

# Stringy Confining Wilson Loops

Jacob Sonnenschein \*

School of Physics and Astronomy

Beverly and Raymond Sacler Faculty of exact Sciences

Tel Aviv University, Ramat Aviv, Tel Aviv 69978, Israel E-mail: [cobi@ccsg.tau.ac.il](mailto:cobi@ccsg.tau.ac.il)

ABSTRACT: The extraction of Wilson loops of confining gauge systems from their SUGRA (string) duals is reviewed. I start with describing the basic classical setup. A theorem that determines the classical values of the loops associated with a generalized background is derived. In particular sufficient conditions for confining behavior are stated. I then introduce quadratic quantum fluctuations around the classical configurations. I discuss in details the following models of confining behavior: (i) Strings in flat space-time, (ii)  $AdS_5$  black hole and its correspondence with pure YM theory in three dimensions. In particular an attractive Luscher term is shown to be the outcome of the quantum fluctuations. (iii) Type 0 string model (iv) The Polchinski Strassler  $N = 1^*$  model. In the latter case we show that SUGRA alone is not enough to get the correct nature of the loops, and only by incorporating the worldvolume phenomena of the five branes a coherent qualitative picture can be derived.

KEYWORDS: Wilson loop, Confinement, Gauge/SUGRA duality.

## A brief reminder on Wilson loops

- In  $SU(N)$  gauge theories one defines the following set of gauge invariant operators

$$W(C) = \frac{1}{N} \text{Tr} P e^{\oint_C A_\mu \dot{x}^\mu(\tau) d\tau}$$

where  $C$  is some contour.

- In this talk I restrict myself to  $C$  which is an infinite strip as is shown in figure 1.
- The quark anti-quark potential  $E(L)$  can be extracted from the infinite strip Wilson loop as follows

$$\langle W(C) \rangle = A(L) e^{-TE(L)}$$

- The natural (bosonic) stringy candidate for the Wilson loop (which obeys the loop equation) is

$$\langle W(C) \rangle \sim e^{-S_{NG}^{ren}}$$

\*This talk is based on works done in collaboration with A. Armoni, A. Brandhuber, E. Fuchs, N. Itzhaki, Y. Kinar, A. Loevy, E. Schreiber, N. Weiss and S. Yankielowicz.

where  $S_{NG}^{ren}$  is the renormalized NG action [1, 2, 3], which is the world sheet area of the string. The renormalization has a simple physical interpretation of subtracting the quark masses.

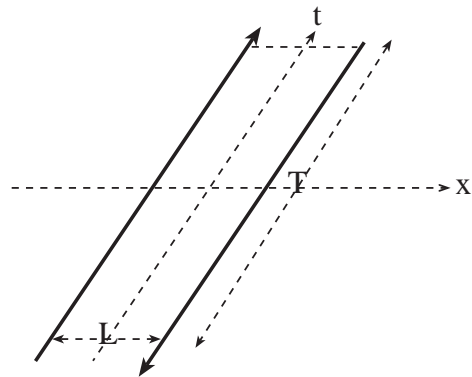


Figure 1: The basic setup of the Wilson loop

## Stringy Wilson loop- general setup

We now introduce the basic setup which will serve us in analysing Wilson loops of various string backgrounds [4]. Consider a 10d space-time met-

ric

$$ds^2 = -G_{00}(s)dt^2 + G_{x_{||}x_{||}}(s)dx_{||}^2 + G_{ss}(s)ds^2 + G_{x_Tx_T}(s)dx_T^2 \quad (1)$$

where  $x_{||}$ -  $p$  space coordinates on a  $D_p$  brane  $s$  and  $x_T$  are the transverse coordinates  
The corresponding Nambu-Goto action is

$$S_{NG} = \int d\sigma d\tau \sqrt{\det[\partial_\alpha x^\mu \partial_\beta x^\nu G_{\mu\nu}]}$$

Upon using  $\tau = t$  and  $\sigma = x$ , where  $x$  is the space coordinate of the loop which is one of the  $x_{||}$ , the action for a static configuration reduces to

$$S_{NG} = T \cdot \int dx \sqrt{f^2(s(x)) + g^2(s(x))} (\partial_x s)^2$$

where

$$f^2(s(x)) \equiv G_{00}(s(x))G_{xx}(s(x))$$

$$g^2(s(x)) \equiv G_{00}(s(x))G_{ss}(s(x))$$

and  $T$  is the time interval.

