Four–Dimensional BPS Black Holes: Macroscopic and Microscopic Correspondence.

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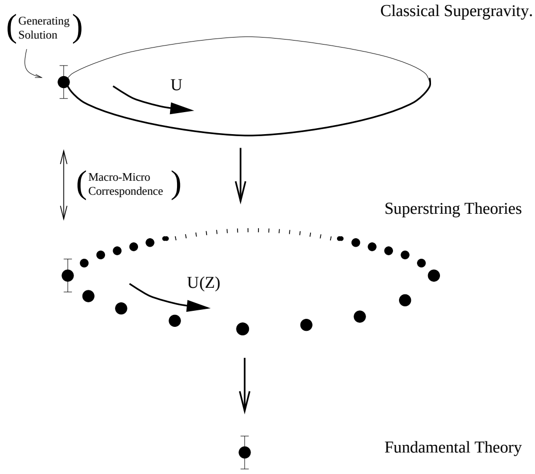
Abstract: I review some recent results in the study of regular four dimensional BPS black holes in toroidally compactified type II string (or M) theory. One of these achievements consists in having written the generating solution of 1/8 BPS black holes in the N=8 theory in a form which could be easily described by suitable configurations of D-branes in the weak string coupling regime of both type IIA and IIB theories. The microscopic parameters characterizing these configurations have been related in a precise way to the supergravity (macroscopic) parameters of the solution. This achievement opens up the possibility of systematically studying the microscopic properties of any regular BPS black hole solution of the N=8 theory.

One of the main issues of the "second string revolution" (1995) is the concept of string dualities which provided a new insight into the non–perturbative side of the known superstring theories. These dualities are mappings between regimes of different superstring theories (some of them have been verified while other just conjectured). Their existence naturally induces to consider the known superstring theories as perturbative realizations on different backgrounds of a fundamental theory of gravity (FTG) whose general formulation however is still missing. It is known that the low energy limit of superstring theory is described by supergravity. Although supergravity in this picture is regarded just as a *macroscopic* theory, it is expected to possess important informations about the FTG. Indeed, it has been argued [1] that the largest (continuous) global symmetry group U of the supergravity field equations and Bianchi identities at classical level should encode the definition, as a suitable discrete group $U(Z)$, of the conjectured superstring U–duality, namely the ultimate duality connecting all superstring theories realized on various backgrounds. This duality is thus expected to be an exact symmetry of the FTG. Unfortunately not much is known about the group $U(Z)$, starting from the very definition and its action on superstring states. On the other hand the action of the group U on the supergravity solutions is, in principle, known.

A fundamental role in probing superstring dualities has been played so far by the BPS black hole solutions of supergravity. These solutions are characterized by the property of preserving a fraction of the original supersymmetries, and this feature protects their physical quantities, to a certain extent, from quantum corrections. As a consequence of their supersymmetry, BPS black holes in supergravity are expected to correspond to exact solutions of superstring theory. The BPS condition moreover is U–duality invariant. This allows to characterize these supergravity solutions within orbits of the continuous U–duality group, defined by a certain number of U–invariants $\{\mathcal{I}_k\}$ (e.g. the entropy). All the physical properties of the BPS solutions entering a same U–duality orbit are expected to be encoded in the corresponding *generating solution.* The generating solution of BPS black holes is defined, within a certain supergravity theory, as the solution depending on the least number of parameters such that the invariants $\{\mathcal{I}_k\}$ are free for a certain choice of the boundary conditions. As a consequence of its definition, by acting on the generating solution by means of U one recovers the whole U–duality orbit. A suitable discrete set of points within this orbit should correspond to superstring black holes (non–perturbative solutions) connected by the action of $U(Z)$ and which therefore represent different descriptions of a same solution within the FTG (see figure below). The



microscopic degrees of freedom described by the FTG are indeed related to invariants of the group $U(Z)$. Pinpointing the *exact* correspondence between the macroscopic (supergravity) and microscopic descriptions (e.g. in terms of D–branes in a suitable regime) of a generating solution, one would in principle be able to study systematically the microscopic realization of a generic solution in the same orbit. Moreover this could be the first step in order to unravel the action of $U(Z)$ on stringy objects in higher dimensions and to ultimately deduce their fundamental degrees of freedom.

