}\,. \tag{3.2}$$

This means that K^3 can be split further:

$$K^3 = \hat{K}^3 - \frac{i}{\sqrt{2}}\partial\tilde{H}_0 - i\partial H_2\,, \tag{3.3}$$

where $\hat{K}^3$ has a regular OPE with $\tilde{H}_0$ (and of course H_2). Similarly, the currents $K^\pm$ also split into a "bosonic" part $\hat{K}^\pm$ which realizes an affine $\mathrm{SU}(2)_{k'-3}$, an $\mathrm{SU}(2)_1$ part built from $\tilde{H}_0$ and the usual fermionic $\mathrm{SU}(2)_2$ piece:

$$K^\pm = \hat{K}^\pm \mp e^{\mp i\sqrt{2}\tilde{H}_0} \mp e^{\mp iH_2}\left(e^{iH_3} + e^{-iH_3}\right). \tag{3.4}$$

We now decompose the supercurrent of the CFT on $\mathcal{N}$ into an SU(2) part and a $\mathcal{N}/\,\mathrm{SU}(2)$ one. It can then be used to directly find all of the 8 physical spin-fields, 6 of one chirality and 2 of the other. The SU(2) part of the supercurrent is:

$$G_{\mathrm{SU}(2)} = \frac{2}{k'}\left(\frac{1}{2}\chi^+\tilde{K}^- + \frac{1}{2}\chi^-\tilde{K}^+ + \chi^3\tilde{K}^3 - \frac{2i}{k'}\chi^1\chi^2\chi^3\right). \tag{3.5}$$

Using (3.1), (3.3) and (3.4), and the bosonization, the relevant part of $G_{\mathrm{SU}(2)}$ for the BRST condition (i.e. the one that might lead to $(z-w)^{-3/2}$ singular terms in the OPE with the spin-fields) is:

$$G = \frac{1}{\sqrt{k'}}\Big\{ -i\left(\partial H_2 + \frac{1}{\sqrt{2}}\partial\tilde{H}_0\right)\left(e^{iH_3} + e^{-iH_3}\right) - \\ -\left(e^{iH_2 - i\sqrt{2}\tilde{H}_0} - e^{-iH_2 + i\sqrt{2}\tilde{H}_0}\right)\Big\} + \cdots. \tag{3.6}$$

The first piece will give rise to a $(z-w)^{-3/2}$ singularity only when $\epsilon_2 = \tilde{\epsilon}_0$, while the second piece will do so only when $\epsilon_2 = -\tilde{\epsilon}_0$. Choosing $\epsilon_2 = \tilde{\epsilon}_0$, we get 4 physical spin-fields of the same chirality. For $\epsilon_2 = -\tilde{\epsilon}_0$ we get 4 physical spin-fields, two of each chirality. It thus accordingly leads to the $N = 3$ or $N = 1$ superalgebra.

4. The chiral spectrum

We now compute the chiral spectrum of the spacetime theories in the two coset model backgrounds $AdS_3 \times SU(3)/U(1)$ and $AdS_3 \times SO(5)/SO(3)$, and show that it agrees, thus suggesting that the two models belong to the same moduli space. Note that in the case of the $N = 3$ superconformal algebra the R-charge is quantized to be in $\frac{1}{2}\mathbf{Z}$ because the $U(1)$ is actually the J^3 of the $SU(2)$ R-symmetry. Therefore the real issue is that of the multiplicity of chiral states in each energy level.

The chirality condition of the $N = 3$ superconformal algebra is the saturation of the bound $2L_0 \geq T_0^3$, i.e. that the spacetime weight equals half the spacetime R-charge which in our case arises from the unbroken $SU(2)$ spin:

$$h_{ST} = \frac{1}{2}j_{ST} \tag{4.1}$$

which can, of course, occur only for the highest weight state inside each $SU(2)$ multiplet.

We start by considering the general form of an NS vertex operator. The construction of such vertex operators in curved backgrounds of the form $AdS_3 \times G/H$ goes along similar lines to that of the flat case. Here however we have to replace the plane wave zero modes with the vertex operators $V_{j,m}$ and $U_{r,j',Q}$ correponding to the zero modes on AdS_3 and G/H. The vertex operators on AdS_3 are labeled by j, m according to the representation of $SL(2)$ [2], and those on G/H by the representation of G that we denote here by r, by the representation of the unbroken $SU(2)$ denoted by j' and possibly by other quantum numbers of G/H that are denoted here by Q.

It turns out that the following vertex operators are the only candidates:

$$e^{-\phi}\left(\psi^3 - \frac{1}{2}\gamma\psi^- - \frac{1}{2\gamma}\psi^+\right)V_{j,m}U_{r,j',Q} \quad (4.2)$$

which have spacetime scaling dimension $h_{ST} = j$. Using $k' = 4k$ the physicality condition reads:

$$j(j+1) = \frac{1}{4}\left(C^G - C^H\right), \quad (4.3)$$

where C^G and C^H are the Casimirs of the representations of G and H respectively (see [16] for all the details).