The equation of motion (geodesic line)

$$\frac{ds}{dx} = \pm \frac{f(s)}{g(s)} \cdot \frac{\sqrt{f^2(s) - f^2(s_0)}}{f(s_0)}$$

A static string configuration connecting quarks which are separated by a distance

$$L = \int dx = 2 \int_{s_0}^{\infty} \frac{g(s)}{f(s)} \frac{f(s_0)}{\sqrt{f^2(s) - f^2(s_0)}} ds$$

To have a finite separation distance the slope  $\frac{ds}{dx}$  has to diverge on the boundary.

The NG action and corresponding energy  $E = \frac{S_{NG}}{T}$  are divergent. The action is renormalized by[1]

- (i) regularizing the integral  $\int^{\infty} \rightarrow \int^{s_{max}}$
- (ii) subtracting the quark masses

$$m_q = \int_0^{s_{max}} g(s) ds$$

So that the renormalized quark anti-quark potential is

$$E = f(s_0) \cdot L$$

$$+ 2 \int_{s_0}^{\infty} \frac{g(s)}{f(s)} (\sqrt{f^2(s) - f^2(s_0)} - f(s)) ds$$

$$- 2 \int_0^{s_0} g(s) ds$$

The behavior of the potential is determined by the following theorem [4].

**Theorem 1** Let  $S_{NG}$  be the NG action defined above, with functions  $f(s), g(s)$  such that:

1.  $f(s)$  is analytic for  $0 < s < \infty$ . At  $s = 0$ , ( we take here that the minimum of  $f$  is at  $s = 0$  ) its expansion is:

$$f(s) = f(0) + a_k s^k + O(s^{k+1})$$

with  $k > 0$ ,  $a_k > 0$ .

2.  $g(s)$  is smooth for  $0 < s < \infty$ . At  $s = 0$ , its expansion is:

$$g(s) = b_j s^j + O(s^{j+1})$$

with  $j > -1$ ,  $b_j > 0$ .

3.  $f(s), g(s) \geq 0$  for  $0 \leq s < \infty$ .

4.  $f'(s) > 0$  for  $0 < s < \infty$ .

5.  $\int^{\infty} g(s)/f^2(s) ds < \infty$ .

Then for (large enough)  $L$  there will be an even geodesic line asymptoting from both sides to  $s = \infty$ , and  $x = \pm L/2$ . The associated potential is

1. if  $f(0) > 0$ , then

$$(a) \text{ if } k = 2(j+1),$$

$$E = f(0) \cdot L - 2\kappa + O((\log L)^\beta e^{-\alpha L})$$

$$(b) \text{ if } k > 2(j+1),$$

$$E = f(0) \cdot L - 2\kappa - d \cdot L^{-\frac{k+2(j+1)}{k-2(j+1)}} + O(L^\gamma).$$

where  $\gamma = -\frac{k+2(j+1)}{k-2(j+1)} - \frac{1}{k/2-j}$  and  $\beta$  and  $\kappa, \alpha, d$  and  $C_{n,m}$  are positive constants determined by the string configuration.

In particular, there is

linear confinement

2. if  $f(0) = 0$ , then if  $k > j + 1$ ,

$$E = -d' \cdot L^{-\frac{j+1}{k-j-1}} + O(L^{\gamma'})$$

where  $\gamma' = -\frac{j+1}{k-j-1} - \frac{2k-j-1}{(2k-j)(k-j-1)}$  and  $d'$  is a coefficient determined by the classical configuration.

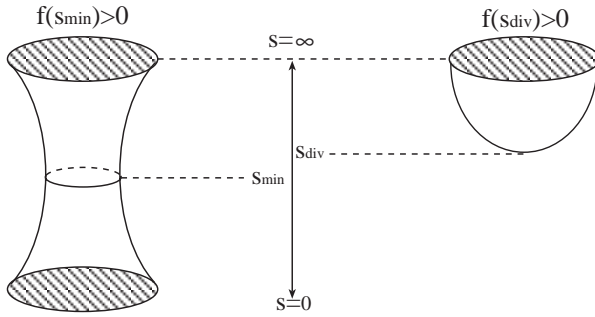
In particular,

there is no confinement

As a corollary of this theorem[4] we find that if one of the following two conditions is obeyed :

(i)  $f$  has a minimum at  $s_{min}$  and  $f(s_{min}) > 0$ ,

(ii)  $g$  diverges at  $s_{div}$  and  $f(s_{div}) > 0$  then the corresponding Wilson loop **confines**.