Here we review some recent results in the study of four dimensional regular BPS black holes within toroidally compactified type II superstring (or M) theory. The relevant low–energy description for these solutions is four dimensional $N = 8$ supergravity. In section 1 we shall start addressing the question: *how much can we learn at classical supergravity level about the microscopic description of a BPS solution?* A possible answer will lead us to discuss a mathematical analysis carried out in [2] which provides an intrinsic group theoretical characterization of the scalar and vector fields in the $D = 4$, $N = 8$ theory in terms of dimensionally reduced type II fields. The geometrical framework so defined turns out to provide the convenient "laboratory" in which to systematically study the microscopic descriptions of BPS solutions and their duality relations. Using these tools one can then characterize R–R charged generating solutions of regular BPS black holes as elements of a suitable equivalence class defined with respect to the action of S and T dualities. This result is discussed in section 2 and allows us to formulate the precise correspondence, worked out in [3], between the parameters defining two T–dual R–R charged microscopic descriptions (in type IIA and IIB settings) of the generating solution and the supergravity quantities related to its macroscopic description. This final goal is dealt with in section 3.

1. Supergravity Laboratory.

The only prediction which may be drawn at classical supergravity level on the microscopic description of a BPS solution is clearly limited just to the background fields which couple to it. This can be done for instance by associating each superstring scalar and vector zero–mode with quantities intrinsic to the U–duality group of the low–energy supergravity (see [4, 2]).

The $D = 4$, $N = 8$ supergravity is a maximally extended supersymmetric theory, i.e. it has 32 supercharges. Its bosonic sector consists of the graviton, 70 scalar fields, spanning the homogeneous manifold $\mathcal{M}_{scal} = E_{7(7)}/SU(8)$, and 28 vector fields. The latter are related to a vector of 56 *quantized charges* (p^Λ, q_Σ), which transforms in the $Sp(56)$ of $E_{7(7)}$, and a *central charge* matrix Z_{AB} entering the local realization on the moduli space of the supersymmetry algebra and transforming in the **28** of $SU(8)$. The former charges are moduli–independent and should be regarded just as supergravity parameters, while the latter are moduli dependent and are related to the physical charges, i.e. the actual charges one would measure in the asymptotically flat radial infinity of a black hole solution[1].

The U–duality group of the classical theory is $U = E_{7(7)}$ [5]. It acts as a generalized electro–magnetic duality, i.e. it has a non–linear action on the scalar fields and a linear (symplectic) action on the vector of quantized charges. As previously mentioned, the $D = 4, N = 8$ theory describes the low–energy limit of type II superstring theory on T_6 (or M–theory on T_7). The first step towards a group theoretical characterization of the ten–dimensional origin of the scalars and charges in this supergravity model is to use a linear algebraic description of the scalar fields. This is achieved by adopting the *solvable Lie algebra* (SLA) parameterization of the scalar manifold [4, 6, 7], which consists in describing the scalar fields as local parameters of a solvable Lie algebra which generates (globally) the scalar manifold as a solvable Lie group. Homogeneous non–compact manifolds of symmetric type like $\mathcal{M}_{scal}$ do admit such a representation:

$$\mathcal{M}_{scal} = \mathrm{Exp}(Solv(U)) \qquad (1.1)$$

The algebra $Solv(U)$ is defined by the Iwasawa decomposition of $E_{7(7)}$ and can be written as $Solv(U) = \mathcal{C} \oplus \mathcal{N}$, where $\mathcal{C}$ is the Cartan subalgebra of $E_{7(7)}$ while $\mathcal{N}$ is a nilpotent subalgebra of $E_{7(7)}$ generated by all the shift generators corresponding to positive roots. In this framework a one to one correspondence between the scalar fields and the generators of $Solv$ is defined.

