Life and death of Schrődinger cats in a Luttinger liquid

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Abstract: Schrődinger cat states built from quantum superpositions of left or right Luttinger fermions located at different positions in a spinless Luttinger liquid are considered. Their decoherence rates are computed within the bosonization approach using as environments the quantum electromagnetic field or two or three dimensional acoustic phonon baths. Emphasis is put on the differences between the electromagnetic and acoustic environments.

Keywords: condensed matter, mesoscopic systems, decoherence.

1. Introduction

Because of the recent advances in microelectronic technology, it is now possible to probe the quantum regime of electron transport in solids. Many exotic manifestations of quantum coherence appear in mesoscopic experimental devices : for example Aharonov-Bohm effects such as permanent currents in a mesoscopic ring and conductance oscillations as a function of the external magnetic field, but also the reduction of shot noise in quantum point contacts [1].

Nevertheless, real systems often involve impurities or interaction with an external environment. In a quantum system, as explain by Zurek [2], interaction of a quantum system with an external environment destroys quantum coherence. Therefore, the following question naturally arises in condensed matter systems : can we use these ideas to understand the crossover between the quantum transport regime where interference effects play a dominant role and the classical transport regime which we are used to? The problem of electron decoherence in metals at zero temperature is indeed an active area of research both from the theoretical and experimental point of view. Recent experiments claim to observe a saturation of the dephasing time τ_ϕ at very low temperatures [3]. Strong discussions among theorists arose from these observations. The heart of the debate, summarized for example in Mohanty's contribution [4], is to determine whether the conventional theory of dephasing in Fermi liquids [5, 6] could explain the saturation of τ_ϕ or whereas one should reconsider the theory completely [7]. Whereas these authors consider weakly disordered conductors in the diffusive regime, we shall present a model for studying *the decay of Schrődinger cat states in 1D ballistic conductors*.

Our line of thought is very close to the one used in atomic physics, for example in works [8] on decoherence experiments in cavity QED [9]. In atomic physics, decoherence is extremely well controlled and simplified models such as the Caldeira-Leggett model [10] apply directly. On the contrary, mesoscopic conductors are complex interacting systems. Nevertheless, 1D ballistic conductors can be described by an effective 2D Conformal Field Theory: the Luttinger Liquid CFT [11]. In this effective theory, interactions between electrons are described by means of an electrostatic short-ranged potential. Obviously, within the approximation of a linear dispersion relation for particle-hole excitations, the Luttinger effective theory contains no source for decoherence. This Luttinger CFT provides a bosonic description of the low energy spectrum of the whole interacting electron fluid. An external quantum environment is necessary to introduce decoherence: we have considered the quantized electromagnetic field and a 2D or 3D bath of longitudinal phonons.

This point of view throws a different light on the question of decoherence by taking into account the electron fluid as a whole and studying the coupling of this many-body system to the external reservoirs. It is well suited to the 1D case because of interactions which break the Fermi liquid picture. The orthogonality catastrophe [12] ruins any attempt based on a quasi-particle approach. Moreover, because of the drastic effects of interactions in 1D fermion systems, electromagnetic or acoustic radiation emitted by one part of the system and absorbed by another part may alter considerably the strongly correlated electron state. These remarks motivated our approach. Following general ideas of Stern, Aharonov and Imry [13], we have computed the decoherence rate of Schrődinger cat states built from localized excitations (*e.g* the so-called Luttinger fermions) introduced at different places in the system or at the same place but moving in different directions (left and right moving components). As expected, we have found that such linear superpositions decay into statistical mixtures even at zero temperatures. This is the main result of this paper and it means that *a 1D pure ballistic conductor exhibits decoherence at absolute zero* in the sense of Schrődinger cat states decay.

More precisely, we have shown that the electromagnetic decoherence time is much larger, although not infinite, than the natural time associated with the Luttinger system. We have also shown that the acoustic decoherence is much stronger than the electromagnetic one. This difference comes from the fact that many bosonic modes of the Luttinger liquid have an efficient acoustic radiation rate in contrast to the electromagnetic case. In the acoustic case, decoherence takes place over a much shorter time scale than dissipation in contrast to the electromagnetic case.

This paper is organized as follows: the model and the Feynman-Vernon and Keldysh basic tools are briefly recalled in section 2. The CFT description of the Luttinger liquid is reviewed in section 3. Our results concerning decoherence in the Luttinger liquid are presented in section 4. Comparison with other work and possible extensions are discussed in the conclusion.

