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# Emitter size as a function of mass and transverse mass

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ABSTRACT: Bose-Einstein and Fermi-Dirac correlations show that the emitter dimension $r$ decreases as the hadron mass increases. Same behaviour is seen for the longitudinal dimension $r_z$ dependence on the transverse mass $m_T$. In both cases the Heisenberg uncertainty relations yield the same expression for $r(m)$ and $r_z(m_T)$. This $r$ behaviour also describes the interatomic separation of Bose condensates. If $r$ represents the emitter radius then its energy density reaches for baryon masses the high value of $\sim$100 GeV/fm$^3$.

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One dimensional (1-D) Bose-Einstein correlations (BEC) of identical bosons, and in particular the pairs $\pi^\pm\pi^\pm$, have been utilised over several decades to estimate the emitter size. These analyses used in many cases the kinematic variable $Q = \sqrt{-(q_1 - q_2)^2}$ where $q_i$ are the four momenta of the two identical bosons. As $Q \to 0$ a BEC enhancement can be observed in the experimental distribution by comparing it to a similar distribution void of BEC like e.g., a Monte Carlo generated event sample. The ratio of these two distributions is then described by the correlation function $C_2(Q) = 1 + \lambda e^{-Q^2 r^2}$ to yield a value for $r$, which is taken to be the emitter dimension. The factor $\lambda$, known as the chaoticity parameter, measures the strength of the BEC effect and can assume the values $0 \le \lambda \le 1$.

More recently it has been proposed [1] to extract a similar emitter dimension for pairs of equal baryons by utilising the so called Fermi-Dirac correlations (FDC), that allows identical fermions at very near phase space, when they are in an s-wave, only to be in a total spin S=0 state (the Pauli exclusion principle). To this end a method has been proposed in reference [1] for the direct measurement, as function of $Q$, of the fraction of $[S = 1]/([S = 0] + [S = 1])$ in pairs of spin 1/2 weakly decaying baryons, like the $\Lambda\Lambda$ system. Alternatively one can apply the method used in BEC of identical bosons and look at the distribution of baryon pairs as $Q$ approaches zero. If a depletion is observed then, by assuming its origin to be due to the Pauli exclusion principle, an $r$ value can be deduced. The measured baryon $r$ values can directly be compared to those obtained for bosons as they also measure the distance between the two hadrons as the set on of a pure s-wave

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state occurs when they approach threshold.

The existing vast data of hadronic $Z^0$ decays, three to four million per LEP experiment, provide an excellent material for BEC and FDC studies at the same $\sqrt{s_{ee}}$ and in a high multiplicity, $\langle n_{ch} \rangle \simeq$21 hadrons, final state. In particular it was possible to measure $r$ as a function of the hadron mass. The results of these analyses are shown in Fig. 1 where average LEP $r$ values [2] are given for charged pion and kaon pairs, for $\Lambda$ pairs in addition to an OPAL preliminary $r$ value [3] for antiproton pairs. As seen, the average $r$ values

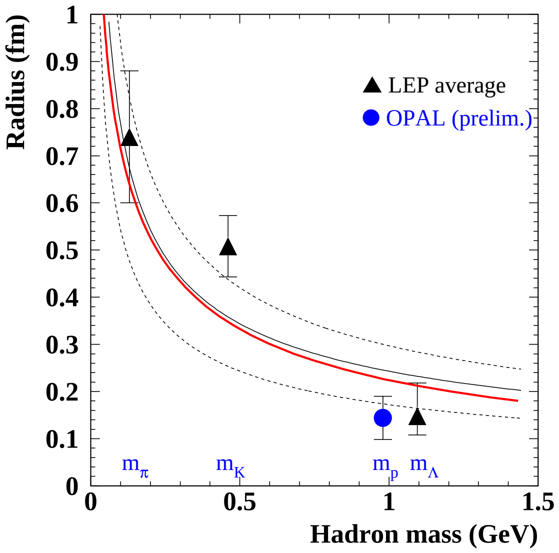


**Figure 1:** The $r(m)$ values (triangles) obtained from 1-D BEC analyses of the hadronic $Z^0$ decays at LEP [2] and an OPAL preliminary [3] value (circle) for antiprotons. The thin lines are from Eq. 1 for $\Delta t$ values of $10^{-24}$ sec (central thin line) and 0.5x$10^{-24}$ and 1.5x$10^{-24}$ sec (thin dashed lines). The thick central line is from the virial theorem using a general QCD potential [2].

