

Black Hole Radiation On and Off the Brane

Roberto Emparan*

*Departamento de Física Teórica
Universidad del País Vasco, Apdo. 644, E-48080 Bilbao, Spain
E-mail: wtpemgar@lg.ehu.es*

ABSTRACT: After a brief review of the description of black holes on branes, we examine the evaporation of a small black hole on a brane in a world with large extra dimensions. We show that, contrary to previous claims, most of the energy is radiated into the modes on the brane. This raises the possibility of observing Hawking radiation in future high energy colliders if there are large extra dimensions.

1. Introduction

It has been realized that space may have extra compact dimensions as large as a millimeter, in a way that can be consistent with all current observations [2]. To achieve this, the Standard Model fields are required to be confined on a three-brane, and only gravity propagates into the extra dimensions. The validity of the Standard Model up to energies around a TeV requires the thickness of the brane to be less than $1 \text{ TeV}^{-1} \sim 10^{-16} \text{ mm}$. On the other hand, the four-dimensional character of gravity has been tested only down to the centimeter scale. This sets an upper bound on the size L of the extra dimensions¹. Since the effective four-dimensional Newton's constant G_4 is related to its d -dimensional (fundamental) counterpart G_d by $G_4 = G_d/L^{d-4}$, if the fundamental scale of gravity in the bulk is of order a TeV, and we take $d = 6$, then G_4 has the observed value provided $L \sim 1 \text{ mm}$, consistent with the bound mentioned above, and falsifiable in the near future. This value of L can be lowered, while keeping a fundamental TeV energy scale, by taking higher values for d , which leads to smaller extra dimensions. In what follows, our results will hold for any number of large

extra dimensions.

For an observer that lives on the brane (*e.g.*, ourselves), the main effect of these large extra dimensions is to introduce a number of light (and heavy) fields, which come from the decomposition of the bulk metric into the four-dimensional graviton and an infinite tower of Kaluza-Klein modes. The latter act like four-dimensional spin-two fields with masses that, for $d = 6$, start at as low as $1/L \sim 10^{-4} \text{ eV}$. So, at a given energy scale $E < \text{TeV}$, the light fields on the brane consist of the Standard Model fields, a four dimensional graviton (the zero mode in the Kaluza-Klein decomposition of the d -dimensional graviton), and a large number, of order $(EL)^{d-4}$, of light Kaluza-Klein modes. The latter, however, couple very weakly, with four-dimensional gravitational strength, to the matter fields that are confined on the brane.

The existence of a low fundamental Planck scale implies that the strength of gravity becomes comparable to other interactions at around the TeV scale. One of the most striking consequences of this is the possibility of forming semiclassical black holes at rather low energies, say of order 100 TeV. Suppose one collapses matter (or collides particles) on the brane to form a black hole of size $\ell_{\text{fun}} \ll r_0 \ll L$ (where $\ell_{\text{fun}} = G_d^{1/(d-2)}$ is the fundamental, *i.e.*, d -dimensional, Planck length). This black hole has a temperature $T \sim 1/r_0$ which is much larger than the mass of the light Kaluza-Klein modes. Since gravity couples

*Based on work in collaboration with G.T. Horowitz and R.C. Myers [1].

¹This is in the simplest models, where the extra dimensions are taken to be flat. In models such as the one of Randall and Sundrum [3], the bound is on the curvature of the extra dimensions.

to everything, and there are so many Kaluza-Klein modes with mass less than the Hawking temperature, it has been claimed [4, 5] that the Hawking radiation will be dominated by these Kaluza-Klein modes, with only a tiny fraction of the energy going into standard model particles. In other words, most of the energy would be radiated off of the brane into the bulk. Since the Kaluza-Klein modes couple so weakly to matter, they would escape our detectors and therefore, if this argument were correct, the Hawking radiation from these small black holes would be essentially unobservable.

However, we have proven, in work with Gary Horowitz and Rob Myers [1], that this argument is incorrect, and most of the Hawking radiation goes into the Standard Model fields on the brane. While the detection of this Hawking radiation would likely not be the first experimental signature of large extra dimensions, such measurements would provide a dramatic new window on black hole microphysics.