**Figure 2:** Sufficient conditions for confinement.

Quantum fluctuations

Introduce quantum fluctuations around the classical configuration

$$x^\mu(\sigma, \tau) = x_{cl}^\mu(\sigma, \tau) + \xi^\mu(\sigma, \tau)$$

The quantum corrections to the Wilson line to quadratic order is [5]

$$\langle W \rangle = e^{-TE_{cl}(L)} \int \prod_a d\xi_a \exp \left( - \int d^2\sigma \sum_a \xi^a \mathcal{O}^a \xi^a \right)$$

where  $\xi^a$  are the fluctuations left after gauge fixing. The corresponding correction to the free energy is

$$F_B = -\log \mathcal{Z}_{(2)} = - \sum_a \frac{1}{2} \log \det \mathcal{O}_a$$

**general form of the bosonic determinant**

In the  $\sigma = u$  gauge ( after a change of variables) the free energy is given by

$$F_B = -\frac{1}{2} \log \det \mathcal{O}_x - \frac{(p-1)}{2} \log \det \mathcal{O}_{x_{II}} - \frac{(8-p)}{2} \log \det \mathcal{O}_{x_T} \quad (2)$$

where

$$\begin{aligned} \hat{\mathcal{O}}_x &= \left[ \partial_x \left( \left( 1 - \frac{f^2(u_0)}{f^2(u_{cl})} \right) \partial_x \right) + \frac{G_{xx}(u_{cl})}{G_{tt}(u_{cl})} \left( \frac{f^2(u_{cl})}{f^2(u_0)} - 1 \right) \partial_t^2 \right] \\ \hat{\mathcal{O}}_{x_{II}} &= \left[ \partial_x \left( \frac{G_{y_i y_i}(u_{cl})}{G_{xx}(u_{cl})} \partial_x \right) + \frac{G_{y_i y_i}(u_{cl})}{G_{tt}(u_{cl})} \frac{f^2(u_{cl})}{f^2(u_0)} \partial_t^2 \right] \\ \hat{\mathcal{O}}_{x_T} &= \dots \end{aligned} \quad (3)$$

where  $\hat{\mathcal{O}} = \frac{2}{f(u_0)} \mathcal{O}$  and the boundary conditions are  $\hat{\xi}(-L/2, t) = \hat{\xi}(L/2, t) = 0$ . The fermionic fluctuations will be discussed for each model separately.

model 1

Wilson loop from string in flat space-time

- Consider the bosonic string in flat space-time with the boundary conditions

$$x(\sigma = 0) = 0 \quad x(\sigma = \pi) = L$$

The static NG action takes the form

$$S_{NG} = T_{st} \int dx \sqrt{1 + (\partial_x u)^2}$$

where  $T_{st} = \frac{1}{2\pi\alpha'}$ .

The classical static configurations are flat,  $u' = 0$

The quark anti-quark potential that follows from the NG action is

$$V(L) = T_{st} L$$

Thus, the classical stringy ‘‘Wilson loop’’ implies a linear confining potential.

**Bosonic fluctuations in flat space-time**

Let us turn now on quantum fluctuations[5].  
The action in this case takes this simple form

$$S_{(2)} = \frac{1}{2} \int d\sigma d\tau \sum_{i=1}^{D-2} \left[ (\partial_\sigma \xi_i)^2 + (\partial_\tau \xi_i)^2 \right]$$

The corresponding eigenvalues are

$$\lambda_{n,m} = \left(\frac{n\pi}{L}\right)^2 + \left(\frac{m\pi}{T}\right)^2$$

and the free energy is given by

$$-\frac{2}{D-2} F_B = \log \prod_{nm} \lambda_{n,m} = T \frac{\pi}{2L} \sum_n n + O(L)$$

Regulating this result using Riemann  $\zeta$  function we find that the quantum correction to the linear quark anti-quark potential is

$$\Delta V(L) = -\frac{1}{T} F_B = -(D-2) \frac{\pi}{24} \cdot \frac{1}{L}$$

which is the so-called **Lüscher term**[6].

**The fermionic fluctuations in flat space-time**

- We use here the Green Schwarz action since in cases which will discuss later the NSR action is not known.
- The fermionic part of the  $\kappa$  gauged fixed GS-action is

$$S_F^{flat} = 2i \int d\sigma d\tau \bar{\psi} \Gamma^i \partial_i \psi$$

where  $\psi$  is a Weyl-Majorana spinor,  $\Gamma^i$  are the SO(1,9) gamma matrices,  $i, j = 1, 2$  and we explicitly considered a flat classical string.