Two relevant duality groups for our discussion are the $S = SL(2,R)$ and $T = O(6,6)$ subgroups of U, defined as the continuous counterparts at the classical level of the discrete S and T superstring dualities. Since these dualities are the largest preserving the R–R and NS–NS identities of the fields, decomposing $Solv(U)$ with respect to $Solv(S) \times Solv(T)$ one may achieve an intrinsic characterization of the R–R and NS–NS fields at classical supergravity level. On the other hand the dimensional reduction of type II superstring to four dimensions may be performed through intermediate steps which define, in the low–energy limit, higher dimensional maximal supergravities, with their own U–duality group at tree level. Fixing then the embedding of $Solv(U_{D>4})$ within $Solv(U)$ for various $D > 4$ allows to identify in a consistent way the scalar fields of the $N = 8$ theory, as associated with the corresponding generators of $Solv(U)$, with dimensionally reduced type II zero–modes.

On the vector field side, it is convenient to work with a set of physical charges (y^Λ, x_Σ) (transforming under $SU(8)$) which are expressed in the same basis of weights $\{\vec{\lambda}\}$, generating the **56** of U, as the quantized charges (p,q). These charges are obtained from the vector $(\mathrm{Re}Z_{AB}, \mathrm{Im}Z_{AB})$ through a suitable rotation and are related to the quantized charged (p,q) by a *moduli–dependent* symplectic transformation which makes them quantized as well [2]. Decomposing the weight basis $\{\vec{\lambda}\}$ with respect to the action of the higher dimensional U–dualities $U_{D>4}$ it was possible to associate consistently with each weight $\vec{\lambda}$ a one–form electric of magnetic potential in four dimensions deriving from suitable ten dimensional type II zero–modes.

As a result of this first group theoretical analysis an $N = 8$ *algebraic dictionary* [2] could be established on the weight lattice $\Lambda_W(U)$ of U in which the *directions* (namely Cartan generators in $\mathcal{C}$) and the *positive roots* are associated with scalar fields (through the SLA parameterization) and the weights $\{\vec{\lambda}\}$ with electric and magnetic one–form potentials, each of these fields having a specific ten dimensional characterization.

The generating solution of regular BPS black holes in the $D = 4, N = 8$ model has been shown to be a solution of a smaller $N = 2$ truncation, namely the STU model [8]. The classical U–duality group of the latter is $U_{STU} = SL(2,R)^3 \subset U$, the scalar manifold has the form $\mathcal{M}_{STU} = U_{STU}/SO(2)^3$ and is generated by a solvable Lie algebra $Solv_{STU}$ parametrized by just three dilaton fileds b_i and three axions a_i. In the light of the previously defined algebraic dictionary, different microscopic descriptions of the generating solution can be put in correspondence with different embeddings of the STU model within the $N = 8$ one (defined by the embedding of the corresponding solvable Lie algebras and charge weights[2]).

[1]Our analysis is restricted just to static, spherically symmetric black holes.