2. Black Holes on Branes

We start by reviewing the description of black holes in a brane-world with extra dimensions of size L . We are far from having any exact, analytic description that accounts for all the effects involved. In fact, such a detailed description would be strongly model-dependent, but if we make several approximations we will be able to obtain generic results in the regimes of most interest. First, we will take the brane to have negligible thickness. This is indeed reasonable, since the actual thickness of the brane is likely to be of order the fundamental scale ℓ_{fun} , and a black hole will behave semi-classically only if its size is $r_0 \gg \ell_{\text{fun}}$. Therefore, we will work in a sort of low energy effective approach where we do not probe the details of the structure of the brane. The latter is simply represented as a four-dimensional Lorentzian hypersurface in the full spacetime, and, in principle, acts as a distributional source for gravity.

Even in this approximation, it has not been possible to find a full analytic description of a black hole on a brane in any realistic model, although it has been possible to work out in de-

tail a useful toy model in low dimensions [6, 7]. In general, the self-gravity of the brane introduces severe complications, and makes the analysis strongly dependent on the number of extra dimensions. When describing large black holes (relative to the compactification scale; see below) this difficulty can be easily overcome, but the representation of small black holes proves still too hard in general. Small black holes are in fact much more effective at probing the extra dimensions, and will be the main focus here. If we take the horizon size to be sufficiently smaller than the length scale of the curvature induced by the brane, then it will be reasonable to neglect brane self-gravity. Also, if the black hole size is much smaller than the compactification length, then, close to the black hole, finite-size effects will be negligible, while at distances much larger than L we will be able to integrate over the internal space. In this way our results will be largely independent of the precise compactification scheme. Nevertheless, for definiteness and simplicity, we will be mainly considering the extra dimensions to be wrapped on a square $(d-4)$ -dimensional torus.

Let us then consider a general dimension d for the bulk spacetime, and assume that we live on a $(3+1)$ -dimensional brane. A black hole horizon on a three-brane arises from the intersection of a higher dimensional horizon with the brane worldvolume. This horizon can originate from two different sorts of higher dimensional objects: one is the “black brane,” (in fact, a $(d-4)$ -brane) obtained by taking the product of the four dimensional Schwarzschild solution and the $(d-4)$ -dimensional internal space \mathcal{M}_{d-4} ,

$$ds^2 = ds_{\text{Schwarzschild}_4}^2 + ds_{\mathcal{M}_{d-4}}^2. \quad (2.1)$$

An observer on the brane (which is localized at a certain point in the internal space) perceives exactly the four-dimensional Schwarzschild solution, without any corrections arising from the existence of extra dimensions. In other words, no Kaluza-Klein modes are excited in this solution, only the massless zero mode of the bulk graviton that yields four dimensional gravity on the brane.

On the other hand, one can also envisage a different configuration that results in a horizon

on the brane, namely, a d -dimensional black hole intersected by the three-brane. This is a localized black hole, as opposed to the black brane which is delocalized in the extra dimensions. Now, we are taking the extra dimensions to be compactified with characteristic length L . In the simplest case of toroidal compactification this is equivalent to regarding space in the directions transverse to the brane as being periodic with period L . While this periodicity can be readily imposed on the black brane solution above, a localized black hole will instead require for its description a $(d - 4)$ -dimensional array of d -dimensional black holes. No such exact periodic solutions are known in dimensions $d > 4$. Nevertheless, if we restrict to black holes of size r_0 much less than L , then the geometry *near the black hole* will be very well approximated by a d -dimensional Schwarzschild solution,

$$ds^2 = -f(r) dt^2 + f^{-1}(r) dr^2 + r^2 d\Omega_{d-2}^2 \quad (2.2)$$

with $f(r) = 1 - (r_0/r)^{d-3}$. The event horizon is at $r = r_0$, and has area $A_d = r_0^{d-2} \Omega_{d-2}$ where Ω_n denotes the volume of a unit n -sphere. Since we are neglecting its self-gravity, the brane is to be identified as a surface of vanishing extrinsic curvature, which, by symmetry, cuts through the equator of the black hole. Then the induced metric on the brane will be

