# $\varepsilon'/\varepsilon$ in the Standard Model

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ABSTRACT: We overview the detailed analysis of $\varepsilon'/\varepsilon$ within the Standard Model, presented in ref. [1]. When all sources of large logarithms are considered, both at short and long distances, it is possible to perform a reliable Standard Model estimate of $\varepsilon'/\varepsilon$. The strong S–wave rescattering of the final pions has an important impact on this observable [1, 2]. The Standard Model prediction is found to be [1] $\mathrm{Re}\,(\varepsilon'/\varepsilon) = (1.7 \pm 0.9) \cdot 10^{-3}$, in good agreement with the most recent experimental measurements. A better estimate of the strange quark mass would reduce the uncertainty to about 30%.

## 1. Introduction

In recent times the determination of $\mathrm{Re}\,(\varepsilon'/\varepsilon)$ has stimulated a lot of work both on the theoretical and experimental sides. The latest has been recently clarified by the new NA48 [3], $\mathrm{Re}\,(\varepsilon'/\varepsilon) = (15.3 \pm 2.6) \cdot 10^{-4}$, and the KTEV [4], $\mathrm{Re}\,(\varepsilon'/\varepsilon) = (20.7 \pm 2.8) \cdot 10^{-4}$, results. The present experimental world average is [3]–[6]

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = (17.2 \pm 1.8) \cdot 10^{-4} \ . \tag{1.1}$$

The theoretical prediction has been the subject of many debates since different groups, using different methods or approximations obtained different results [7]–[12]. Recently however it has been observed [1] that once all essential ingredients are taken into account, including final state interactions (FSI) [2], one can give a reliable estimate of $\mathrm{Re}\,(\varepsilon'/\varepsilon)$

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which is in perfect agreement with the experimental value. In ref. [1] a detailed analysis is presented, which includes the evaluation of all large logarithmic corrections both at short and long distances; the resulting Standard Model prediction is

$$\text{Re}\left(\varepsilon'/\varepsilon\right) = (17 \pm 9) \cdot 10^{-4} \ . \tag{1.2}$$

The subject of this talk is a review of the main ingredients in the calculation of $\varepsilon'/\varepsilon$.

The physical origin of $\varepsilon'/\varepsilon$ is at the electroweak scale, where the flavor–changing processes can be described in terms of quarks, leptons and gauge bosons with the usual gauge coupling perturbative expansion. At the scale $M_Z$ the heavy gauge bosons $W^{\pm}$ and $Z$, and the top quark are integrated out of the theory. The dynamics is then described in terms of Wilson coefficients $C_i(\mu)$ and operators $Q_i(\mu)$, via a Lagrangian of the form

$$\mathcal{L}_{eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) \, Q_i(\mu). \tag{1.3}$$

The values of the coefficients $C_i$ are matched with the underlying theory at the electroweak scale $\sim M_Z$. Then, using the Operator Product Expansion (OPE) [13] and renormalization group equations [14], one can evaluate the Wilson coefficients at any scale $\mu$ summing up the short–distance logarithms. The overall renormalization scale $\mu$ separates the short– $(M > \mu)$ and long– $(m < \mu)$ distance contributions, which are contained in $C_i(\mu)$ and $Q_i$, respectively. The physical amplitudes are independent of $\mu$; thus, the explicit scale (and scheme) dependence of the Wilson coefficients should cancel exactly with the corresponding dependence of the $Q_i$ matrix elements between on-shell states.

Our knowledge of $\Delta S = 1$ transitions has improved qualitatively in recent years, thanks to the completion of the next-to-leading logarithmic order calculation of the Wilson coefficients [15, 16]. All gluonic corrections of $\mathcal{O}(\alpha_s^n t^n)$ and $\mathcal{O}(\alpha_s^{n+1} t^n)$, where $t \equiv \log M/m$ and $M$ and $m$ are any scales appearing in the evolution, are already known. Moreover the full $m_t/M_W$ dependence (to first order in $\alpha_s$ and $\alpha$) has been taken into account at the electroweak scale. We will fully use this information up to scales $\mu \sim \mathcal{O}(1 \text{ GeV})$, without making any unnecessary expansion. At a scale $\mu < m_c$ one has a three–flavor theory described by a Lagrangian of the same general form as in eq. (1.3). The difficult and still unsolved problem resides in the calculation of the hadronic matrix elements. As we will see in the following the large–$N_c$ expansion and Chiral Perturbation Theory ($\chi$PT) allow to estimate those matrix elements with sufficient accuracy for the determination of $\varepsilon'/\varepsilon$. In the following we adopt the usual isospin decomposition:

$$A[K^0 \to \pi^+\pi^-] \equiv \mathcal{A}_0 + \frac{1}{\sqrt{2}}\,\mathcal{A}_2\,, \quad A[K^0 \to \pi^0\pi^0] \equiv \mathcal{A}_0 - \sqrt{2}\,\mathcal{A}_2\,. \tag{1.4}$$

The complete amplitudes $\mathcal{A}_I \equiv A_I \exp\left\{i\delta_0^I\right\}$ include the strong phase shifts $\delta_0^I$. The S–wave $\pi$-$\pi$ scattering generates a large phase-shift difference between the $I = 0$ and $I = 2$ partial waves [17]: $\left(\delta_0^0 - \delta_0^2\right)(M_K^2) = 45° \pm 6°$. There is a corresponding dispersive FSI effect in the moduli of the isospin amplitudes, because the real and imaginary parts are related by analyticity and unitarity. The presence of such a large phase-shift difference

clearly signals an important FSI contribution to $A_I$. In terms of the $K \to \pi\pi$ isospin amplitudes,

$$\frac{\varepsilon'}{\varepsilon} = e^{i\Phi} \frac{\omega}{\sqrt{2}|\varepsilon|} \left[ \frac{\text{Im}(A_2)}{\text{Re}(A_2)} - \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right] . \tag{1.5}$$

Due to the famous "$\Delta I = 1/2$ rule", $\varepsilon'/\varepsilon$ is suppressed by the ratio $\omega = \text{Re}(A_2)/\text{Re}(A_0) \approx 1/22$. The phases of $\varepsilon'$ and $\varepsilon$ turn out to be nearly equal: $\Phi \approx \delta_0^2 - \delta_0^0 + \frac{\pi}{4} \approx 0$. The CP–conserving amplitudes $\text{Re}(A_I)$, their ratio $\omega$ and $|\varepsilon|$ are usually set to their experimentally determined values. A theoretical calculation is then only needed for $\text{Im}(A_I)$. Using the short–distance Lagrangian (1.3), the CP–violating ratio $\varepsilon'/\varepsilon$ can be written as [7]

$$\frac{\varepsilon'}{\varepsilon} = \text{Im}\,(V_{ts}^* V_{td})\, e^{i\Phi} \frac{G_F}{2|\varepsilon|} \frac{\omega}{|\text{Re}(A_0)|} \left[ P^{(0)}\,(1 - \Omega_{IB}) - \frac{1}{\omega}\,P^{(2)} \right] , \tag{1.6}$$

where the quantities $P^{(I)} = \sum_i y_i(\mu)\, \langle (\pi\pi)_I | Q_i | K \rangle$ contain the contributions from hadronic matrix elements with isospin $I$, $\Omega_{IB} = (1/\omega)\,\text{Im}(A_2)_{\text{IB}}/\text{Im}(A_0)$ parameterizes isospin breaking corrections and $y_i(\mu)$ are the CP–violating parts of the Wilson coefficients: $C_i(\mu) = z_i(\mu) + \tau\, y_i(\mu)$ with $\tau = -V_{td}V_{ts}^*/V_{ud}V_{us}^*$. The factor $1/\omega$ enhances the relative weight of the $I = 2$ contributions. In the Standard Model, $P^{(0)}$ and $P^{(2)}$ turn out to be dominated respectively by the contributions from the QCD penguin operator $Q_6$ and the electroweak penguin operator $Q_8$ [9],

$$Q_6 = (\bar{s}_\alpha d_\beta)_{\text{V}-\text{A}} \sum_q (\bar{q}_\beta q_\alpha)_{\text{V}+\text{A}} , \quad Q_8 = \tfrac{3}{2}\,(\bar{s}_\alpha d_\beta)_{\text{V}-\text{A}} \sum_q e_q\,(\bar{q}_\beta q_\alpha)_{\text{V}+\text{A}} . \tag{1.7}$$