[2]In other words, by the embedding of the weight lat-

Dualities relating different embeddings of the STU model can be described in terms of the action on $\Lambda_W(U_{STU})$ of *automorphisms* (Aut) of the relevant duality algebra [2]. In order to characterize the generating solution as charged with respect to R–R or NS–NS fields, we would need then to consider the action of the $S \times T$ dualities through their authomorphism group ($Aut(S \times T)$). The Dynkin diagram of the T algebra is D_6. It has *inner* and *outer* automorphisms, the latter being related, through Weyl transformations, to the only symmetry of D_6 (for a study of Weyl duality transformations in supergravity see [9]). These outer automorphisms are particularly interesting since they are not a symmetry and can be thought of as relating two different descriptions of the same theory, namely the type IIA and type IIB ones. Indeed, using the SLA representation it was shown in [2] that the outer automorphisms of T correspond to a "large $\leftrightarrow$ small radius" T–dualities along an odd number of directions inside T_6. From the algebraic viewpoint these automorphisms map $U = E_{7(7)}$ into an isomorphic algebra U' constructed from different T–weights. The two T–dual weight lattices $\Lambda_W(U)$ and $\Lambda_W(U')$ differ only in the R–R weights (spinorial weights of T) and naturally fit respectively the type IIA and type IIB descriptions of the $N = 8$ theory. Therefore, in order to accommodate all the T–dual descriptions of the generating solution, the $N = 8$ algebraic dictionary ought to be extended to $\Lambda_W(U) \oplus \Lambda_W(U')$ (see tables 2 and 3 of [2]).

2. Regular BPS black holes with R–R charge.

BPS black holes in the $D = 4$, $N = 8$ theory preserving different fractions of the original supersymmetry have been extensively studied in the literature (see for instance [10] and references therein). It has been shown that the only regular ones (i.e. having a finite horizon area) are those preserving a residual $N = 1$ supersymmetry. This property is equivalent to the existence of a *Killing spinor*, i.e. a direction in the spinorial parameter space, along which the supersymmetry shifts of the fermionic fields vanish, on the solution. The latter condition may be in turn restated in terms of a system of *first order* differential equations in the background fields, which, as shown in [11], have a fixed point for the scalar fields at the horizon ($r = 0$) depending only on the quantized charges (p, q).

The physical charges of a BPS black hole solution, as previously mentioned, are related to the (antisymmetric) central charge matrix Z_{AB} which depends on the point on the moduli space ϕ_0, representing the boundary condition at radial infinity of the scalar fields, as well as on the quantized charges. The U–duality invariants $\{\mathcal{I}_k\}$ of the solution are given by all the $SU(8)$ invariants which can be built out of Z_{AB}. Indeed, acting by means of a U–duality transformation on the scalar fields and the quantized charges, the central charge matrix will transform under a corresponding $SU(8)$ transformation. These invariants are *five* and on the orbit of regular BPS black holes they are independent parameters. A way of expressing them is in terms of the norm of the central charge *skew–eigenvalues* Z_α ($\alpha = 0, \dots, 3$) and their overall phase, i.e. $\{\mathcal{I}_k\} = \{|Z_\alpha|, \Theta\}$. By suitably combining them it is possible to obtain a moduli–independent invariant, namely the quartic invariant $J_4(p, q)$ of the **56** of $E_{7(7)}$. This is the only invariant characterizing the near–horizon geometry of the solution. It is indeed related to the area of the horizon and, through the Bekenstein–Hawking formula, to the entropy of the black hole: $S = \pi\sqrt{J_4}$.

Consistently with the definition outlined in the introduction, the generating solution depends on five independent charges of which the invariants $\{\mathcal{I}_k\}$, computed on the corresponding point ϕ_0 of the moduli space at infinity, are independent functions. This solution can be described within a STU model, which may be characterized as the smallest consistent truncation of the $N = 8$ theory on which the four Z_α are independent. This model has, besides the six scalars a_i, b_i previously introduced, four vector fields (one gravi–photon and three matter vectors) which give rise to eight quantized charges (p^α, q_β) and eight physical charges (y^α, x_β). Using the $N = 8$ algebraic dictionary, two T–dual embeddings STU_1, STU_2 of the STU model, for which the charges were

tices: $\Lambda_W(U_{STU}) \subset \Lambda_W(U)$.