$$ds^2 = -f(r) dt^2 + f^{-1}(r) dr^2 + r^2 d\Omega_2^2. \quad (2.3)$$

On the brane then, the event horizon is again at $r = r_0$, and its area is $A_4 = 4\pi r_0^2$. This induced metric on the brane is certainly not the four-dimensional Schwarzschild geometry. Indeed, one can think of it as a black hole with matter fields (*i.e.*, Kaluza-Klein modes) around it. However, the calculation of Hawking evaporation relies mainly on properties of the horizon, such as its surface gravity (*i.e.*, temperature). Since the Hawking temperature is constant over the horizon, it is the same for both the black hole in the bulk and on the brane, and is given by $T = (d - 3)/(4\pi r_0)$.

In the metric (2.3) the g_{tt} component has no $1/r$ term and hence seems to give zero mass in four dimensions. However, this metric only describes the geometry near the black hole. In order to consider the metric at distances large from

the black hole we have to take into account the effects of compactification, *i.e.*, of the full array of black holes, each of mass M and separated by a distance L . From a large distance², the periodic array looks like a “surface density” $\rho = M/L^{d-4}$, where M is the mass of the d -dimensional black hole. Thus, asymptotically the metric will be of the form (2.3), but now with $f(r) = 1 - (2G_d \rho/r)$. However, since $G_d = G_4 L^{d-4}$, this is equivalent to $f(r) = 1 - (2G_4 M/r)$. So, for $r \gg L$ the geometry will be approximated by (2.3) with $f(r) \simeq 1 - (2G_4 M/r)$, and the mass measured on the brane is the same as the mass in the bulk.

So we have two objects, namely, the extended brane and the localized black hole, that can describe a black hole of size r_0 on the brane³. Which of the two is the preferred configuration for a collapsed object? The answer can be easily determined on the basis of entropy arguments, which are furthermore supported by a study of the classical stability of the black brane [10]. It turns out that, for r_0 greater than the compactification size L , the black brane has larger entropy and thus dominates. On the other hand, for r_0 roughly smaller than L , an instability of the black brane sets in so that the localized, higher dimensional black hole becomes the stable (and more entropic) configuration. The transition between these two regimes is very poorly understood, and any progress in the construction of periodic arrays of black holes in dimensions larger than four would be of great help.

Large black holes, therefore, give us virtually no clues as to the presence of extra dimensions. It is by looking at small black holes that we can expect to probe the physics of large extra dimensions. We now turn to study their evaporation through Hawking radiation.

3. Radiation On and Off the Brane

Since we are interested in black holes of radius

²Here, we ignore the gravitational interaction energy of the black holes in the array, which is justified for $r_0 \ll L$.

³However, in the Randall-Sundrum model with a non-compact dimension the extended solution—a black string—is unphysical [8], and both large and small black holes are localized on the brane. The above description of small black holes is still essentially valid, but large black holes are rather like pancakes on the brane [6, 9].

much less than the size of the internal space, we can treat, as far as the radiation process is concerned, the extra dimensions as non-compact. Then, for a single massless bulk field, the rate at which energy is radiated is of order

$$\frac{dE}{dt} \sim A_d T^d \sim \frac{r_0^{d-2}}{r_0^d} \sim \frac{1}{r_0^2} \quad (3.1)$$

where A_d denotes the area of the higher dimensional black hole. For a single massless four-dimensional field on the brane, the rate of energy loss is of order

$$\frac{dE}{dt} \sim A_4 T^4 \sim \frac{r_0^2}{r_0^4} \sim \frac{1}{r_0^2} \quad (3.2)$$

and hence is the same. That is, with a single relevant scale r_0 determining the Hawking radiation, bulk and brane fields must both have $dE/dt \sim r_0^{-2}$. Hence the Hawking evaporation must emit comparable amounts of energy into the bulk and brane. However, with the typical assumption that there are many more fields on the brane than in the bulk, one would conclude that most of the energy goes into the observable four-dimensional fields.