A recent improved calculation of $\Omega_{IB}^{\pi^0\eta}$ at $\mathcal{O}(p^4)$ in $\chi$PT has found the result [18]

$$\Omega_{IB}^{\pi^0\eta} = 0.16 \pm 0.03 . \tag{1.8}$$

## 2. Chiral Perturbation Theory

Below the resonance region and using global symmetry considerations one can define an effective field theory in terms of the QCD Goldstone bosons ($\pi, K, \eta$). The $\chi$PT formulation of the SM [19, 20, 21] describes the meson–octet dynamics through a perturbative expansion in powers of the ratio of momenta and quark masses over the chiral symmetry breaking scale ($\Lambda_\chi \sim 1\text{GeV}$). The operator content of the theory is fixed by chiral symmetry. At lowest order, the most general effective bosonic weak Lagrangian, with the same $SU(3)_L \otimes SU(3)_R$ transformation properties and quantum numbers as the short–distance Lagrangian (1.3), contains three terms transforming as $(8_L, 1_R)$, $(27_L, 1_R)$ and $(8_L, 8_R)$ whose corresponding couplings are denoted by $g_8$, $g_{27}$ and $g_{ew}$.

The isospin amplitudes $\mathcal{A}_I$ have been computed up to next–to–leading order in the chiral expansion [22]–[27]. Decomposing the isospin amplitudes according to their representation components $\mathcal{A}_I = \Sigma_R \mathcal{A}_I^{(R)}$, the results of those calculations can be written in the form (the expressions for $\mathcal{A}_0^{(ew)}$, $\mathcal{A}_0^{(27)}$, $\mathcal{A}_2^{(27)}$ can be found in ref. [1]):

$$\mathcal{A}_0^{(8)} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^*\, \sqrt{2}\, f_\pi\, g_8\, (M_K^2 - M_\pi^2) \left[ 1 + \Delta_L \mathcal{A}_0^{(8)} + \Delta_C \mathcal{A}_0^{(8)} \right] ,$$

$$\mathcal{A}_2^{(ew)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^*\, \frac{2}{3}\, e^2\, f_\pi^3\, g_8 \left[ g_{ew} \left( 1 + \Delta_L \mathcal{A}_2^{(ew)} \right) + \Delta_C \mathcal{A}_2^{(ew)} \right] . \tag{2.1}$$

These formulae contain the chiral one–loop corrections $\Delta_L \mathcal{A}_I^{(R)}$, and local contributions $\Delta_C \mathcal{A}_I^{(R)}$ from $\mathcal{O}(p^4)$ $\chi$PT counterterms.

It is convenient to rewrite these amplitudes in the form $\mathcal{A}_I^{(R)} = \mathcal{A}_I^{(R)\infty} \times \mathcal{C}_I^{(R)}$, where $\mathcal{A}_I^{(R)\infty}$ is the contribution at leading order in the large–$N_c$ expansion while the factors $\mathcal{C}_I^{(R)}$ represent the next–to–leading order (NLO) correction in the same expansion. The chiral loop contributions are NLO corrections in $1/N_c$. In order to determine $\mathcal{A}_I^{(R)\infty}$ one needs only to match properly $\chi PT$ with the effective short distance Lagrangian in eq. (1.3) and so determine the $\chi PT$ couplings. As an example we have (a more complete list can be found in ref. [1]):

$$g_8^\infty \left[ 1 + \Delta_C \mathcal{A}_0^{(8)} \right]^\infty =$$
$$\left\{ -\frac{2}{5} C_1(\mu) + \frac{3}{5} C_2(\mu) + C_4(\mu) - 16 L_5 C_6(\mu) \left[ \frac{M_K^2}{(m_s + m_q)(\mu) f_\pi} \right]^2 \right\} f_0^{K\pi}(M_\pi^2), \quad (2.2)$$