related to suitable R–R one–forms, were worked out in [2]. These two descriptions of the fields in the STU model in terms of $E_{7(7)}$ weights are mapped into each other through an outer automorphism of T, which is interpreted, in the SLA formalism, as a "large $\leftrightarrow$ small radius" duality along the directions x^5, x^7, x^9 of T_6 (in our notation the compact directions are $x^4, \dots, x^9$ while the non–compact are $x^0, \dots, x^3$). One embedding (STU_1) can be indeed consistently described in the type IIA setting while the other (STU_2) in the type IIB one. In particular, from the $N = 8$ algebraic dictionary, it is possible to characterize the *axions* of the STU_1 embedding as deriving from the antisymmetric tensor B_{MN} ($\{a_i\} = \{B_{45},\, B_{67},\, B_{89}\}$) while those in STU_2 as deriving from the metric G_{MN} ($\{a_i\} = \{G_{45},\, G_{67},\, G_{89}\}$). As far as the vector fields are concerned, in an analogous way the charges (y^α, x_β) in the type IIA embedding STU_1 are associated with 1–form (magnetic and electric) potentials deriving from the following components of the ten dimensional R-R fields A_M, A_{MNP}:

$$\begin{aligned}(y^\alpha) &\leftrightarrow (A_{\mu 456789}, A_{\mu 6789}, A_{\mu 4589}, A_{\mu 4567}) \\ (x_\beta) &\leftrightarrow (A_\mu, A_{\mu 45}, A_{\mu 67}, A_{\mu 89}) \end{aligned} \tag{2.1}$$

while for the type IIB embedding STU_2 this correspondence between charges and components of the R–R forms A_{MN}, A_{MNPQ} reads:

$$\begin{aligned}(y^\alpha) &\leftrightarrow (A_{\mu 468}, A_{\mu 568}, A_{\mu 478}, A_{\mu 469}) \\ (x_\beta) &\leftrightarrow (A_{\mu 579}, A_{\mu 479}, A_{\mu 569}, A_{\mu 578}) \end{aligned} \tag{2.2}$$

From this background field prediction and from the values of the physical charges of the generating solution at infinity (for a suitable choice of the boundary conditions), two T–dual D–brane descriptions, corresponding to the embeddings discussed above, can be consistently worked out and precise relations established between the parameters defining the macroscopic (supergravity) and microscopic (D–brane) descriptions of the generating solution [3]. Finally, acting on $STU_{1,2}$ by means of $Aut(S \times T)$ one could define an equivalence class of R–R charged embeddings of the STU model (yielding all the R–R charged generating solutions) within the $N = 8$ theory.

3. The Miscoscopic Description.

We refer to [3] for the explicit macroscopic description of the generating solution in terms of harmonic functions, while in the present section we shall discuss its two previously mentioned microscopic realizations. For a suitable choice of the point on the moduli space at infinity the five independent physical charges of the generating solution have the following form: $(0, y^1, y^2, y^3, x_0, x_1, x_2, 0)$ with $x_1 = -x_2$. As previously anticipated, bound states of D–branes in the type IIA and IIB pictures coupled to the forms in eqs. (2.1) and (2.2) respectively and giving rise to the above effective charges, were found. On the type IIB front the microscopic system consists of N_0, N_1, N_2, N_3 $D3$–branes arranged within T_6 in such a way to preserve $N = 1$ supersymmetry, this happening if the relative rotation between each couple of D3–brane is a $SU(3)$ rotation. The configuration is depicted in table 1.

	ϕ_1	ϕ_2	ϕ_3
N_0	$\pi/2$	$\pi/2$	$\pi/2$
N_1	$\pi/2$	π	0
N_2	0	$\pi/2$	π
N_3	$\pi+\theta$	$-\theta$	$\pi/2$

Table 1: The position of the D3–branes on the compactifying torus; ϕ_i $(i = 1, 2, 3)$ is the angle on the (x^{2i+2}, x^{2i+3}) torus and θ is a generic non–trivial angle.