Thus the fermionic operator is

$$\hat{O}_F = D_F = \Gamma^i \partial_i$$

and squaring it we get

$$(\hat{O}_F)^2 = \Delta = \partial_x^2 - \partial_t^2$$

The total free energy is

$$F = 8 \times \left( -\frac{1}{2} \log \det \Delta + \log \det D_F \right) = 0$$

since for D=10, we have 8 transverse coordinates and 8 components of the unfixed Weyl-Majorana spinor.

- Thus, in flat space-time the classical stringy Wilson loop is not corrected by quadratic quantum fluctuations.

**Can the Wilson line be evaluated exactly?**

Let us consider the bosonic string with the boundary conditions given above[7].

- The energy of any string state is given by  $E^2 = P^2 + 4(L_0 - a) = (LT_{st})^2 + 4(L_0 - a)$

Thus for the lowest **tachyonic** state ( $L_0 = 0$ ) it is given by

$$E^2 = P^2 + m_{tach}^2 = (LT_{st})^2 - T_{st} \frac{\pi(D-2)}{12}$$

If we assign the potential with this energy we have

$$V(L) = T_{st} L \sqrt{1 - \frac{\pi(D-2)}{12} \frac{1}{T_{st} L^2}}$$

which can be expanded

$$\sim T_{st} L - \pi \frac{(D-2)}{24} \frac{1}{L} + \dots$$

Thus this expansion yields the leading confining behavior as well as the Luscher quadratic fluctuation term.

- Moreover for a bosonic string in Flat space-time O.Alvarez[8] showed that in the large D limit

$$D \rightarrow \infty \quad \frac{\pi}{24T_{st}L^2} \rightarrow 0 \quad \frac{D\pi}{24T_{st}L^2} \rightarrow finite$$

using the variables  $\sigma_{\alpha\beta} = \partial_\alpha x^\mu \partial_\beta x_\mu$  the exact effective action is

$$S_{NG}^{exact} = T_{st} L \sqrt{1 - \frac{\pi(D-2)}{12} \frac{1}{T_{st} L^2}}$$

- for  $L < \sqrt{\frac{\pi(D-2)}{12} \frac{1}{T_{st}}}$  this approach fails.

**The Ads black hole and pure YM theory in 3d**

- Can we detect the confining nature of pure YM theory

- In Field theory  $YM_3$  can be reached from  $\mathcal{N} = 4$  SYM by:
  - (i) Compactifying the Euclidean time direction ( introducing temperature)
  - (ii) Imposing anitperiodic boundary conditions.
- Recall that due to such boundary conditions

**supersymmetry is broken** ,

and in the case of the  $\mathcal{N} = 4$  SYM the fermions and scalars become massive[9]

$$m_{fermions} \sim T \quad m_{scalars} \sim g_{YM4}^2 T$$

- In the limit of  $T \rightarrow \infty$  the Euclidean 4d theory turns into a 3d theory, and since the fermions and scalars decouple it is pure  $YM_3$  with the coupling

$$g_{YM3}^2 = g_{YM4}^2 T .$$

- In the SUGRA picture the introduction of temperature translates into the use of near extremal  $AdS_5 \times S^5$  solution
- The metric of near extremal D3 branes in the large N limit is

$$\begin{aligned} \frac{ds^2}{\alpha'} &= \frac{U^2}{R^2} [-f(U)dt^2 + dx_i^2] \\ &+ R^2 f(U)^{-1} \frac{dU^2}{U^2} + R^2 d\Omega_5^2 \\ f(U) &= 1 - U_T^4/U^4 \\ R^2 &= \sqrt{4\pi gN}, \quad U_T^4 = \frac{2^7}{3} \pi^4 g^2 \mu, \end{aligned}$$

where  $\mu$  is the energy density.

- The idea is thus to consider the Wilson loop along two space directions for the case of the near extremal D3 brane solution[10].
- We take  $Y \rightarrow \infty$  which will be the 3d Euclidean time direction and the other direction,  $L$ , to be finite.
- For such a setup we have

$$f^2 = \left(\frac{U}{R}\right)^4 \quad g^2 = \frac{1}{(1 - (\frac{U_T}{U})^4)}$$

since  $g^2$  diverges at  $U = U_T$  we must have confinement.

- Indeed, let us insert the metric of above to the NG action

$$S = \frac{Y}{2\pi} \int dx \sqrt{\frac{U^4}{R^4} + \frac{(\partial_x U)^2}{1 - U_T^4/U^4}} \quad (4)$$

- The distance between the quark and the anti-quark is

$$L = 2 \frac{R^2}{U_0} \int_1^\infty \frac{dy}{\sqrt{(y^4 - 1)(y^4 - \lambda)}}, \quad (5)$$

where  $\lambda = U_T^4/U_0^4 < 1$  and  $U_0$  is the minimal value of  $U$ .