$$e^2 g_8^\infty \left[ g_{ew} + \Delta_C \mathcal{A}_2^{(ew)} \right]^\infty = -3 C_8(\mu) \left[ \frac{M_K^2}{(m_s + m_q)(\mu) f_\pi} \right]^2 \left[ 1 + \frac{4L_5}{f_\pi^2} M_\pi^2 \right]$$
$$+ \frac{3}{2} \left[ C_7 - C_9 - C_{10} \right](\mu) \frac{M_K^2 - M_\pi^2}{f_\pi^2} f_0^{K\pi}(M_\pi^2), \quad (2.3)$$

where $f_0^{K\pi}(M_\pi^2) \approx 1 + 4L_5 M_\pi^2 / f_\pi^2$ is the $K\pi$ scalar form factor at the pion mass scale, $L_5$ is a coupling of the strong $\mathcal{O}(p^4)$ scalar Lagrangian and $m_q \equiv m_u = m_d$. In the limit $N_c \to \infty$, $L_5^\infty = (1/4) f_\pi^2 (f_K/f_\pi - 1)/(M_K^2 - M_\pi^2) \approx 2.1 \cdot 10^{-3}$ and $f_0^{K\pi}(M_\pi^2) \approx 1.02$.

These results are equivalent to the standard large–$N_C$ evaluation of the usual bag parameters $B_i$. In particular, for $\varepsilon'/\varepsilon$, where only the imaginary part of the $g_i$ couplings matter [i.e. Im$(C_i)$], the leading order large–$N_c$ estimate amounts to $B_8^{(3/2)} \approx B_6^{(1/2)} = 1$. Therefore, up to minor variations of some input parameters, the corresponding $\varepsilon'/\varepsilon$ prediction, obtained at lowest order in both the $1/N_C$ and $\chi$PT expansions, reproduces the published results of the Munich [7] and Rome [8] groups. Thus at this order there is a large numerical cancellation between the $I = 0$ and $I = 2$ contributions, leading to an accidentally small value of $\varepsilon'/\varepsilon$.

Notice that the strong phase shifts are induced by chiral loops and, thus, they are exactly zero at this leading order approximation.

The large–$N_C$ limit has been only applied to the matching between the 3–flavor quark theory and $\chi$PT. The evolution from the electroweak scale down to $\mu < m_c$ has to be done without any unnecessary expansion in powers of $1/N_C$; otherwise, one would miss large corrections of the form $\frac{1}{N_C} \ln(M/m)$, with $M \gg m$ two widely separated scales [28]. Thus, the Wilson coefficients contain the full $\mu$ dependence.

At large–$N_c$ the operators $Q_i$ ($i \neq 6, 8$) factorize into products of left– and right–handed vector currents, which are renormalization–invariant quantities. The matrix element of each single current represents a physical observable which can be directly measured; its $\chi$PT realization just provides a low–energy expansion in powers of masses and momenta. Thus, the large–$N_C$ factorization of these operators does not generate any scale dependence. Since the anomalous dimensions of $Q_i$ ($i \neq 6, 8$) vanish when $N_C \to \infty$ [28], a very important

ingredient is lost in this limit [29]. To achieve a reliable expansion in powers of $1/N_C$, one needs to go to the next order where this physics is captured [29, 30]. This is the reason why the study of the $\Delta I = 1/2$ rule has proved to be so difficult. Fortunately, these operators are numerically suppressed in the $\varepsilon'/\varepsilon$ prediction.

The only anomalous dimension components which survive when $N_C \to \infty$ are the ones corresponding to $Q_6$ and $Q_8$ [28, 31]. One can then expect that the matrix elements of these two operators are well approximated by this limit [29, 30, 32]. These operators factorize into color–singlet scalar and pseudoscalar currents, which are $\mu$ dependent. This generates the factors $\langle \bar{q}q \rangle^{(2)}(\mu) \approx -M_K^2 f_\pi^2/(m_s + m_q)(\mu)$ which exactly cancel the $\mu$ dependence of $C_{6,8}(\mu)$ at large–$N_C$ [28, 29, 30, 31, 32, 33]. It remains a dependence at next-to-leading order. While the real part of $g_8$ gets its main contribution from $C_2$, $\mathrm{Im}(g_8)$ and $\mathrm{Im}(g_8\, g_{ew})$ are governed by $C_6$ and $C_8$, respectively. Thus, the analyses of the CP–conserving and CP–violating amplitudes are very different. There are large $1/N_C$ corrections to $\mathrm{Re}(g_i)$ [29, 30, 32], which are needed to understand the observed enhancement of the $(8_L, 1_R)$ coupling. On the contrary, the large–$N_C$ limit can be expected to give a good estimate of $\mathrm{Im}(g_i)$.

