# Comments on String (Non) Decoupling in Noncommutative Theories *

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ABSTRACT: We comment on subtle decoupling properties of the string theory regularization of noncommutative gauge theories. We focus on two examples: the identification of closed-string winding modes in thermal noncommutative field theories, and the non-decoupling of string oscillators in time-like noncommutative strings.

KEYWORDS: noncommutative geometry, strings.

## 1. Introduction

The relation between noncommutative field theory (NCFT) and string theory proposed in [1, 2] and extended in various works [3] has proven a very fruitful source of insight for both string theory and noncommutative field theory. The basic issue is that D-brane dynamics in the presence of a Neveu–Schwarz $B$-field introduces a noncommutativity in the spacetime coordinates [4]. The so-called Seiberg–Witten (SW) limit [5] isolates this effect by specifying a low-energy $\alpha' \to 0$ limit of the open-string dynamics given by the sigma-model action

$$S = \frac{1}{4\pi\alpha'} \int g_{jk} dx^j \wedge *dx^k + \frac{i}{4\pi} \int B_{jk} dx^j \wedge dx^k, \tag{1}$$

in such a way that the sigma-model metric is negligible compared to the nonvanishing $B$-field, *i.e.* $|g_{ij}| \ll \alpha' |B_{ij}|$. Thus, in those directions the world-sheet action is dominated by the topological term

$$\int_\Sigma B_{ij}\, dx^i \wedge dx^j = \oint_{\partial\Sigma} B_{ij}\, x^i\, \partial_t\, x^j, \tag{2}$$

where $\partial_t$ denotes derivative with respect to world-sheet time. The momentum conjugate to $x^i$ is $\pi_j = B_{ij} x^i$ and canonical quantization $[\pi_j, x^k] = -i\delta_j^k$ implies (3) in the form

$$[x^j, x^k] = i \left(\frac{1}{B}\right)^{jk} \equiv i\, \theta^{jk}. \tag{3}$$

Finally, combining the resulting conjugate indeterminacies $\Delta x^j\, \Delta x^k \sim |\theta^{jk}|$ with Heisenberg's $\Delta x^j\, \Delta p_k \sim \delta_k^j$, we obtain $\Delta x^j \sim \theta^{jk}\, \Delta p_k$, as if the noncommutative particles of momentum $p$ were associated to a rigid rod of length

$$L^j \sim p_k\, \theta^{jk}. \tag{4}$$

This is the famous UV/IR correspondence of NCFT. In this context, it means that NCFT retains some degree of the stringy nonlocality.

More precisely, the SW limit takes $\alpha' \to 0$ with

$$g_{ij} \sim (\alpha')^2, \qquad B_{ij} \sim \text{fixed}, \tag{5}$$

including a scaling of the string coupling constant (we set $b \equiv 2\pi\alpha' B$ throughout), in such a way that the effective open-string coupling $G_s$:

$$G_s^2 = g_s^2 \left(\frac{-\det(g+b)}{-\det(g)}\right). \tag{6}$$

remains fixed. This scaling ensures that the effective metric for open-string dynamics $G_{ij}$, as well as the noncommutativity parameters $\theta^{ij}$ remain fixed in the low-energy limit:

$$G^{ij} + \frac{\theta^{ij}}{2\pi\alpha'} = \left(\frac{1}{g+b}\right)^{ij}, \tag{7}$$

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so that the effective dynamics of the massless gauge fields on the brane is described by the noncommutative action

$$S_{\rm NCYM} = \frac{1}{4g_{\rm YM}^2} \int {\rm Tr}\, \sqrt{-\det(G)} G^{ij} G^{kl} \widehat{F}_{ik} \star \widehat{F}_{jl}, \tag{8}$$

where $\star$ is the Moyal product compatible with (3) and $\widehat{F}_{\mu\nu} = \partial_\mu \widehat{A}_\nu - \partial_\nu \widehat{A}_\mu + i\, \widehat{A}_\mu \star \widehat{A}_\nu - i\, \widehat{A}_\nu \star \widehat{A}_\mu$ is the corresponding noncommutative field strength.

From the physical point of view the NC low-energy limit decouples the open-string oscillators as $\alpha' \to 0$. However, the stringy nature of the fundamental degrees of freedom survives to some extent as particles behave as 'rigid dipoles' [6]. Namely the large $B$-field polarizes the open strings and this feature is crucial in the physical properties of the theory described by (8).