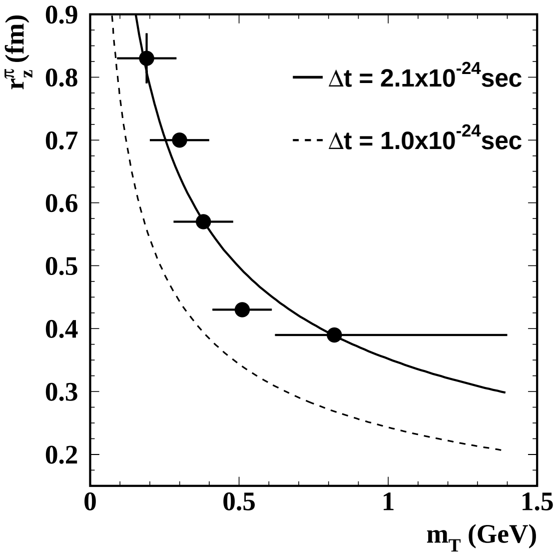


**Figure 2:** Preliminary DELPHI results [5], obtained from a 2-D BEC analysis, for the longitudinal emitter length $r_z^\pi$ dependence on the transverse mass $m_T$ in $Z^0$ decays. The solid and dashed lines are from Eq. 1 using $\Delta t$ values of 2.1x$10^{-24}$ and 1.0x$10^{-24}$ sec respectively.

decreases from ∼0.75 fm for pions down to ∼0.15 fm for antiprotons and $\Lambda$ hyperons. Whereas the experimental findings that $r(m_\pi)$ is somewhat larger than $r(m_K)$, but equal within errors, may still be consistent with the string fragmentation model although in its basic form it expects $r(m)$ to increase with $m$, the much smaller value obtained for $r(m_\Lambda)$ and $r(m_{\bar{p}})$ poses a challenge to the model [4]. On the other hand it has been shown [2] that by applying the Heisenberg uncertainty relations one can derive an expression for $r(m)$ that decreases with $m$, namely:

$$r(m) \;=\; \frac{c\sqrt{\hbar \Delta t}}{\sqrt{m}} \;. \qquad (1)$$

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The prediction of Eq. 1 is drawn in Fig. 1 and is seen to follow the general trend of the experimental values when $\Delta t$ is set to $\sim 10^{-24}$ sec, to represent a typical time scale for strong interactions.

The effective range of two-pion source was also estimated in 2-dimensional (2-D) BEC analyses, in hadronic interactions as well as in the hadronic $Z^0$ decays [5, 6], as a function of the pion-pair transverse mass $m_T$. This transverse mass is defined as $m_T = 0.5 \times \sum_{i=1}^{2} \sqrt{m^2 + p_{i,T}^2}$ where $p_{1,T}^2$ and $p_{2,T}^2$ are the transverse momenta of the two bosons defined in the longitudinal centre of mass system (LCMS) [7]. The longitudinal and transverse dimensions $r_z$ and $r_T$ are then obtained from a fit of an expression of the type $C_2(Q_z, Q_T) = 1 + \lambda e^{-r_z^2 Q_z^2 + r_T^2 Q_T^2}$ to the data. The DELPHI preliminary results [5] for the longitudinal dimension $r_z$ of two identical charged pion pairs are seen in Fig. 2 to depend on $m_T$ in a very similar way to the $r(m)$ dependence on $m$ (see Fig. 1). In fact, when substituting in Eq. 1, $r$ by $r_z$ and $m$ by $m_T$ one obtains the lines shown in Fig. 2 for two chosen values of $\Delta t$. This similarity can be understood if one remembers that $r_z$ and the longitudinal momentum $p_z$ are conjugate observable [8]. Thus one has $\Delta p_z \Delta r_z = 2\mu v_z r_z = p_z r_z = \hbar c$ where $\mu$ is the reduced mass of the two hadrons, so that

$$r_z = \hbar c / p_z \ . \tag{2}$$

Simultaneously we can also use the uncertainty relation given in energy and time i.e., $\Delta E \Delta t = \hbar$, where the energy is given in GeV and $t$ in seconds utilising the fact that in the LCMS, $p_{1,z} = -p_{2,z}$. In as much that the total energy of the boson-pair system is predominantly determined by the sum of their relativistic mass values, one has