## 3. Chiral loop corrections

The large–$N_c$ amplitudes in eq. (2.3) do not contain any strong phases $\delta_0^I$. Those phases originate in the final rescattering of the two pions and, therefore, are generated by chiral loops which are of higher order in the $1/N_C$ expansion. Since the strong phases are quite large, specially in the isospin–zero case, one should expect large higher–order unitarity corrections. The multiplicatively correction factors $\mathcal{C}_I^{(R)}$ contain the chiral loop contributions we are interested in. At the one loop, they take the following numerical values ($\mathcal{C}_I^{(R)} \approx 1 + \Delta_L \mathcal{A}_I^{(R)}$; see ref. [1] for a complete list):

$$\mathcal{C}_0^{(8)} = 1.27 \pm 0.05 + 0.46\,i\,, \quad \mathcal{C}_2^{(27)} = 0.96 \pm 0.05 - 0.20\,i\,, \quad \mathcal{C}_2^{(ew)} = 0.50 \pm 0.24 - 0.20\,i\,. \tag{3.1}$$

The central values have been evaluated at the chiral renormalization scale $\nu = M_\rho$. To estimate the corresponding uncertainties we have allowed the scale $\nu$ to vary between 0.6 and 1 GeV. The scale dependence is only present in the dispersive contributions and should cancel with the corresponding $\nu$ dependence of the local $\chi$PT counterterms. However, this dependence is next-to-leading in $1/N_C$ and, therefore, is not included in our large–$N_C$ estimate of the $\mathcal{O}(p^4)$ and $\mathcal{O}(e^2 p^2)$ chiral couplings. The $\nu$ dependence of the chiral loops would be cancelled by the unknown $1/N_C$–suppressed corrections $\Delta_C \mathcal{A}_I^{(R)}(\nu) - \Delta_C \mathcal{A}_I^{(R)\infty}$, that we are neglecting in the factors $\mathcal{C}_I^{(R)}$. The numerical sensitivity of our results to the scale $\nu$ gives then a good estimate of those missing contributions.

The numerical corrections to the 27–plet amplitudes do not have much phenomenological interest for CP–violating observables, because $\mathrm{Im}(g_{27}) = 0$. Remember that the CP–conserving amplitudes $\mathrm{Re}(A_I)$ are set to their experimentally determined values. What is relevant for the $\varepsilon'/\varepsilon$ prediction is the 35% enhancement of the isoscalar octet amplitude $\mathrm{Im}[A_0^{(8)}]$ and the 46% reduction of $\mathrm{Im}[A_2^{(ew)}]$. These destroy the accidental lowest–order

cancellation between the $I = 0$ and $I = 2$ contributions, generating a sizeable enhancement of $\varepsilon'/\varepsilon$.

A complete $\mathcal{O}(p^4)$ calculation [18, 24] of the isospin–breaking parameter $\Omega_{IB}$ is not yet available. The value 0.16 quoted in eq. (1.8) only accounts for the contribution from $\pi^0$–$\eta$ mixing [18] and should be corrected by the effect of chiral loops. Since $|\mathcal{C}_2^{(27)}| \approx 0.98 \pm 0.05$, one does not expect any large correction of $\mathrm{Im}(A_2)_{IB}$, while we know that $\mathrm{Im}[A_0^{(8)}]$ gets enhanced by a factor 1.35. Taking this into account, one gets the corrected value $\Omega_{IB} \approx \Omega_{IB}^{\pi^0\eta} \left| \mathcal{C}_2^{(27)}/\mathcal{C}_0^{(8)} \right| = 0.12 \pm 0.05$, where the quoted error is an educated theoretical guess. This value agrees with the result $\Omega_{IB} = 0.08 \pm 0.05 \pm 0.01$, obtained in ref. [34] by using three different models [9, 27, 30, 35, 36, 37] to estimate the relevant $\mathcal{O}(p^4)$ chiral couplings.