- The energy is

$$\begin{aligned} E &= \frac{U_0}{\pi} \int_1^\infty dy \left( \frac{y^4}{\sqrt{(y^4 - 1)(y^4 - \lambda)}} - 1 \right) \\ &+ \frac{U_T - U_0}{\pi}, \end{aligned} \quad (6)$$

- Notice that in the limit  $U_0 \rightarrow U_T$  ( $\lambda \rightarrow 1$ ) we get  $L \rightarrow \infty$ . In this limit the main contribution to the integrals of  $L$  and  $E$  comes from the region near  $y = 1$ .
- Therefore, we get for large  $L$

$$E = T_{QCD} L$$

$$T_{QCD} = \frac{\pi}{2} R^2 T^2,$$

- Notice that the string tension diverges in the SUGRA limit since

$$R = \sqrt{g_{YM}^2 N} \rightarrow \infty \quad T \rightarrow \infty$$

- If there are no phase transitions in going from the SUGRA limit  $R \rightarrow \infty$  to the full stringy description of  $YM_3$  then indeed the latter predicts **confinement**

**The determinant for “confining scenarios”**

- Let us consider now the quantum fluctuations in this SUGRA setup which is dual to

the pure YM theory in 3d ( AdS black hole in the  $T \rightarrow \infty$  limit)[5]. Now we have

$$\begin{aligned} f(u) &= u^2/R^2 \\ g(u) &= (1 - (\frac{u_T}{u})^4)^{-1/2} \end{aligned} \tag{8}$$

- In the large  $L$  limit the classical string is very flat with  $u \sim u_0$ . In fact as  $L$  grows  $u_0 \rightarrow u_T$ . In this limit

$$\begin{aligned} \hat{O}_y &\longrightarrow \frac{u_T^2}{2} [\partial_x^2 + \partial_t^2] \\ \hat{O}_\theta &\longrightarrow \frac{R^2}{2} [\partial_x^2 + \partial_t^2] \\ \hat{O}_z &\longrightarrow 2u_T^2 e^{-2u_T L} [\partial_x^2 + \partial_t^2] \\ \hat{O}_n &\longrightarrow \left[ \frac{4u_T^2}{2R^4} + \frac{1}{2} \partial_x^2 + \frac{1}{2} \partial_t^2 \right] \end{aligned} \tag{9}$$

- We see that the operators for transverse fluctuations,  $\hat{O}_y, \hat{O}_z$ , turn out to be simply the Laplacian in flat spacetime, multiplied by overall factors, which are irrelevant. Therefore, the transverse fluctuations yield the standard Lüscher term proportional to  $1/L$ .
- The longitudinal normal fluctuations give rise to an operator  $\hat{O}_n$  corresponding to a scalar field with mass  $2u_T/R^2 = \alpha$ . Such a field contributes a Yukawa like term

$$\approx -\frac{\sqrt{\alpha} e^{-\alpha L}}{\sqrt{L}}$$

to the potential.

- Thus, altogether there are 7 Lüscher type modes and one massive mode.
- Now we have to turn on the fermionic fluctuations. Had the fermionic modes been those of flat space-time then the total coefficient in front of the Lüscher term would have been  $+8-7 = +1$ , namely, a repulsive Coulomb like potential[11, 12]. This contradicts gauge dynamics[13].
- There is a GS formulation for the  $AdS_5 \times S^5$  background[14, ?], but the analog for

the AdS black hole has not been written down. Nevertheless, we argue that the coupling of the fermion to the RR field is the same as for the extremal  $AdS_5 \times S^5$  background. (Since the dilaton, the RR field and  $det(G_{\mu\nu})$  are unchanged)

- For that case we found that in the large  $L$  limit the square of the fermionic operator is

$$\hat{O}_\psi^2 = \frac{u_T^2}{2} \left[ \partial_x^2 + \partial_t^2 + \left( \frac{u_T}{R^2} \right)^2 \right]$$

- If the assumption about the coupling to the RR is correct, the quark anti-quark potential is corrected by an attractive Luscher term

$$-7 \frac{\pi}{24} \frac{1}{L}$$

model 3.