Let us just notice that, due to the non–trivial angle θ, the fourth set of D3–branes, N_3, induces charges along the 3–cycles (469), (579), (479) and (569). The corresponding (T–dual) type IIA system consists of a set of D0–branes and three sets of D4–branes along the four–cycles (6789), (4589) and (4567) . In addition, there is a magnetic flux switched on the world volume of the latter (i.e. along (4567)) which is proportional to a rational number $\gamma = p/q$, where the integers p, q are related to the angle θ characterizing the type IIB configuration by the condition: $q \sin\theta = p \cos\theta$. This flux induces an effective D0 charge and effective D2 charges along the two–cycles (45) and (67)[3]. Notice that the presence of this flux is also consistent with the fact that the axions in the type IIA embedding are interpreted as coming from the B_{MN} tensor. Indeed, this tensor couples to the $D4$–brane world volume trough a gauge invariant combination with the flux density: $\mathcal{F} = 2\pi\alpha' F + \hat{B}$ ($\hat{B}$ being the pull–back of

[3]These effective charges derive from Chern–Simons couplings of the magnetic flux to the one and three–form potentials in the theory on the $D4$–brane world volume.

the B field).

The macroscopic generating solution [3] is expressed in terms of harmonic functions depending on the quantized charges (p^Λ, q_Σ) and hence, via a symplectic transformation, for given asymptotic values of the moduli, on the charges (y^Λ, x_Σ) (see section 2). Therefore, taking into account relations (2.1) and (2.2), one can express the values of the brane charges along the different cycles in terms of the four dimensional physical charges (y^Λ, x_Σ) and achieve a precise matching between the parameters characterizing the microscopic and the macroscopic configurations:

$$N_0\,,\,N_1\,,\,N_2\,,\,N_3\,,\,\theta\;\longleftrightarrow\;x_0\,,\,x_1\,,\,y_1\,,\,y_2\,,\,y_3$$

The precise correspondence is illustrated in table 2.

type IIB	Charge	type IIA	
D3(468)	0	D6	y^0
D3(568)	N_1	D4(6789)	y^1
D3(478)	N_2	D4(4589)	y^2
D3(469)	$N_3 q^2$	D4(4567)	y^3
D3(579)	$N_0 + p^2 N_3$	D0	x_0
D3(479)	$-p\,q\,N_3$	D2(45)	x_1
D3(569)	$p\,q\,N_3$	D2(67)	x_2
D3(578)	0	D2(89)	x_3

Table 2: The correspondence between type IIB and type IIA charges on the different cycles of the compactifying torus.

The important feature of our solution is that its *axionic* nature (i.e. a_i not identically zero) is related to the non vanishing of the charge $x \equiv x_2 = -x_1$. The latter is proportional to the magnetic flux density in the type IIA picture (which is essentially γ) or to $\mathrm{tg}\,\theta$ in the type IIB framework: $x \propto \gamma = \mathrm{tg}\,\theta$. As $x \to 0$ we end up with a *purely dilatonic* four parameter solution described by a system of D0 and D4–branes without magnetic flux ($\gamma \to 0$) or by a system of orthogonal D3–branes as in table 1 with $\theta = 0$.

Upon use of table 2 one may express the quartic invariant of $E_{7(7)}$, J_4 (which is generally expressed in terms of the quantized charges (p^Λ, q_Σ) or equivalently of (y^Λ, x_Σ)) as a function of the microscopic parameters. Hence, remembering that for the entropy the following relation holds, $S = \pi\sqrt{J_4}$, one may express the latter in terms of the microscopic parameters, giving a prediction for its microscopic evaluation, [3] :

$$S_{micro} = 2\,\pi\sqrt{N_1 N_2 N_3\, q^2\left[N_0 + p^2\, N_3 - \frac{1}{4} p^2\, N_3 \frac{(N_1+N_2)^2}{N_1 N_2}\right]} \tag{3.1}$$

This is the predicted expression for the entropy of the generating solution and should clearly be derived via microscopic counting techniques. This important goal has been recently achieved in [12].

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