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## QCD radiative corrections to $\gamma^*\gamma^* \rightarrow$ hadrons at LEP2

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ABSTRACT: In this talk we describe the order- $\alpha_s$  corrections to the total cross section and to jet rates in  $\gamma^*\gamma^* \rightarrow$  hadrons for the process  $e^+e^- \rightarrow e^+e^- +$  hadrons. We use a next-to-leading order general-purpose partonic Monte Carlo event generator that allows the computation of a rate differential in the produced leptons and hadrons. We compare our results with the experimental data for  $e^+e^- \rightarrow e^+e^- +$  hadrons at LEP2.

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Strong interaction processes, characterised by a large kinematic scale, are described in perturbative QCD by a fixed-order expansion in  $\alpha_s$  of the parton cross section, complemented, if the scattering process is initiated by strong interacting partons, with the Altarelli-Parisi evolution of the parton densities. In many scattering processes, the state-of-the-art computation of production rates is at the next-to-leading order (NLO). However, in kinematic regions characterised by two large and disparate scales, a fixed-order expansion may not suffice: large logarithms of the ratio of the kinematic scales appear, which may have to be resummed. In processes where the centre-of-mass energy  $S$  is much larger than the typical transverse scale  $Q^2$ , the leading logarithms of type  $\ln(S/Q^2)$  are resummed by the BFKL equation. Several observables, like the scaling violations of the  $F_2$  structure

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function, forward-jet production in DIS, dijet production at large rapidity intervals and  $\gamma^*\gamma^* \rightarrow$  hadrons in  $e^+e^-$  collisions have been measured [1, 2, 3, 4] and analysed [5, 6] in this fashion.

In this talk we report on a NLO calculation [6] of the total cross section and of jet rates in  $\gamma^*\gamma^* \rightarrow$  hadrons for the process  $e^+e^- \rightarrow e^+e^- +$  hadrons at LEP2, and we compare the NLO calculation to the data from the CERN L3 [1, 2], OPAL [3] and ALEPH [4] Collaborations. Namely, we consider

$$e^+ + e^- \longrightarrow e^+ + e^- + \underbrace{\gamma^* + \gamma^*}_{\rightarrow \text{hadrons};} \quad (1)$$

selecting those events in which the incoming leptons produce two photons which eventually initiate the hard scattering that produces the hadrons. However, it is clear that the process in Eq. (1) is non physical; rather, it has to be understood as a shorthand notation for a subset of Feynman diagrams contributing to the process that is actually observed,

$$e^+ + e^- \longrightarrow e^+ + e^- + \text{hadrons}. \quad (2)$$

Other contributions to the process in Eq. (2) are, for example, those in which the incoming  $e^+e^-$  pair annihilates into a photon or a  $Z$  boson, eventually producing the hadrons and a lepton pair, or those in which one (or both) of the two photons in Eq. (1) is replaced by a  $Z$  boson. However, one can devise a set of cuts such that the process in Eq. (1) gives the only non-negligible contribution to the process in Eq. (2). One can tag both of the outgoing leptons, and retain only those events in which the scattering angles of the leptons are small: in such a way, the contamination due to annihilation processes is safely negligible. Furthermore, small-angle tagging also guarantees that the photon virtualities are never too large (at LEP2, one typically measures  $Q_i^2 = \mathcal{O}(10 \text{ GeV}^2)$ ); therefore, the contributions from processes in which a photon is replaced by a  $Z$  boson are also negligible. Thus, it is not difficult to extract the cross section of the process  $\gamma^*\gamma^* \rightarrow$  hadrons from the data relevant to the process in Eq. (2).

Our calculation was performed in the massless limit for the final state quarks. We compared our LO result to the massless limit of the JAMVG program of Ref. [7], and found perfect agreement. To study the effect of the NLO corrections, we used the experimental cuts employed by the L3 Collaboration. The scattered electron and positron are required to have energy  $E_{1,2}$  larger than 30 GeV and scattering angle  $\theta_{1,2}$  between 30 and 66 mrad. Furthermore, the rapidity-like variable  $Y$ , defined by

$$Y = \log \frac{y_1 y_2 S}{\sqrt{Q_1^2 Q_2^2}}, \quad (3)$$

is required to lie between 2 and 7 ( $y_i$ , with  $i = 1, 2$ , is proportional to the light-cone momentum fraction of the  $i^{\text{th}}$  virtual photon, and is precisely defined in Ref. [6], where a discussion on the properties of  $Y$  can also be found). The cross sections have been evaluated at  $\sqrt{S} = 200 \text{ GeV}$ , including up to five massless flavours.