### Wilson loops in type 0 string theory

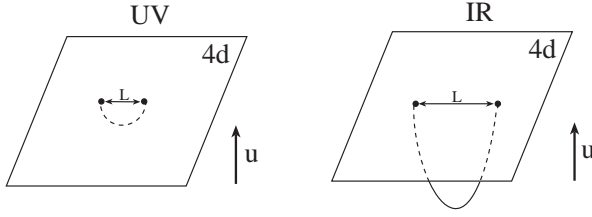
- **What is type 0 string**

Type 0 string is supersymmetric on the world sheet but not in space-time due to a non-chiral GSO projection. The type  $0_A$  and type  $0_B$  differ from the type  $II_A$  and type  $II_B$  (i) No space-time fermions, (ii) Doubling of the RR fields, (iii) Tachyons.

- A type 0 model can be made consistent only provided (i) The Tachyon  $m_{tach}^2$  can be shifted to  $m_{tach}^2 > \frac{c}{R^2}$  (ii) No dilaton ( and possible other massless fields) tadpoles (iii) The low energy effective theory is reliable if  $g_{st} \ll 1$   $\mathcal{R} \ll 1$  where  $\mathcal{R}$  is the scalar curvature in the string frame.
- The Wilson loops were discussed both in the critical string and in Polyakov's non-critical string model.
- The equations of motion of the low energy effective theory guarantee that [21]

$$\partial_s^2 f(s) \geq 0$$

- The interpretation of an IR and UV domains may be in terms of the structure of the Wilson line is as is shown in figure 4.



**Figure 3:** The IR and UV regimes

so that the large  $u$  regime corresponds to the gauge theory UV regime and the small  $u$  regime to the IR.

- In the IR the generic solution has

$$\partial_s f(s) = 0 \quad \text{with} \quad f(s_{min}) \neq 0$$

So that generically the solution in the IR admits a linear confinement behavior.

- This can also be verified from arguments based on the 5d bulk theory and in particular also from the screening nature of the 't Hooft loop[22].
- In the UV a fixed point in the form of  $AdS_5 \times S^5$  was observed. Moreover around the fixed point  $f \sim \log L$  so that it was argued that[19]

$$\Delta V_1 \sim \frac{1}{\log \frac{L_0}{L}} \frac{1}{L}$$

It was further found that the higher order correction produces a Wilson line[20]

$$\Delta V_2 \sim \frac{1}{(\log \frac{L_0}{L} - c \log \log \frac{L_0}{L})} \frac{1}{L}$$

which resembles the 2 loop correction in the gauge theory picture. Note however that in the UV generically the curvature in the string frame is not negligible and thus the assertions have to be made with a grain of salt.

model 4.

**Loops in the Polchinski Strassler  $N = 1^*$  theory**

We start with a brief review of the theory[23]

**The field theory picture**

- Perturb the  $\mathcal{N} = 4$   $SU(N)$  SYM theory by adding a mass term to the superpotential.

$$W + \Delta W = \frac{2\sqrt{2}}{g_{YM}^2} \text{tr}([\phi_1, \phi_2]\phi_3) + \frac{m}{g_{YM}^2} \sum_{i=1}^3 \phi_i^2$$

where  $\phi_i$  are the 3 complex scalars of the  $\mathcal{N} = 4$ .

- The *classical vacua* are given by  $N$  dimensional reducible reps. of  $SU(2)$  since the equation of motion is

$$[\phi_i, \phi_j] = \frac{-m}{\sqrt{2}} \epsilon_{ijk} \phi_k$$

- The *quantum vacua* correspond to order  $N$  subgroups of

$$Z_N \times Z_N$$

- The perturbation also turns on a mass term to 3 out of the 4 Weyl fermions

$$m^{\alpha\beta} \lambda_\alpha \lambda_\beta + h.c$$

**The SUGRA picture**

- The perturbing fermionic mass term,  $\bar{1}0$  of  $SU(4)$  corresponds to turning on a *magnetic 3 form* obeying

$$*_6 T = iT$$

- Explicitly, denoting the 6 transverse coordinates by 3 complex coordinates  $z_i$   $i = 1, 2, 3$ , the 3 form  $T$  is

$$T_3 = m[dz_1 \wedge d\bar{z}_2 \wedge d\bar{z}_3 + d\bar{z}_1 \wedge dz_2 \wedge d\bar{z}_3 + d\bar{z}_1 \wedge d\bar{z}_2 \wedge ddz_3]$$

- Further breaking to  $N = 0^*$  can be achieved by adding a term  $m' dz_1 \wedge dz_2 \wedge dz_3$  to  $T$ .