$$E = \sum_{i=1}^{2} \sqrt{m^2 + p_{i,x}^2 + p_{i,y}^2 + p_{i,z}^2} = \sum_{i=1}^{2} m_{i,T} \sqrt{1 + \frac{p_z^2}{m_{i,T}^2}} \approx \sum_{i=1}^{2} \left( m_{i,T} + \frac{p_z^2}{2m_{i,T}} \right) \ ,$$

where $m_{1,T}$ and $m_{2,T}$ are the transverse mass of the first and second hadron. At small $Q_z$, the difference $\delta m_T = |m_{1,T} - m_{2,T}|/2$ is much smaller than the transverse mass $m_T = (m_{1,T} + m_{2,T})/2$, and therefore can be neglected, so after a few algebraic steps one obtains $E = 2m_T + p_z^2/m_T$. Since $2m_T$ is not a function of $Q_z$ it may be considered to stay fixed as $Q_z \to 0$ so that

$$\Delta E \Delta t \approx (p_z^2/m_T) \times \Delta t = \hbar \ . \tag{3}$$

Combining Eqs. 2 and 3 one finds

$$r_z(m_T) \ \approx \ \frac{c\sqrt{\hbar \Delta t}}{\sqrt{m_T}} \ , \tag{4}$$

which is identical to Eq. 1 when replacing $r$ and $m$ respectively by $r_z$ and $m_T$. Here it is worthwhile to note that in heavy ion collisions it was found out [9] that $r_z \approx 2/\sqrt{m_T(GeV)}$. Experimentally a decrease of $r_T$ with the increase of $m_T$ was also observed [6] but unlike $r_z$ which is a geometrical quantity, $r_T$ is a mixture of the transverse radius and the emission time so that an application of the uncertainty relations is not straightforward.

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An alternative approach for the description of $r_z(m_T)$ can be achieved by the so called Bjorken-Gottfried conjecture that the momentum-energy 4-vector, $q_\mu$, is proportional to the space-time 4-vector, $x_\mu$. In this method one did find [10] that $r_z(m_T)$ moves from a typical value of $\sim 1.1$ fm for $m_T = 0.14$ GeV to $\sim 0.25$ fm for an $m_T$ of about 1 GeV.

Another consequence of the Bose-Einstein statistics of identical bosons is the existence of Bose condensates of bosonic atoms. These condensates, which have been discovered in 1995, are formed by bosonic atoms when cooled down to temperatures in the typical range of 500nK to 2$\mu$K, bellow a critical temperature $T_B$, where the interatomic separation, $d_{BE}$, is of the order of the de Broglie wave length, $\lambda = \sqrt{h^2/(2\pi mkT)}$. Specific calculations [8] show that at a very low temperature $T_0$ where $T_0/T_B \ll 1$, the average $d_{BE}$ is equal to

$$d_{BE}(m) \approx \frac{\sqrt{2\pi}}{1.378}\left(\frac{\hbar^2}{mkT_0}\right)^{1/2} . \tag{5}$$

From this follows that when two different condensates having atomic mass $m_1$ and $m_2$ are at the same temperature $T_0$, way below their individual $T_B$ values, the ratio of their interatomic separation will be equal to $d_{BE}(m_1)/d_{BE}(m_2) = \sqrt{m_2/m_1}$ similarly to the dependence of $r$ ($r_z$) on $m$ ($m_T$). It is further interesting to note that in as much that it is permissible to replace, at very low temperatures, $kT_0$ by $\Delta E$ and use the uncertainty relation $\Delta E = \hbar/\Delta t$, one derives for $d_{BE}(m)$ the expression given in Eq. 1 for $r(m)$ multiplied by the factor $\sqrt{2\pi}/1.378$. This similarity between interatomic separation and emitter dimension may well be traced back to the close connection between the de Broglie wave length and the $\Delta p \Delta x \approx \hbar$ Heisenberg uncertainty relation. Caution should however be exercised when trying to relate the Bose condensates to the production of hadrons at high energy reactions. Common to both systems is their bosonic nature which allows all hadrons (atoms) to occupy the same lowest energy state. Furthermore the condensates are taken to be in a thermal equilibrium state. Among the various models proposed for the hadron production some attempts [11] have also been made to explore the application of a statistical thermal-like models however if these will survive is presently questionable. Finally condensates are taken to be in a coherent state whereas hadron pairs systems for which an $r$ value can be measured must be at least partially not coherent, i.e. $\lambda \neq 0$.