We discuss briefly the dependence of our predictions on the electromagnetic and strong couplings; our cross sections are  $\mathcal{O}(\alpha_{\text{em}}^4)$  and we chose to evolve  $\alpha_{\text{em}}$  (using one-loop  $\overline{\text{MS}}$  running) on an event-by-event basis to the scales set by the virtualities of the exchanged photons; hence, we replace the Thomson value  $\alpha_0 \simeq 1/137$  by  $\alpha_{\text{em}}(Q_i^2)$ . We treat independently the two photon legs: thus,  $\alpha_{\text{em}}^4$  has to be understood as  $\alpha_{\text{em}}^2(Q_1^2)\alpha_{\text{em}}^2(Q_2^2)$ . As for the strong coupling  $\alpha_s$ , we define a default scale  $\mu_0$  so as to match the order of magnitude of the (inverse of the) interaction range,

$$\mu_0^2 = \frac{Q_1^2 + Q_2^2}{2} + \left( \frac{k_{1T} + k_{2T} + k_{3T}}{2} \right)^2. \quad (4)$$

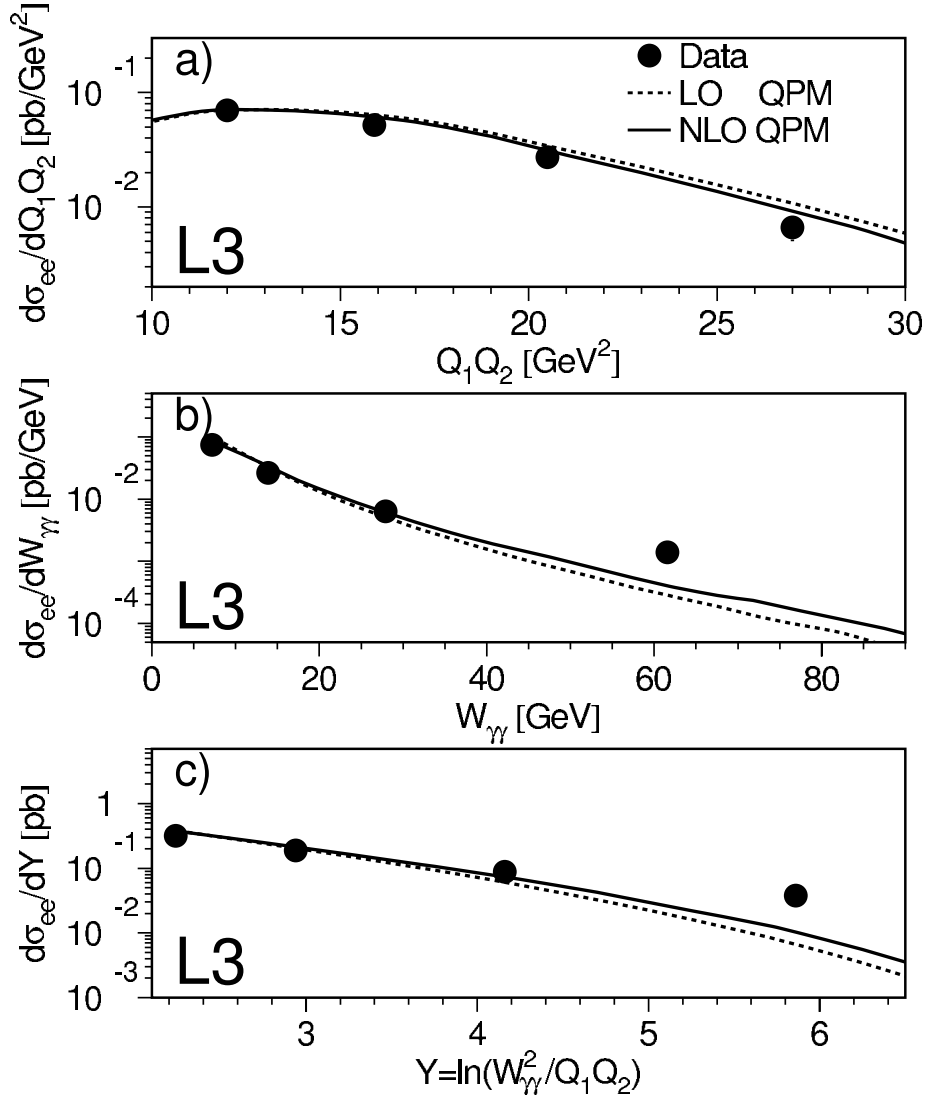
The renormalization scale  $\mu$  entering  $\alpha_s$  is set equal to  $\mu_0$  as a default value, and equal to  $\mu_0/2$  or  $2\mu_0$  when studying the scale dependence of the cross section. In Eq. (4), the  $k_{iT}$  are the transverse energies of the outgoing quarks and, for three-particle events, the emitted gluon. Since the hard process is initiated by the two virtual photons, we study its properties in the  $\gamma^*\gamma^*$  center-of-mass frame. We evolve  $\alpha_s$  to next-to-leading log accuracy, with  $\alpha_s(M_Z) = 0.1181$  [8] (in  $\overline{\text{MS}}$  at two loops and with five flavours, this implies  $\Lambda_{\overline{\text{MS}}}^{(5)} = 0.2275$  GeV). In all of the distributions examined [6], we found that the uncertainty related to  $\mu$  is always smaller than the net effect of including the NLO corrections. As for the effect of the NLO corrections themselves, we found that, apart from slightly increasing the cross section with respect to the LO calculation, they induce visible shape modifications in the  $Y$  distribution, their effect changing from almost nil at the lowest end of the  $Y$  spectrum to a more than 50% increase at the highest end.