- The coupling of the  $D_3$  branes to the magnetic 3 form produces via the *Myers Polarization* mechanism[24] *five branes* that wrap  $S^2$ .

- The metric of the  $N = 1^*$  models takes the form

$$ds^2 = Z_x^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + Z_y^{1/2} (dy^2 + y^2 d\Omega_y^2 + dw^2) + Z_\Omega^{1/2} w^2 d\Omega_w^2 \tag{10}$$

$$\text{denoting by } Z_0 = \frac{R^4}{\rho_+^2 \rho_-^2} \quad \rho_\pm = \sqrt{[y^2 + (w \pm r_0)^2]}$$

- The  $D_5$  solution has

$$Z_x = Z_y = Z_0 \quad Z_\Omega = Z_0 \left[ \frac{\rho_-^2}{\rho_-^2 + \rho_c^2} \right]^2$$

where  $\rho_c = \frac{2gr_0\alpha'}{R^2}$  and  $r_0 = \pi\alpha' mN$

- The  $NS5$  solution has

$$Z_x = Z_\Omega = Z_0 \frac{\rho_-^2}{\rho_-^2 + \rho_c^2} \quad Z_y = Z_0 \frac{\rho_-^2 + \rho_c^2}{\rho_-^2}$$

where  $\rho_c = \frac{2r_0\alpha'}{R^2}$  and  $r_0 = g\pi\alpha' mN$

### Wilson loops

- To check whether the Wilson loops are of area law behavior we return to our criterion stated in terms of

$$f^2 = Z_x^{-1} \quad g^2 = \frac{Z_y^{1/2}}{Z_x^{1/2}}$$

so that  $f^2$  and  $g^2$  take for the  $D_5$  and the  $NS5$  cases the following values respectively,  $\frac{\rho_+^2 \rho_-^2}{R^4}$ ,  $\frac{\rho_+^2 (\rho_-^2 + \rho_c^2)}{R^4}$ ,  $1$ ,  $\frac{\rho_c^2 + \rho_-^2}{\rho_-^2}$

- **Wilson loop of the  $D_5$  case**

$f^2$  has a minimum with  $f_{min} = 0$ , and  $g^2 = 1$  does not diverge thus *there is no confinement*. In fact an explicit calculation shows that there is **Screening** behavior.

- **Wilson loop of the  $NS5$  case**

$g^2$  diverges at  $y = 0, w = r_0$  where  $f(y = 0, w = r_0) > 0$  *there is confinement*

- **Can we get in a similar manner the Wilson loop associated with the rest of the possible vacua?**

- Consider for example the case of  $p$   $D_5$  branes that corresponds to an  $SU(p) \in SU(N)$  gauge theory[25].
- Naively we expect the strings ( $F1$ ) to end on the  $D_5$  branes and hence to have screening.
- This is also the outcome of the use of the general theorem applies to the metric of the  $p$   $D_5$  branes ( $f(u_{min}) = 0$ )

- However, from the field theory we know that quarks of the  $SU(p)$  must confine.

- How do we resolve this contradiction?

- Recall that the world volume theory of the  $p$   $D_5$  branes is a  $SU(p)$  gauge theory.

- A fundamental string ending on the  $D_5$  branes is a “quark” of the  $wv$   $SU(p)$  theory and thus can “end” only provided

- (i) if it is connected to an anti- quark string
- (ii) if  $p$  quarks combine to form a Baryon

- In this way of incorporating the  $wv$  theory we get that indeed  $(1, 0)$  quarks confine and  $(p, 0)$  are screened.

- For the case of unbroken  $SU(p)$  field theory also tells us that a magnetic monopoles with charge  $q = \frac{N}{p}$  has to be screened. The naive use of the theorem tells us that any  $D1$  string confines.

- Again we have to use the full SUGRA background. Indeed in the SUGRA picture there are  $D3$  branes filling the  $S^2$  sphere on which the 5-branes are wrapped which behave as *baryon vertices*.

- Those baryon vertices arise through the Hanany–Witten effect, when baryon vertex of the unperturbed  $\mathcal{N} = 4$  theory, which is a 5-brane wrapping an  $S^5$ , contracts and moves through the sphere.

- Now each  $D5$  brane has a dissolved  $D3$  charge of  $q$  the junction of a  $D3$  ball with a wrapped  $D5$  must support strings with total  $D1$  charge of  $q$  but  $D1$ s with different charge cannot end and thus are confined.

- One can account for the various loops associated with the other vacua.

### Summary and open questions

- Indeed in all the stringy setups that supposed to be associated with confining dynamics we detect an area law Wilson loop.