Notice that it would be incorrect to think of brane fields as bulk fields confined to a limited phase space. The brane fields are intrinsically four-dimensional, and their emission is governed by the four-dimensional relation (3.2), and *not* the d -dimensional formula (3.1) with a restricted area.

Another important point worth stressing is that even if there are a large number (of order $(L/r_0)^{d-4}$) of light Kaluza-Klein modes with masses below the scale of the Hawking temperature, they do *not* dominate the evaporation. The pitfall here is to think of the individual Kaluza-Klein modes of the bulk graviton as massive spin two fields on the brane with standard (minimal) gravitational couplings. Rather, since the Kaluza-Klein modes are excitations in the full transverse space, their overlap with the small (d -dimensional) black holes is suppressed by the geometric factor $(r_0/L)^{d-4}$ relative to the brane fields. Hence this geometric suppression precisely compensates for the enormous number of modes, and the total contribution of all Kaluza-Klein modes is only the same order as that from a single brane field.

In order to see in more detail how this geometric suppression factor appears, it is instructive to look into the calculation of the emission rate of a massless bulk field from the four dimensional perspective, *i.e.*, by decomposing it into Kaluza-Klein modes. Thus, let us separate the modes of the bulk field according to the momentum \mathbf{k} which they carry into the $d-4$ transverse dimensions. On the brane, this Kaluza-Klein momentum is identified with the *four-dimensional* mass of these modes, which we denote $m = |\mathbf{k}|$. If we then sum over all other quantum numbers, we will find the emission rate corresponding to a Kaluza-Klein mode with momentum \mathbf{k} . In this way, we get, for the emission rate per unit frequency interval, of modes with momenta in the interval $(\mathbf{k}, \mathbf{k} + d\mathbf{k})$,

$$\frac{dE}{d\omega dt}(\omega, \mathbf{k}) \simeq (\omega^2 - m^2) \frac{\omega A_d}{e^{\beta\omega} - 1} d^{d-4}k. \quad (3.3)$$

Here, A_d is the area of the black hole in the d -dimensional bulk. We are neglecting purely numerical factors since we have found that they do not play any significant role.

Consider a light Kaluza-Klein mode, with a mass much smaller than the black hole temperature, $m \ll 1/r_0$. We set $d^{d-4}k \sim (1/L)^{d-4}$ for an individual mode, and $A_d \sim r_0^{d-4} A_b$, with A_b the sectional area on the brane. Then,

$$\frac{dE}{d\omega dt}(\omega, m) \simeq \left(\frac{r_0}{L}\right)^{d-4} (\omega^2 - m^2) \frac{\omega A_b}{e^{\beta\omega} - 1}. \quad (3.4)$$

which is identical to the emission rate of a massive field in four dimensions, except for a suppression factor of $(r_0/L)^{d-4}$. (Note that this formula applies equally well for $m = 0$.) So we see that the Hawking radiation into each Kaluza-Klein mode (among these, the massless graviton) is much smaller, by a factor of $(r_0/L)^{d-4}$, than the radiation into any other minimally coupled field that propagates only in four dimensions. Still the total radiation into a bulk field is comparable to that into a field on the brane, because there are of order $(L/r_0)^{d-4}$ light modes with $m < T \sim 1/r_0$. So if we integrate the emission rate over all Kaluza-Klein modes, and then over frequency, we recover eq. (3.1), as it must be for consistency.

Since the number of relevant fields on the brane may be only a factor of ten or so larger than the number of bulk fields, one might worry that the claim that the Hawking radiation is dominated by brane fields could still be thwarted by large numerical factors coming from the higher dimensional calculation. To check this, we have considered two improvements over the rough estimate of the radiation rates given in (3.1) and (3.2). The first is to include the dimension dependent Stefan-Boltzman constant that appears in the black body radiation formula. A second improvement concerns the effective area of the radiating black body, which is slightly larger than the horizon area. We have found there are no unexpected large factors to ruin the naive estimate that a Hawking evaporation emits as much energy into a typical brane field as into a typical bulk field. A definitive comparison of the bulk and brane radiation rates would require a detailed analysis, with a specific brane-world model to determine the exact black hole geometry and the precise multiplicity of bulk and brane fields.