2. Electron systems coupled to external reservoirs (QED or phonons)

Studying an out of equilibrium quantum system consists in computing its density matrix evolution. An approach to this problem has been given long ago in the context of perturbative field theory by Keldysh [14]. For a matter system coupled to an external reservoir, we are interested in the evolution of the reduced density matrix for the matter system. Integrating over the reservoir's degrees of freedom gives a non-local Feynman-Vernon influence functional [15].

It is convenient to choose the Coulomb gauge, in which dynamical degrees of freedom of the quantum electromagnetic field (transverse photons) are decoupled from the Coulomb interaction. The latter is taken into account through the effective Luttinger CFT description for the matter system. Integrating out transverse photons gives the electromagnetic Feynman-Vernon influence functional. It can be evaluated in terms of Keldysh's Green function of the EM field is Gaussian in terms of the current density (for the EM field initially at thermal equilibrium).

Within the elastic approximation, coupling to acoustic phonons can be treated along the same line. They create a potential proportional to the compression rate of the bulk material and their dynamics is described by a quadratic action. The resulting phonon influence functional is a Gaussian in terms of the electric charge density

In a generic electron system, the environment influence functional *a priori* contains quartic fermion terms. Thanks to bosonization and neglecting environment-induced *umklapp* processes, the influence functional is Gaussian (although non local) in terms of the bosonic field underlying the Luttinger CFT. This trick makes the problem exactly solvable. It is worth mentionning here that bosonization techniques also exist in 2D and 3D and could, although they are not exact, be used to analyze these cases.

3. Effective CFT for the Luttinger Liquid

We consider a system of spinless interacting electrons on a $1D$ circle of length L. As was shown by Haldane [11] low energy properties of gapless systems can be described using an effective theory with only two parameters: a renormalized Fermi velocity v_S and a dimensionless interacting constant α. This 1D effective description can conveniently be formulated in terms of a 2D Conformal Field Theory [16] (CFT), the Hilbert state of which can be decomposed according to a $\widehat{U(1)}_R \times \widehat{U(1)}_L$ symmetry algebra. The electric charge and current densities are linearily related to these modes. Therefore, coupling the Luttinger liquid to the electromagnetic of acoustic environment boils down to a set of independent harmonic oscillators, each of them being linearly coupled to a bath of quantum oscillators. This problem is known under the name of *Quantum Brownian Motion* (QBM) [17].

Irreducible highest weight states of the symmetry algebra are associated with vertex operators indexed by $(n,m) \in (\mathbf{Z}/2) \times \mathbf{Z}$ such that $2n \equiv m \pmod 2$. Physically $2n$ is the total charge and $mev_S/L\alpha$ is the total current circling around the system. The $l \neq 0$ modes are called *hydrodynamic* in reference to Wen's pioneering work [18] on edge states in the FQHE. The Luttinger parameter is related to the filling fraction by $\alpha\nu = 1$.

From the 1D point of view, the original fermion operators renormalize (orthogonality catastrophe) to specific vertex operators. For example the left (resp. right) moving renormalized fermion operators $\psi_R^\dagger$ (resp. $\psi_L^\dagger$) corresponds to $V_{1/2,1}$ (resp. $V_{1/2,-1}$). In the case of a FQH fluid, edge fermions carrying unit charge on one of the two edges appear in the spectrum: $V_{1/2,\nu^{-1}}$ and $V_{1/2,-\nu^{-1}}$. In contrast to them, the Luttinger fermions carry a fractional charge on each edge $q_R = (1+\nu)/2$ and $q_L = (1-\nu)/2$ for $V_{1/2,1}$.

Elementary excitations of the Luttinger liquid can be created using these vertex operators. States created this way are nothing but coherent states. Since the evolution of coherent states in QBM can be exactly computed [19], so can the evolution of any superposition of these elementary excitations in the Luttinger liquid.

4. Mutual decoherence of elementary excitations in a Luttinger liquid

4.1 Statement of the problem

The Schrődinger cat states considered here are superpositions of left or right moving moving Luttinger fermions at different places around the circle:

$$|\psi_{RR}(0)\rangle = \frac{1}{\sqrt{2}}\left(\psi_R^\dagger(\sigma_1)|0\rangle + \psi_R^\dagger(\sigma_2)|0\rangle\right) \quad (4.1)$$
$$|\psi_{RL}(0)\rangle = \frac{1}{\sqrt{2}}\left(\psi_R^\dagger(\sigma_1)|0\rangle + \psi_L^\dagger(\sigma_2)|0\rangle\right) \quad (4.2)$$

When coupled to an environment, this Schrődinger cat is expected to decohere into a statistical mixture. Here, two questions will be addressed: what is the strength of the decoherence process and on which time scale does it take place?