In this lecture we review two instances in which the decoupling of string oscillators involves some subtlety.

## 2. No Decoupling for Time-like Noncommutativity

This section is based on work with E. Rabinovici in [7].

The previous discussion of the SW limit assumes implicitly that the $B$-field (hence the noncommutativity) is purely space-like. For a $B$-field with timelike components, letting the $B$-field dominate over the sigma-model metric $|g| \ll |b|$ will induce zero eigenvalues of the matrix $g+b$, rendering some of the previous formulas meaningless. In particular, the mapping from sigma-model parameters $(g, b, g_s)$ to open-string parameters $(G, \theta, G_s)$ is singular whenever

$$\det(g+b) = 0. \tag{1}$$

We shall call these the G-singularities. Conversely, the inverse mapping from open-string to closed-string variables is given for example by

$$g + b = \frac{1}{G^{-1} + \frac{\theta}{2\pi\alpha'}} = \frac{1}{1 + \frac{G\theta}{2\pi\alpha'}}\, G, \tag{2}$$

and thus it is singular at the locus

$$\det\left(1 + \frac{G\theta}{2\pi\alpha'}\right) = 0. \tag{3}$$

These are denoted $g$-singularities. Scaling arguments based on [8, 9] show that perturbative physics at the two kinds of singularities is slightly different.

The $G$-singularities, with smooth sigma-model metric, have been studied in the past from various points of view [10, 11]. In a $T$-dual picture [12], they correspond to the limiting speed of light of a dual D-brane. The salient properties of $G$-singularities are a vanishing effective coupling $G_s$, so that the effective theory of open strings becomes classical. On the other hand, the effective tension of the D-brane vanishes, so that nonlinear effects are out of control. From the point of view of defining a NCFT with time-like $\theta$, these singularities are not very interesting, since the effective metric $G_{ij}$ that should appear in (8) is singular.

On the other hand, at $g$-singularities we have a blowing-up $g, b$ with fixed $G, \theta$. This is a more interesting case characterized by a divergent brane tension

$$T_{\rm eff} \sim \sqrt{-\det(g+b)} = \sqrt{\frac{-\det(G)}{-\det\left(1 + \frac{G\theta}{2\pi\alpha'}\right)}} \to \infty. \tag{4}$$

Open-string loops are effectively weighted by

$$G_s = g_s \sqrt{-\det\left(1 + \frac{G\theta}{2\pi\alpha'}\right)} \to 0, \tag{5}$$

and nonlinear effects are weighted by $\lambda_{\rm eff} \sim G_s^{1/2} \to 0$. Thus, we have a free classical theory at the $g$-singular points, with fixed $g_s$ and $|G\theta| = 2\pi\alpha'$. We may define an interacting theory by scaling $g_s \to \infty$, so that $G_s$ remains fixed and nonzero. The resulting analysis must be done in the $S$-dual frame of the underlying closed-string theory. This is the celebrated NCOS theory introduced in [13] and much studied in recent works. Its most important features are the correlation between the scales of noncommutativity, $|\theta|$, and the stringy fuzziness, $\alpha'$, and a decoupling from closed-string modes off the brane, hence defining a noncommutative open-string theory without gravity.

For our purposes today, we want to emphasize the fact that, keeping a non-singular set of open-string data $(G, \theta, G_s)$, and requiring the time-like noncommutativity to be encoded in Moyal products at the level of the effective action, means that we cannot decouple $|\theta|$ and $\alpha'$, while at the same time keeping control over perturbation theory. The best we can do is to set the maximum noncommutativity to a value $|\theta_{\rm max}| \sim 2\pi\alpha'$ [14, 13, 7]. We conjecture that this is probably the way in which string theory protects itself from pathologies such as the lack of unitarity and acausality that seem to plague time-like noncommutative *field* theories [14, 7, 15].

For example, insisting in the formal SW limit takes us into the $\det(g + b) > 0$ region, with imaginary coupling $G_s$. Performing a formal analytic continuation over $g_s$ and $g_{ij}$ allows us to define the NCFT perturbation theory with time-like Moyal products. From here we may deduce the AdS/CFT large-$N$ master field of the theory along the lines of [16, 17], which turns out to have a naked singularity at the physical scale of noncommutativity. Hence all evidence based on diagrammatics (perturbative and large-$N$) points to the lack of unitarity of such theories [7, 15].