So far we have considered small black holes with $r_0 < L$. Larger black holes (which are described by (2.1)) also radiate mainly on the brane. The essential feature now is that the Hawking temperature is lower than the mass of all Kaluza-Klein modes, so their contribution to the Hawking radiation is clearly suppressed. Also, the massless mode of the bulk field radiates in this regime identically to a brane field. So for large black holes, a bulk field still carries essentially the same energy as a field on the brane, but the latter again dominate the Hawking radiation since they are more numerous.

We have also shown in [6] that black holes in the Randall-Sundrum scenario [3] with an infinite extra dimension still radiate mainly on the brane.

4. The Evaporation Process

We are thus led to the following picture for the evaporation of a black hole that initially has a radius $r_0 > L$. At the beginning, as a result of Hawking evaporation, it will decrease its size, at a rate determined by four dimensional physics. When $r_0 \sim L$, the solution (2.1) becomes unstable [10], and is believed to break up into d -

dimensional black holes which coalesce and form a single higher dimensional black hole. It can be shown [1] that this final black hole will lie in the brane, and not in the bulk, since it feels a restoring force due to the brane tension. From this point on, the evaporation of the small black hole starts to differ from the way predicted by the four dimensional law. The radiation rate, as we have seen, varies smoothly across the transition between regimes, and the evaporation slows down. This is a consequence of the fact that, in this regime, the black brane has given way to another configuration, the localized black hole, that, for a given mass, has higher entropy. Although the area is larger, the temperature is lower, and the evaporation rate is slower. As a consequence, the lifetime of a small black hole is longer (possibly enormously longer) by a factor $(L/r_0)^{2(d-4)}$ than would have been expected from four-dimensional Einstein gravity. Finally, notice that the black hole can be described semiclassically down to a mass scale (say, of order 100 TeV) much smaller than the four dimensional Planck mass of 10^{19} GeV [5].

So we conclude that the brane-world scenario has the potential to make interesting observable predictions about small black holes appearing either in collider experiments or in the early universe. Much of their detailed phenomenology is still to be investigated.

Acknowledgments

I would like to thank the organizers, and in particular Bernard Julia, for the invitation to present this work at this TMR conference. I would also like to thank Gary Horowitz and Rob Myers, with whom the results reported here were obtained. Work supported by UPV grant 063.310-EB187/98 and CICYT AEN99-0315. Attendance to the conference was partially supported by TMR program under EC contract number FMRX-CT96-0012.

References

- [1] R. Emparan, G. T. Horowitz and R. C. Myers, Phys. Rev. Lett. **85** (2000) 499 [hep-th/0003118].

- [2] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. **B429** (1998) 263; Phys. Rev. **D59** (1999) 086004; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. **B436** (1998) 257.
- [3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690; Phys. Rev. Lett. **83** (1999) 3370.
- [4] P. Argyres, S. Dimopoulos, and J. March-Russell, Phys. Lett. **B441** (1998) 96.
- [5] T. Banks and W. Fischler, hep-th/9906038.
- [6] R. Emparan, G.T. Horowitz and R.C. Myers, J. High Energy Phys. **1** (2000) 7.
- [7] R. Emparan, G. T. Horowitz and R. C. Myers, J. High Energy Phys. **0001** (2000) 021.
- [8] A. Chamblin, S. W. Hawking and H. S. Reall, Phys. Rev. **D61** (2000) 065007.
- [9] S. B. Giddings, E. Katz and L. Randall, J. High Energy Phys. **0003** (2000) 023
- [10] R. Gregory and R. Laflamme, Phys. Rev. Lett. **70** (1993) 2837; Nucl. Phys. **B428** (1994) 399.