Let us recall that in QED's case, the relevant coupling constant is

$$g = 4\pi\,\frac{\alpha_{QED}}{\alpha}.\left(\frac{v_S}{c}\right)^2 \quad (4.3)$$

whre α_{QED} is the fine structure constant, c the speed of light. In the acoustic case

$$g_{ph}(L) = \frac{D^2}{\alpha\rho_M L^{d-1}\,\hbar\, c_S^3} \quad (4.4)$$

where c_S is the sound velocity, ρ_M the volumic mass and D the typical electron/phonon coupling energy.

4.2 Evolution of zero modes

The zero modes evolution at finite temperature contains two features : (1) a dynamical renormalization of the Luttinger liquid's parameters, (2) a decoherence factor.

(1) The velocity and interacting parameters of the Luttinger liquid get renormalized as $v'_S = v_S.\zeta$ and $\alpha' = \alpha/\zeta$. The the renormalization constant ζ depends on time and equal to:

$$\zeta(t)^2 = 1 + \frac{c}{\pi v_S}\int_0^{+\infty} \mathcal{I}_0(\omega)\left(\frac{\sin(\omega t)}{\omega t} - 1\right)\frac{d\omega}{\omega} \quad (4.5)$$

The renormalization effect is of course strongly cut-off dependent through the bath's spectral density for zero modes $\mathcal{I})0(\omega)$. The dimensionless coupling constant appearing in the here is $\frac{\alpha_{QED}}{\alpha}.\frac{v_S}{c}\simeq 10^{-5}$.

(2) The EM decoherence factor between two different highest weight states $|n,m\rangle$ and $|n',m'\rangle$ of the LL is given by $e^{-d(t)(m-m')^2/\alpha}$ where :

$$d(t)=\frac{ct^2}{L}\int_0^{+\infty}d\omega\,\mathcal{I}_0(\omega)\,\coth\left(\frac{\beta\hbar\omega}{2}\right)\frac{1-\cos(\omega t)}{(\omega t)^2}. \tag{4.6}$$

Decoherence takes place in a time of the order of the UV cutoff time and then reaches an asymptotic value:

$$d(+\infty)=\int_0^{+\infty}\frac{c}{L\omega^2}\,\mathcal{I}_0(\omega)\,\coth\left(\frac{\beta\hbar\omega}{2}\right)d\omega \tag{4.7}$$

Since acoustic zero modes couple to the total charge of the Luttinger system, the acoustic decoherence factor between states $|n,m\rangle$ and $|n',m'\rangle$ is found to be $\exp\left(-2\,\alpha\,d(t)(n-n')^2)\right)$ where $d(t)$ is obtained by using the acoustic spectral density and sound velocity c_S instead of their electromagnetic counterparts. The Luttinger parameters α and v_S also get renormalized but only $v_S\alpha$ is renormalized.

In both cases, the final decoherence exponent is proportional to the square of the difference between the total current (*resp.* charge), quantities which measure the "distance" between the two quantum states. Besides this, the zero mode decoherence exponent is of order of the coupling constant to the environment. Let us notice that a Schrődinger cat obtained by superposing the *same* excitation of the Luttinger liquid at two different positions along the ring has all its decoherence due to hydrodynamic modes !

4.3 Spatial dependence of decoherence

The Caldeira-Legett model has local evolution kernels and can therefore be solved in closed form. For Luttinger hydrodynamic modes, this is not the case (except for the $l=1$ modes in the electromagnetic case or in the 2D acoustic case). Nevertheless, if the coupling to the bath is weak enough ($\gamma L << v_S$), a Breit-Wigner approximation can be performed. This is equivalent to using an effective Caldeira-Leggett model in order to estimate the decoherence properties of all Luttinger modes.