The L3 [1, 2], OPAL [3] and ALEPH [4] Collaborations have analysed data for hadron production in  $e^+e^-$  collisions (through  $\gamma^*\gamma^*$  scattering) at a center-of-mass energy around 200 GeV. L3 made use of the previously mentioned set of experimental cuts. Recently L3 re-analysed their data [2, 9]. The new analysis of L3 is reported in Fig. 1, where the cross section is presented as a function of the geometric mean of the photon virtualities  $Q_1Q_2$ , the hadron energy  $W_{\gamma\gamma}$  and the  $Y$  variable, and is compared to our leading and next-to-leading order predictions [6]), evaluated at  $\sqrt{S} = 200$  GeV. We note that in the distribution as a function of  $Q_1Q_2$  there is a fairly good agreement between theory and data; in the  $W_{\gamma\gamma}$  and  $Y$  distributions the agreement between theory and data is good at the low end of the spectrum, but the data tend to lie above the theory at the high end of the spectrum<sup>1</sup>. Thus we find a noticeable difference in shape between theory and data which, if confirmed, could be interpreted as the onset of important higher order effects, perhaps of BFKL type.

We have also studied the effect of the finite mass of the outgoing heavy quarks in the charm and bottom case, by comparing our results with the ones obtained with the JAMVG [7] code. Within the L3 set of cuts, such mass effects can be seen to decrease the LO massless cross section by an amount of the order of 10-15%.

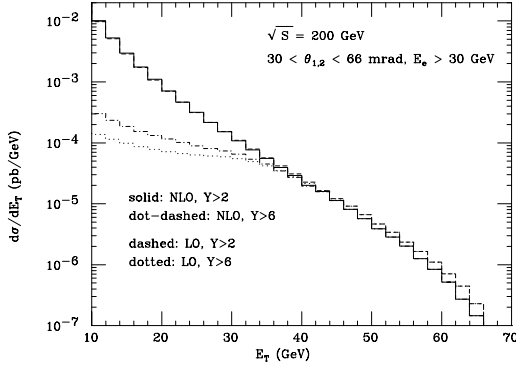
The data the OPAL Collaboration has taken at  $\sqrt{S} = 189 - 202$  GeV, making use of a slightly different set of cuts are analysed in Ref. [3], and compared there to our NLO

<sup>1</sup>It is to be noted that the scale uncertainties affecting our predictions are much smaller than the experimental errors.