On the other hand, an analysis of very specific nonperturbative effects in NCQED shows no pathologies provided the right space-time frame is used to parametrize the physics [18]. For example, the pair-production rate takes the Schwinger form [19], with the simple replacement of $F_{\mu\nu}$ by its noncommutative counterpart $\widehat{F}_{\mu\nu}$. In addition, a formal hamiltonian formalism was proposed for NCFT with time-like noncommutativity [20], and no pathologies were found in a study of the superficially similar case of light-like noncommutativity [21]. Thus, it will be interesting to determine to what extent full nonperturbative physics can overcome the pathologies inherent to time-like noncommutativity.

## 3. Non-Decoupling of Closed-string Winding Modes?

This section is based on work with G. Arcioni, J. Gomis and M.A. vázquez-Mozo in [22].

It was noticed in [24] that the UV/IR connection has radical consequences for the decoupling of noncommutativity effects at low energies. Namely in NCFT perturbation theory, the presence of the Moyal phases in Feynman diagrams renders many of them finite in the UV, since $\sqrt{|\theta|}$ and $|\theta p_{\rm ext}|$ for some external momentum $p_{\rm ext}$ act as effective UV cutoffs [23]. This means that, on renormalizing the theory for nonvanishing $\theta$, we subtract less UV divergences than at $\theta = 0$. As a result, the limit $(\Lambda_{\rm eff})^{-1} \sim |\theta p_{\rm ext}| \to 0$ is singular in general. This singularity is identifiable as a non-analiticity in $\theta$ at finite $p$, or as an IR singularity at $p \to 0$ and finite $\theta$. Since these IR singularities originate in the ultraviolet fluctuations of the noncommutative field, they are largely independent of the mass of the noncommutative particles. It must be stressed that these effects are independent of the special features of time-like noncommutativity, discussed in the previous section. We consider purely space-like $\theta$ throughout this section.

Since the UV/IR effects violate naive decoupling, in trying to maintain the standard Wilsonian language, the authors of [24, 25] introduced extra degrees of freedom $\psi$ with effective propagators of the form

$$\langle \psi(p)\, \psi(-p) \rangle = \frac{1}{(\theta p)^2 + {\bf z}_\perp^2}, \qquad (1)$$

where $(\theta p)^\mu = \theta^{\mu\nu} p_\nu$ and ${\bf z}_\perp$ represent $d_\perp$ 'transverse' momentum variables, making $\psi$ propagate in a $(d + d_\perp)$-dimensional bulk. The tree exchange of the $\psi$-fields mimics the non-standard infrared singularities caused by the UV quanta of the original fields.

The interpretation of these poles as light closed-string modes in a double-twist diagram is very suggestive (for the one-loop case), given the formal resemblance of (1) with a closed-string propagator in the bulk. Indeed, the particular combination $(\theta p)^2$ is nothing but the SW limit of $p_\mu g^{\mu\nu} p_\nu$, up to powers of $\alpha'$. Since $g_{\mu\nu}$ is the metric felt by the closed strings, the interpretation is rather natural.

Still, it is very odd to find a kinematical situation where both dual channels (open and closed) are saturated by massless poles. Normally, whenever the open-string channel is dominated by massless exchange (Yang–Mills), the closed-string description *of the same diagram* involves all the tower of closed-string oscillator states. Conversely, whenever supergravity is a good approximation, the open-string picture is not simple. A partial counterexample may be found in the case of the AdS/CFT duality, although the 't Hooft coupling $g_{\rm YM}^2 N = g_s N$ plays there the role of control parameter separating the ranges of aplication of both descriptions.

In particular, the SW limit of the double-twist diagrams in question has been studied recently in great detail [26] with the result that the whole NCFT perturbation theory comes from the region of string moduli space where open-string massless modes dominate.