## 4. FSI at higher orders

Given the large size of the one-loop contributions, one should worry about higher–order chiral corrections.

The large one-loop FSI correction to the isoscalar amplitudes is generated by large infrared chiral logarithms involving the light pion mass [2]. These logarithms are universal, i.e. their contribution depends exclusively on the quantum numbers of the two pions in the final state [2]. As a result, they give the same correction to all isoscalar amplitudes. Identical logarithmic contributions appear in the scalar pion form factor [20], where they completely dominate the $\mathcal{O}(p^4)$ $\chi$PT correction.

Using analyticity and unitarity constraints [38], these logarithms can be exponentiated to all orders in the chiral expansion [2]. The result can be written as: $\mathcal{C}_I^{(R)} \equiv \mathcal{C}_I^{(R)}(M_K^2) = \Omega_I(M_K^2, s_0)\, \mathcal{C}_I^{(R)}(s_0)$. The Omnès [38, 39, 40] exponential

$$\Omega_I(s, s_0) \equiv e^{i\delta_0^I(s)}\, \Re_I(s, s_0) = \exp\left\{ \frac{(s - s_0)}{\pi} \int \frac{dz}{(z - s_0)} \frac{\delta_0^I(z)}{(z - s - i\epsilon)} \right\} \qquad (4.1)$$

provides an evolution of $\mathcal{C}_I^{(R)}(s)$ from an arbitrary low–energy point $s_0$ to $s \equiv (p_{\pi_1} + p_{\pi_2})^2 = M_K^2$. The physical amplitudes are of course independent of the subtraction point $s_0$. Intuitively, what the Omnès solution does is to correct a local weak $K \to \pi\pi$ transition with an infinite chain of pion–loop bubbles, incorporating the strong $\pi\pi \to \pi\pi$ rescattering to all orders in $\chi$PT. The Omnès exponential only sums a particular type of higher–order Feynman diagrams, related to FSI. Nevertheless, it allows us to perform a reliable estimate of higher–order effects because it does sum the most important corrections. Moreover, the Omnès exponential enforces the decay amplitudes to have the right physical phases.

The Omnès resummation of chiral logarithms is uniquely determined up to a polynomial (in $s$) ambiguity [2, 38, 41], which has been solved with the large–$N_C$ amplitude $\mathcal{A}_I^{(R)\infty}$. The exponential only sums the elastic rescattering of the final two pions, which is responsible for the phase shift. Since the kaon mass is smaller than the inelastic threshold, the virtual loop corrections from other intermediate states ($K \to K\pi, K\eta, \eta\eta, K\bar{K} \to \pi\pi$) can be safely estimated at the one loop level; they are included in $\mathcal{C}_I^{(R)}(s_0)$.

Taking the chiral prediction for $\delta_0^I(z)$ and expanding $\Omega_I(M_K^2, s_0)$ to $\mathcal{O}(p^2)$, one should reproduce the one-loop $\chi$PT result. This determines the factor $\mathcal{C}_I^{(R)}(s_0)$ to $\mathcal{O}(p^4)$ in the

chiral expansion. It remains a local ambiguity at higher orders [2, 38, 41]. To estimate the remaining sensitivity to those higher order corrections, we have changed the subtraction point between $s_0 = 0$ and $s_0 = 3M_\pi^2$ and have included the resulting fluctuations in the final uncertainties. At $\nu = M_\rho$, we get the following values for the resummed loop corrections $(\left|\mathcal{C}_I^{(R)}\right| = \Re_I(M_K^2, s_0)\,\mathcal{C}_I^{(R)}(s_0))$:

$$\left|\mathcal{C}_0^{(8)}\right| = 1.31 \pm 0.06 \;, \quad \left|\mathcal{C}_2^{(27)}\right| = 1.05 \pm 0.05 \;, \quad \left|\mathcal{C}_2^{(ew)}\right| = 0.62 \pm 0.05 \;. \tag{4.2}$$

These results agree within errors with the one-loop chiral calculation of the moduli of the isospin amplitudes, indicating a good convergence of the chiral expansion.