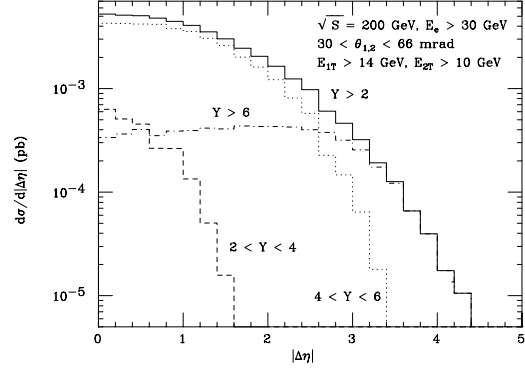


**Figure 1:** Differential cross sections with respect to  $Q_1^2Q_2^2$ ,  $W_{\gamma\gamma}$  and  $Y$  from the L3 Collaboration, compared to leading and next-to-leading order predictions. The data are taken at  $\sqrt{S} = 189 - 202$  GeV. The theoretical simulation is always run at  $\sqrt{S} = 200$  GeV.

predictions [6]. For the differential distribution in the variable  $\bar{Y}$  (a variant of  $Y$ , which tends to it in the high-energy limit [6]), a generally good agreement within errors can be observed, even though in the largest  $\bar{Y}$  bin the data tend to lie above the prediction. In the analysis of the data and in the comparison with our NLO predictions that ALEPH [4] has performed, there is a good agreement between theory and data in the  $W_{\gamma\gamma}$  distribution, while a difference between theory and data is present in the  $Y$  distribution, but only in normalisation and not in shape. Thus we find in general a good agreement between theory and LEP2 data, with a discrepancy between theory and L3 data at the highest end of the distributions in  $W_{\gamma\gamma}$  and  $Y$ . It would therefore be of the utmost importance to measure as accurately as possible the  $Y$  spectrum, in order to perform a precise study of effects



**Figure 2:** Transverse energy in single-inclusive jet production.



**Figure 3:** Distribution of the rapidity difference between the jets in dijet production.

beyond NLO (such as BFKL dynamics).

Other production rates of interest are the jet distributions, because they give us information on different kinematic regions. We define the jets by means of a  $k_T$  clustering algorithm, and set the jet-resolution parameter  $D = 1$ . In Fig. 2 we show the transverse energy distribution of single-inclusive jets, considering the cuts  $Y > 2$  and  $Y > 6$ . The first striking feature of this observable is that the curves relevant to  $Y > 2$  and  $Y > 6$  coincide for  $E_T > 40$  GeV. In fact, at the threshold (where the jets are produced at zero rapidity),  $W^2 = 4E_T^2$ , we obtain  $Y \simeq 6$  [6]. Therefore, the region  $2 < Y < 6$  simply does not contribute to events with  $E_T > 40$  GeV: at  $E_T = 40$  GeV, the two-photon system has just enough energy, at  $Y = 6$ , to produce the jets. Larger values of  $Y$  do not contribute much, since the  $Y$  spectrum is very rapidly falling at large  $Y$ 's. When considering larger transverse momenta, the situation is exactly the same. We are led to the conclusion that the tail of the  $E_T$  spectrum is dominated by threshold production, and therefore cannot be reliably predicted by a fixed-order computation, like ours; a resummation of large threshold logarithms is necessary. At smaller transverse energies the behaviour of the radiative corrections displays a pattern similar to that of total rates. For  $Y > 2$ , NLO and LO results are very close to each other. For  $Y > 6$ , the radiative corrections increase sizably the LO result; this is in agreement with the behaviour of the  $Y$  spectrum shown. The increase is related to the appearance of large logarithms in the cross section, as it is always the case when two scales (here, the small  $E_T$  and the large hadronic energy) are present. Next, we argue that the large logarithms in the large- $Y$  region are of BFKL type.

In Fig. 3, we show the distributions in the rapidity interval  $\Delta\eta$  between the two tagged jets in dijet events, for various cuts on  $Y$ . We select the jets by imposing a  $E_T > 14$  GeV cut on the transverse energy of the most energetic jet and requiring  $E_T > 10$  GeV for at least another jet (in this case, only the NLO results are shown), in order to avoid the problems that arise in the case in which such cuts are chosen to be equal. The most interesting feature of this plot is that it shows that the large- $Y$  and the large- $\Delta\eta$  regions select the same events: the distributions relevant to  $Y > 2$  (solid line) and to  $Y > 6$  (dot-dashed line) exactly coincide for  $\Delta\eta > 3.5$ . This is the same behaviour we observe in the case of the transverse energy distribution, but the underlying physics is different. In fact, in this case

we also get sizable contributions away from the threshold; thus, at fixed  $E_T$ , part of the energy of the two-photon system contributes to the longitudinal degrees of freedom, and jets can be produced away from the central region. The large- $Y$  region is thus naturally suitable to study BFKL physics.

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