In as much that the $r$ values obtained from the 1-D BEC analyses represent the emitter radius one can further try and estimate the experimental measured energy density, $\epsilon_{exp}$, of the emitter by dividing the sum of the hadron-pair masses by a sphere volume of radius $r$, that is

$$\epsilon_{exp} = \frac{2m}{(4/3)\pi r^3} . \tag{6}$$

In Fig. 3 the measured energy density of the emitter of pions, kaons and baryons are shown in units of GeV/fm$^3$. The data points are compared in the figure by the dashed curves with the values expected when $r$ given by Eq. 1 is inserted in Eq. 6 to give

$$\epsilon_{model} = \frac{3}{2\pi}\frac{m^{5/2}}{c^3(\hbar\Delta t)^{3/2}} . \tag{7}$$



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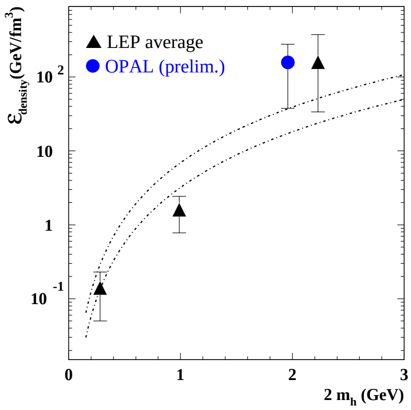


**Figure 3:** Emitter energy density as a function of the hadron-pair mass sum. The dashed lines are the expectation of Eq. 7 with $\Delta t$=(1.2 $\pm$ 0.3)x10$^{-24}$ sec.

As can be seen, the energy density values for kaon and pion pairs are lying in a reasonable range of $\sim$1 GeV/fm$^3$ and below. On the other hand the energy density of the baryon pairs reaches an average value of the order of 100 GeV/fm$^3$, very high even in comparison to the energy density required for the formation of a quark-gluon plasma. A similar energy density evaluation of the hadron emitter, deduced from 2-D BEC analyses, is problematic if not only for the fact that $r_T$ is not a pure geometrical quantity. In as much that $r$ does in fact represent the emitter radius then the resulting high energy density poses a challenge to the existing production and hadronisation models for hadrons and in particular baryons, emerging from high energy collisions.

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## References

[1] G. Alexander, H.J. Lipkin, *Phys. Lett.* **B 352** (1995) 162.

[2] G. Alexander, I. Cohen, E. Levin, *Phys. Lett.* **B 452** (1999) 159.

[3] OPAL Collaboration, *Fermi-Dirac Correlations Between Antiprotons in Hadronic $Z^0$ Decays*, PN-486 (2000), submitted to this conference, # 173.

[4] B. Andersson, *Some remarks on Bose-Einstein correlations*, Proc. XXXV Recontres de Moriond, QCD, March 18-25, 2000, Les Arcs, France, in press.

[5] B. Lörstad, O.G. Smirnova, *Transverse mass dependence of Bose-Einstein correlation radii in $e^+e^-$ annihilation at LEP energies*, Proc. 7th Workshop on Multiparticle Production: Correlations and Fluctuations, Nijmegen 1996, World Scientific, 1997, p. 42.

[6] L3 Collaboration, *A Two-Dimensional Study of Bose-Einstein correlation in Z decays at LEP*, Nijmegen report HEN-408, presented at the XXIX Int. Conf. on HEP, Vancouver, 1998.

[7] T. Csörgő, S. Pratt, Proc. Workshop on Relativistic Heavy-Ion Physics, eds. Csörgő et al., (KFKI-1991-28/A. Budapest, 1991) p. 75.

[8] G. Alexander, *Phys. Lett.* **B 506** (2001) 45 and references therein.

[9] See e.g., U. Heinz, B.V. Jacak, *Ann. Rev. Nucl. Part. Sci.* **49** (1999) 529 and references therein.

[10] A. Bialas et al., *Phys. Rev.* **D 62** (2000) 114007 and references therein.

[11] See e.g., F. Becattini et al., *Nucl. Phys. Proc. Suppl.* **92** (2001) 137 and references therein.

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