Nevertheless, a clear smoking gun for closed-string modes would be the identification of topologically nontrivial configurations such as winding modes. Interestingly, evidence for thermal winding modes was recently reported in [27]. These authors found that the two-loop free energy of various NCFTs could be written in terms of modes effectively living at the '$T$-dual' temperature $1/\theta T$. More generally, the basic phenomenon can be understood directly at the level of the nonplanar one-loop self-energy. Performing the momentum integral and a Poisson resummation in the thermal frequencies, we can write:

$$\Pi_{\rm NP}(p) = -\frac{g^2}{\beta}\sum_n \int d\mathbf{q}\, \frac{e^{i\theta(p,q)}}{\frac{4\pi^2 n^2}{\beta^2} + \mathbf{q}^2 + M^2} = -\sum_\ell \int d\mathbf{z}_\perp \frac{|g_{\rm eff}|^2}{\beta^2\ell^2 + (\theta\mathbf{p})^2 + \mathbf{z}_\perp^2}, \quad (2)$$

where we have introduced extra $4-d$ 'gaussian momenta' $\mathbf{z}_\perp$ and an effective coupling

$$|g_{\rm eff}(\ell,\mathbf{p},\mathbf{z}_\perp)|^2 = \frac{g^2}{4\pi^2}\int_0^\infty ds\, e^{-s-\frac{M^2}{4s}[\beta^2\ell^2+(\theta\mathbf{p})^2+\mathbf{z}_\perp^2]}. \quad (3)$$

Expression (2) has the form of a 'dual channel' representation [22], where the nonplanar loop is replaced by the tree-level exchange of an infinite tower of 'resonances' $\chi_\ell$ of mass $M_\ell \sim |\beta\ell|$, propagating in $4-d$ extra dimensions, with an effective kinetic lagrangian of the form

$$\mathcal{L}_{\rm kin} \sim \chi_\ell \left[-\partial_\perp^2 - (\theta\partial)^2\right] \chi_\ell. \quad (4)$$

These fields generalize the $\psi$-fields introduced in [24, 25]. In particular $\psi_{\rm MRS} \approx \chi_{\ell=0}$, except that the infinite tower replaces the whole nonplanar loop rather than just the UV part.

This 'channel duality' representation can be generalized to arbitrary loops [22]. In the general case, however, the effective Feynman rules for the fields $\chi_\ell$ are nonlocal. In particular, the Feynman and Schwinger parameters of the original diagram are not factorizable in the new vertices, and one ends up with a 'crossed channel duality' operating at the level of the *integrand* over the moduli space of Feynman and Schwinger parameters. This feature is also reminiscent of the way world-sheet duality works in string theory (locally in the moduli space of Riemann surfaces).

However, as noted before, there is a puzzle in any tentative identification of the $\chi_\ell$ fields with closed-string modes, since we do not expect the closed-string exchange to be saturated by the massless fields in the low-energy spectrum. For the simple example of the nonplanar two-point function, we would like to have a relation of the form:

$$\lim_{\rm SW} \langle {\rm D}p, V_p|(\Delta_{cl})^{-1}|{\rm D}p, V_p\rangle = \sum_\ell \int_{\mathbf{z}_\perp} \frac{|g_{\rm eff}|^2}{\beta^2\ell^2 + (\theta\mathbf{p})^2 + \mathbf{z}_\perp^2}, \quad (5)$$

for the full closed-string channel expression. The inverse closed-string propagator is

$$\Delta_{cl} = \frac{\alpha'}{2}\left(g^{\mu\nu}p_\mu p_\nu + M_{cl}^2\right) = \frac{\alpha'}{2}\left(g^{\mu\nu}p_\mu p_\nu + \frac{\beta^2\ell^2}{4\pi^2\alpha'^2} + M_{\rm osc}^2\right), \quad (6)$$

where $M_{\rm osc}^2 \sim N_{\rm osc}/\alpha'$ and the term proportional to $\ell^2$ gives the mass of a thermal winding mode. In the SW limit with $G_{\mu\nu} = \delta_{\mu\nu}$ we have $g^{\mu\nu} \to -\frac{1}{(2\pi\alpha')^2}(\theta^2)^{\mu\nu}$ and we get the following scaling of $\Delta_{cl}$:

$$\frac{1}{8\pi^2\alpha'}\left[\beta^2\ell^2 + (\theta\mathbf{p})^2 + (2\pi\alpha')^2(\mathbf{p}_\perp^2 + M_{\rm osc}^2)\right]. \quad (7)$$