- Each of the models suffers from certain problems and it seems that we have not found yet the optimal stringy laboratory to examine confinement.
- There are indications that there is an attractive Luscher term. To be contrasted with lattice simulations and phenomenology.
- The  $N = 1^*$  case emphasized the fact that (not only the metric) but the full background affects the stringy loops.

### Acknowledgement

Work supported in part by TMR program FMRX-CT96-0012.

### References

- [1] J. Maldacena, "Wilson loops in large  $N$  field theories", Phys. Rev. Lett. 80 (1998) 4859-4862, hep-th/9803002.
- [2] S.-J. Rey and J. Yee, "Macroscopic strings as heavy quarks in the large  $N$  gauge theory and anti-de Sitter supergravity", hep-th/9803001.
- [3] N. Drukker, D. Gross and H. Ooguri "Wilson Loops and Minimal Surfaces", hep-th/9904191
- [4] Y. Kinar, E. Schreiber and J. Sonnenschein, " $Q\bar{Q}$  Potential from Strings in Curved Space-time - Classical Results", hep-th/9811192.
- [5] Y. Kinar, E. Schreiber, J. Sonnenschein, N. Weiss "Quantum fluctuations of Wilson loops from string models" Nucl.Phys. B583 (2000) 76-104, hep-th/9911123
- [6] M. Luscher, K. Symanzik, P. Weisz, "Anomalies of the free loop wave equation in the WKB approximation" Nucl. Phys. B173 (1980) 365.
- [7] J. F. Arvis Phys. Lett. 127B (1983) 106.
- [8] O. Alvarez, Phys. Rev. D24 (1981) 440.
- [9] E. Witten, "Anti-de Sitter Space, Thermal Phase Transition, and Confinement in Gauge Theories", hep-th/9803131.
- [10] A. Brandhuber, N. Itzhaki, J. Sonnenschein, S. Yankielowicz, "Wilson Loops, Confinement, and Phase Transitions in Large  $N$  Gauge Theories from Supergravity", JHEP 9806 (1998) 001, hep-th/9803263
- [11] J. Greensite, P. Olesen, "Remarks on the Heavy Quark Potential in the Supergravity Approach", hep-th/9806235; "Worksheet Fluctuations and the Heavy Quark Potential in the AdS/CFT Approach", hep-th/9901057.
- [12] H. Dorn, V. D. Pershin "Concavity of the  $Q\bar{Q}$  potential in  $\mathcal{N} = 4$  super Yang-Mills gauge theory and AdS/CFT duality" hep-th/9906073
- [13] C. Bachas, Phys. Rev. D33 (1986) 2723.
- [14] R. R. Metsaev, A. A. Tseytlin, "Type IIB superstring action in  $AdS_5 \times S^5$  background", hep-th/9805028.
- [15] I. Pesando, "A  $\kappa$  Gauge Fixed Type IIB Superstring Action on  $AdS_5 \times S^5$ ", hep-th/9808020.
- [16] R. Kallosh, A. A. Tseytlin, "Simplifying Superstring Action on  $AdS_5 \times S^5$ ", hep-th/9808088.
- [17] A.M. Polyakov, "The Wall of the Cave", hep-th/9809057.
- [18] I.R. Klebanov and A.A. Tseytlin, "D-Branes and Dual Gauge Theories in Type 0 Strings", hep-th/9811035.
- [19] J.A. Minahan, "Glueball Mass Spectra and Other Issues for Supergravity Duals of QCD Models", hep-th/9811156.
- [20] I.R. Klebanov and A.A. Tseytlin, "Asymptotic Freedom and Infrared Behavior in the Type 0 String Approach to Gauge Theory", hep-th/9812089.
- [21] Adi Armoni, E. Fuchs, J. Sonnenschein "Confinement in 4D Yang-Mills Theories from Non-Critical Type 0 String Theory" JHEP 9906 (1999) 027 hep-th/9903090
- [22] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, "Confinement and Condensates Without Fine Tuning in Supergravity Duals of Gauge Theories", hep-th/9903026.
- [23] J. Polchinski and M. J. Strassler, "The String Dual of a Confining Four Dimensional Gauge Theory" hep-th/0003136
- [24] R. C. Myers, "Dielectric-Branes", JHEP 9912 (1999) 022, hep-th/9910053
- [25] Y. Kinar, A. Loewy, E. Schreiber, J. Sonnenschein, S. Yankielowicz "Supergravity and Worldvolume Physics in the Dual Description of  $N = 1^*$  Theory" hep-th/0008141