We now turn to calculate the quadratic Casimir eigenvalues in a general representation of $SU(3)$ in order to use (4.3) for the calculation of the chiral spectrum in the $AdS_3 \times SU(3)/U(1)$ case. A representation of $SU(3)$ is denoted by two positive integers $[r, s]$ and has a highest weight

$$\mu = r\mu^1 + s\mu^2 = \left(\frac{r+s}{2}, \frac{r-s}{2\sqrt{3}}\right). \quad (4.4)$$

Its Casimir is:

$$C_{[r,s]} = \frac{(r+s)}{2}\frac{(r+s+4)}{2} + \frac{(r-s)^2}{12}. \quad (4.5)$$

Now since $SU(2) \times U(1) \subset SU(3)$, the Cartan subalgebra of $SU(3)$ is composed of this $U(1)$ (denoted by K^8 by which we mod out) and the Cartan generator K^3 of the $SU(2)$. It is easy to see that the $U(1)$ charge under this K^8 is the y coordinate in weight space (x, y), while the x coordinate is the K^3 eigenvalue. Again, since we are looking for chiral primaries we must take the highest $SU(2)$ weights in a given $SU(2)$ representation.

Since the quadratic Casimir of the $U(1)$ in a given representation is simply the square of the $U(1)$ charge in that representation, we can straightforwardly compute (4.3) on the highest weight of the $SU(3)$ representation, and it is easily solved to give:

$$j = \frac{(r+s)}{4} \quad (4.6)$$

As explained above the x coordinate in weight space (x, y) is the charge j' in the unbroken $SU(2)$ which serves as the R-symmetry in the spacetime superconformal algebra and therefore we can write

$$j_{ST} = j' = \frac{(r+s)}{2}. \quad (4.7)$$

Comparing now between (4.7) and (4.6) we conclude that

$$h_{ST} = j = \frac{(r+s)}{4} = \frac{1}{2}j_{ST}, \quad (4.8)$$

i.e. these states are chiral.

Let us thus recapitulate that every $SU(3)$ representation gives rise to a chiral state in spacetime, through the vertex operator built using the highest weight of the representation. We can thus read off the multiplicity of chiral states at each spacetime spin $j_{ST} = j' = \frac{(r+s)}{2}$ to be $2j'+1$ since we have such a state coming from each $[r, s]$ $SU(3)$ representation with $\frac{(r+s)}{2} = j'$. Moreover, it can be proven that the identification of chiral primaries done so far is complete, i.e. that there are no new chiral primaries arising neither from the NS sector nor from the Ramond sector.

In a similar fashion, one can show that the spacetime chiral spectrum in the $SO(5)/SO(3)$ is identical to the one described above. However one can easily get convinced that the non chiral spectrum is not identical. For example, in the $SO(5)/SO(3)$ the smallest neutral state has spacetime weight $h_{ST}(h_{ST}+1) = \frac{1}{8}$. Such states do not exist in the $SU(3)/U(1)$ spacetime CFT, since there the smallest neutral state has $h_{ST}(h_{ST}+1) = \frac{1}{4}$. We therefore conclude that the agreement of the chiral spectrum is a non trivial fact suggesting that the two models are part of the same moduli space.

5. Conclusion

We conclude by commenting on the geometrical interpretation. It would be nice to translate the conditions we impose on the CFT on $\mathcal{N}$ into conditions on the geometry of the manifold. A related, but different, problem was actually discussed in the literature [8, 9], where conditions on Einstein 7-manifolds $\mathcal{N}$ are found in order to get different amounts of supersymmetry when considering 11 dimensional supergravity on $AdS_4 \times \mathcal{N}$. The condition for getting $N = 3$ in AdS_4 is that $\mathcal{N}$ has a tri-sasakian structure. The above geometries are considered as near horizon geometries of M2-branes at the singularity of the Ricci-flat cone $C(\mathcal{N})$ over such manifolds. The tri-sasakian structure implies the presence of 3 Killing vectors forming an SO(3) algebra which rotates the 3 Killing spinors. It turns out that the only 7-dimensional tri-sasakian manifolds (satisfying some additional regularity conditions) are exactly the cosets $\mathrm{SU}(3)/\,\mathrm{U}(1)$ and $\mathrm{SO}(5)/\,\mathrm{SO}(3) \cong S^7$.

We should nevertheless stress that in spite of the similarity, there are a few differences. For instance, superstring theory on $AdS_3 \times \mathcal{N}$ does not require $\mathcal{N}$ to be an Einstein manifold.[1] Recall that the metric of the coset CFT sigma model, which can be obtained by gauging the WZW model on the group G and integrating out the gauge fields, is not the same as the metric on the homogeneous G/H coset space. Thus presumably the direct relation between the two issues is more algebraic in nature than geometrical.

Another question regards the brane configuration which might lead to the models considered here in the near horizon limit. Since we are dealing with pure NSNS backgrounds in type II theories, we expect such a brane configuration to involve fundamental strings and NS5-branes intersecting on the string, and possibly at nontrivial angles.

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[1] This might explain why in the $\mathrm{SO}(5)/\,\mathrm{SO}(3)$ case we do not find $N = 8$ supersymmetry but only $N = 3$.

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