Thus, we obtain the right scaling if we define $(2\pi\alpha')\mathbf{p}_\perp \equiv \mathbf{z}_\perp$, since $(2\pi\alpha')^2 M_{\rm osc}^2 \sim \alpha' \to 0$, *i.e.* the infinite tower of oscillator states makes a negligible contribution to the effective mass in the SW limit! More precisely, the oscillator states get squeezed into a continuous band, as compared to the gap of the winding modes:

$$\frac{\text{oscillator gap}}{\text{winding gap}} \sim \frac{\alpha'}{\beta^2} \to 0 \quad (8)$$

in the SW limit. This means that, on the scale of the effective $\chi_\ell$ fields, the whole tower of oscillator closed-string states fails to decouple. The best we can hope for is to derive sum rules for $g_{\rm eff}$ in terms of the sum over closed-string oscillator states. Naively

$$|g_{\rm eff}|^2 \sim \alpha' \sum_{\rm osc} \langle {\rm D}p|\Psi_{\rm osc}\rangle \langle \Psi_{\rm osc}|{\rm D}p\rangle\,, \qquad (9)$$

so that the $\chi_\ell$ fields really represent the coherent exchange of the infinite tower of massive string states. In fact, (9) is not precisely correct. As it stands, it is divergent due to the Hagedorn growth of closed-string states. In addition, the identification of the formal transverse space of the $\mathbf{z}_\perp$ variables with the real transverse space to the D-brane is not exactly correct. It is shown in [22] that these deficiencies can be overcome by phrasing the sum rule (9) in terms of the integrand over the moduli space of the Feynman diagram. Then one finds that Feynman parameters map consistently to Koba–Nielsen moduli of D-brane boundary states, whereas Schwinger parameters map to the standard closed-string modular parameters. The resulting sum rule for the effective couplings is convergent despite the presence of open- or closed-string tachyons, and the correct relation between 'formal' and 'real' transverse dimensions depends on the number of external insertions. For an amplitude with $N$ external insertions, the formal number of transverse dimensions $d_\chi^\perp$ is related to the real number of Dirichlet–Dirichlet dimensions, $d_\perp$, of the D-brane by:

$$d_\chi^\perp = d_\perp + 2N + 2 - D, \qquad (10)$$

where $D = 26$ for bosonic D-branes and $D = 10$ for supersymmetric D-branes. We can summarize the basic issues as follows:

- There is a 'dual channel' picture in NCFT with effective 'closed channel' fields $\chi_\ell$ having winding-scaling masses, extra moduli in effective vertices and propagating in extra bulk dimensions.

- This structure descends directly from the corresponding open/closed world-sheet duality in the underlying string theory. In particular modes with effective mass $M_{\chi_\ell} \sim |\beta\ell|$ can be associated to closed-string thermal winding modes in the microscopic string description.

- The literal interpretation of the extra bulk dimensions where the $\chi_\ell$ fields propagate freely as real dimensions transverse to the D-brane is not correct. Rather the relation is rather indirect, and depends on the particular diagram we are looking at.

- Most importantly, the $\chi_\ell$ fields *do not* represent *individual* closed string modes that fail to decouple. In fact, the $\chi_\ell$ fields are a formal device representing the coherent coupling of the infinite tower of closed-string oscillator modes. The fact that they still behave roughly as standard quantum fields is one of the surprises of NCFT.

## 4. Final Remarks

We have discussed two particular instances where the low-energy decoupling of strings in noncommutative systems is somewhat nontrivial. We conclude with a brief summary and a highlight of the differences between the two cases.

In section 2 it is shown that *open*-string oscillators do not decouple at critical electric-field singular points in moduli space. The resulting interacting theory, the so-called NCOS, is a genuine string theory with noncommutativity scale of the order of the string scale. We pointed out that forcing the decoupling at a formal level, by performing analytic continuations of the SW formulas, leads to an inconsistent model. Thus, it seems that time-like NCFT cannot be reached from a consistent string regularization.

In section 3 we consider a purely space-like NCFT and focus on the UV/IR mixing as presented in [24, 25]. Although these theories are decoupled from gravity by construction, we show that some rudiments of the open/closed channel duality of the ultraviolet string theory survive down to low energies in the SW limit. Namely, NCFTs are 'models' of full open-string dynamics. We confirm this by looking at the fate of winding quantum numbers of closed strings at finite temperature. There is no useful way of thinking in terms of 'undecoupled closed strings', but the formalism emulates these in the sense that the quantum number itself does not decouple and is visible in the field theory, by means of our 'channel duality' representation.

These remarks should be distinguished from superficially similar claims in the context of NCOS theories. It was shown in [13] that NCOS *string* theories decouple from closed strings, except in compact volume, where wound closed strings still survive in the NCOS Hilbert space [28]. Although these results are based on the study of the double twist one-loop diagram, just as our analysis in section 3, it must be stressed that both discussions refer to different physical systems in different regimes.

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