Within the Caldeira-Legett model, and provided the system is weakly damped, the decoherence factor $e^{-D(t)}$ of a Schrodinger cat based on two coherent states can be computed in terms of the dissipation rate γ and the coherent state's parameters α and β:

$$D(t)=\frac{|\alpha-\beta|^2}{2}\,(1-e^{-\gamma t}) \tag{4.8}$$

The decoherence rate is therefore given by a very simple formula:

$$\tau_{\mathrm{Dec}}^{-1}=\gamma.\,\frac{|\alpha-\beta|^2}{2} \tag{4.9}$$

The $t\to+\infty$ limit of decoherence is independent of temperature but the decoherence time will scale with temperature according to the factor $\tanh\left(\hbar\omega_l/2k_BT\right)$ (ω_l is the mode's eigenfrequency).

The decoherence rates for the lth hydrodynamic modes are of the form ($\tau_E=L/c$):

$$8\pi^2\Delta_{n,m}\,.\,\frac{\mathcal{I}_l(\omega_l)}{\omega_l\tau_E}\,.\,F(\sigma_1,\sigma_2) \tag{4.10}$$

where $\frac{\mathcal{I}_l(\omega_l)}{\omega_l\tau_E}$ takes into account the bath's spectral density and the geometrical factor is given by ($\sigma_{12}=\sigma_1-\sigma_2$):

$$\begin{aligned}F_{(R/R)}&=\sin^2\left(\frac{l\pi\sigma_{12}}{L}\right)\\ F_{(R/L)}&=1+\frac{m^2-4n^2\alpha^2}{m^2+4n^2\alpha^2}\cos\left(\frac{2\pi l\sigma_{12}}{L}\right)\end{aligned}$$

Here $\Delta_{n,m}$ is the conformal dimension of the vertex operator $V_{n,m}(\sigma)$. The appearance of an odd dependence – in term of σ_{12} – in the R/L decoherence rate is understood by noticing that an appropriate parity operation transforms the R/L Schrődinger cat into an L/R one. Therefore R/L decoherence rate is invariant into simultaneous changes $\sigma_1\leftrightarrow\sigma_2$ and $nm\mapsto -nm$. Not surprisingly, the decoherence time of a R/R Schrődinger cat diverges when $\sigma_{12}\to 0$. This result is obvious since in this limit, we have a single excitation.

4.4 Decoherence time estimations: QED's case

In this case, the $\mathcal{I}_l(\omega_l)/(\omega_l\tau_E)$ factor is equal to:

$$\frac{g}{2\pi^2}\cdot\left(\frac{2\pi v_S}{c}\right)^{2(l-1)}\cdot\frac{l^2(l+1)}{(2l+1)!}$$

Since $v_S/c \simeq 10^{-3}$, decoherence times for the l and $l+1$ modes are related by a typical factor of 10^6. This argument shows that the $l=1$ modes dominate the decoherence process. Physically, higher Luttinger modes contribute to higher electric and magnetic multipoles, for which radiative dissipation is known to be weaker. The decoherence rate of the $l=1$ mode is nothing but the electromagnetic relaxation time ($\tau_L = L/v_S$):

$$\gamma\,\tau_L \simeq \frac{16\pi}{3}\,\frac{\alpha_{\rm QED}}{\alpha}\,\left(\frac{v_S}{c}\right)^2 \simeq 10^{-8} \qquad (4.11)$$

4.5 Decoherence time estimations: acoustic case

We have shown that *all $l \neq 0$ modes can be considered as weakly damped*. But in contrast to the electromagnetic case, these damping rates do not decrease with increasing l. Although in some cases the coupling constant $g_{ph}(L)$ is very small, "decoherence repartition" effects between modes plays a much more important role here than in QED's case since one has to sum up over many mode contributions to decoherence.

The total decoherence exponent in the linear regime is obtained by summing contributions over all the modes up to the Debye frequency. Since we sum over a large number of modes, the decoherence time rapidly decreases when $\sigma_{12}c_S \gg av_S$, a spectacular effect due to $c_S \ll v_S$. Roughly speaking, the Luttinger fermion has the time to circle many times around the loop before emitted phonons escape whereas it barely has the time to move in the electromagnetic case. This "averaging effect" explains why the dependence in the initial relative position is much weaker for acoustic than for electromagnetic decoherence. The maximal inverse decoherence rate at zero temperature can be expressed in the limit $Lc_S \gg av_S$:

$$\tau_L\,.\,\gamma^{(R/R)} = \Delta_{n,m}\,\frac{g_{ph}^{(d)}(a)}{4^{d-2}\pi d}\,\left(\frac{c_S}{v_S}\right)^2\,\frac{L}{a} \qquad (4.12)$$

The temperature dependence is very weak (remember we are typically working in situations where $\omega_D\tau_L > 10^5$ and $k_BT \simeq \hbar v_S/L$). To be precise, it goes like ($k_B\Theta_D = \hbar\omega_D$):

$$\frac{\gamma^{(R/R)}(T) - \gamma^{(R/R)}(0)}{\gamma^{(R/R)}(0)} \simeq \left(\frac{T}{\Theta_D}\right)^d \qquad (4.13)$$

In opposition with the photon bath case, the total acoustic decoherence time scales as L^{-1} in units of L/v_S.

4.6 Asymptotic decoherence

Applying previous results to the *electromagnetic decoherence* ($l=1$ modes), asymptotic decoherence exponents can be computed. They have the same spatial dependance than the decoherence rates. The typical asymptotic value is simply proportional to $\Delta_{n,m}$. Again, this number can be viewed as measuring the "distance" between the two quantum states which built our Schrődinger cat. Using vertex operator with small values of m and n in our Schrődinger cats produces mesoscopically separated coherent states in each mode.

In the *acoustic case (two dimensional phonon bath)*, contributions of all relevant modes should be summed. As before, the σ_{12} dependence disappears as soon as $\sigma_{12} \gg av_S/c_S$. Introducing $\tau^* = 2\pi^2\tau_L v_S/(c_S g_{ac}(a))$, the sum over all Luttinger modes up to the cut-off frequency can be evaluated:

$$\frac{d^{(R/R)}(t)}{d^{(R/R)}(\infty)} = \frac{d^{(R/L)}(t)}{d^{(R/L)}(\infty)} = 1 + \frac{e^{-t/\tau^*}-1}{(t/\tau^*)} \qquad (4.14)$$

where $d^{(R/R)}(\infty) = d^{(R/L)}(\infty) = \Delta_{n,m}\,c_S L/(v_S a)$. For the continuum approximation to be valid, we have assumed that $L/a \gg v_S/c_S$ and therefore $d^{(R/R)}(\infty) \gg 1$. This also implies that most of the decoherence process is accomplished within the previously computed acoustic decoherence time $2\tau^*/d^{(R/R)}(\infty) \ll \tau^*$.

5. Conclusion and discussion

5.1 Related works

Within the bosonization framework, the pioneering work [20] by Martin and Loss investigates the question of equilibrium permanent currents induced by fluctuations of the quantized electromagnetic field. They have shown that coupling the Luttinger liquid to QED leads to a renormalization of the Luttinger liquid parameters. Our discussion of *dynamical* and therefore *non-equilibrium* aspects of the coupled Luttinger & QED system shows how this renormalization appears dynamically. Let us mention that coupling a Luttinger liquid to one dimensional phonons also renormalizes the Luttinger liquid parameters and drives a 1D Fermi liquid to a non Fermi liquid fixed point [21, 22]. But a 1D phonon bath does not introduce any intrinsic decoherence in the Luttinger liquid since it does not have enough modes. That's why 2D and 3D phonon baths are considered in this paper and we have shown that these baths have enough modes to kill Schrődinger cat states. Finally, let us mention the work by Castro-Neto, Chamon and Nayak [23] who have studied equilibrium correlation functions in an infinite open Luttinger liquid coupled to an environment.

5.2 Perspectives

In the present work, the environment was assumed to be initially at equilibrium but, one could imagine changing the state of the environment, taking into account an external radiation. Increasing the incoming radiation power within the range of resonant frequencies should increase decoherence of Schrődinger cat states (enhancement of dissipation by stimulated emission of radiation). With such environmental states, one expects to meet also the problem of "decoherence repartition" between all the modes of the Luttinger system (even in the electromagnetic case). Although this makes computations much harder to control, it may lead to more interesting behaviors. Altshuler *et al.* [24] suggested that the non-equilibrium noise or an external microwave radiation could explain the saturation observed in Mohanty and Webb's experiments. Let us say that our approach, which only deals with ballistic conductors, also enables to study the influence of non-equilibrium noise. It can therefore provide an insight of the interplay between zero point fluctuations of the electromagnetic field and possible external non-equilibrium noise.

These considerations may be interesting from an experimental point of view since the QED decoherence effect is very slow (compared to L/v_S). As pointed out to us by L. Saminadayar, it seems to be possible to put a Ghz-range generator in an experiment and then study the variation of decoherence sensitive quantities. Although this perspective seems quite interesting, such experiments are quite difficult to perform. In particular, although the generator can be quite precisely controled, it is harder to control the electromagnetic noise experienced by the correlated electron gas.

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