## 5. Final results

The infrared effect of chiral loops generates an important enhancement of the isoscalar $K \to \pi\pi$ amplitude. This effect gets amplified in the prediction of $\varepsilon'/\varepsilon$, because at lowest order (in both $1/N_C$ and the chiral expansion) there is an accidental numerical cancellation between the $I = 0$ and $I = 2$ contributions. Since the chiral loop corrections destroy this cancellation, the final result for $\varepsilon'/\varepsilon$ is dominated by the isoscalar amplitude. Thus, the Standard Model prediction for $\varepsilon'/\varepsilon$ is finally governed by the matrix element of the gluonic penguin operator $Q_6$.

A detailed numerical analysis has been provided in ref. [1]. The short–distance Wilson coefficients have been evaluated at the scale $\mu = 1$ GeV. Their associated uncertainties have been estimated through the sensitivity to changes of $\mu$ in the range $M_\rho < \mu < m_c$ and to the choice of $\gamma_5$ scheme. Since the most important $\alpha_s$ corrections appear at the low–energy scale $\mu$, the strong coupling has been fixed at the $\tau$ mass, where it is known [42] with about a few percent level of accuracy: $\alpha_s(m_\tau) = 0.345 \pm 0.020$. The values of $\alpha_s$ at the other needed scales can be deduced through the standard renormalization group evolution.

Taking the experimental value of $\varepsilon$, the CP–violating ratio $\varepsilon'/\varepsilon$ is proportional to the CKM factor $\mathrm{Im}(V_{ts}^* V_{td}) = (1.2 \pm 0.2) \cdot 10^{-4}$ [43]. This number is sensitive to the input values of several non-perturbative hadronic parameters adopted in the usual unitarity triangle analysis; thus, it is subject to large theoretical uncertainties which are difficult to quantify [44]. Using instead the theoretical prediction of $\varepsilon$, this CKM factor drops out from the ratio $\varepsilon'/\varepsilon$; the sensitivity to hadronic inputs is then reduced to the explicit remaining dependence on the $\Delta S = 2$ scale–invariant bag parameter $\hat{B}_K$. In the large–$N_C$ limit, $\hat{B}_K = 3/4$. We have performed the two types of numerical analysis, obtaining consistent results. This allows us to better estimate the theoretical uncertainties, since the two analyses have different sensitivity to hadronic inputs.

The final result quoted in ref. [1] is:

$$\mathrm{Re}\left(\varepsilon'/\varepsilon\right) \;=\; \left(1.7 \pm 0.2\,^{+0.8}_{-0.5} \pm 0.5\right) \cdot 10^{-3} \;=\; (1.7 \pm 0.9) \cdot 10^{-3} \;. \tag{5.1}$$

The first error comes from the short–distance evaluation of Wilson coefficients and the choice of the low–energy matching scale $\mu$. The uncertainty coming from varying the strange quark mass in the interval $(m_s + m_q)(1\,\mathrm{GeV}) = 156 \pm 25\,\mathrm{MeV}$ [45] is indicated

by the second error. The most critical step is the matching between the short– and long–distance descriptions. We have performed this matching at leading order in the $1/N_C$ expansion, where the result is known to $\mathcal{O}(p^4)$ and $\mathcal{O}(e^2 p^2)$ in $\chi$PT. This can be expected to provide a good approximation to the matrix elements of the leading $Q_6$ and $Q_8$ operators. Since all ultraviolet and infrared logarithms have been resummed, our educated guess for the theoretical uncertainty associated with $1/N_C$ corrections is $\sim 30\%$ (third error).

A better determination of the strange quark mass would allow to reduce the uncertainty to the 30% level. In order to get a more accurate prediction, it would be necessary to have a good analysis of next–to–leading $1/N_C$ corrections. This is a very difficult task, but progress in this direction can be expected in the next few years [9, 11, 30, 46, 47, 48].

To summarize, using a well defined computational scheme, it has been possible to pin down the value of $\varepsilon'/\varepsilon$ with an acceptable accuracy. Within the present uncertainties, the resulting Standard Model theoretical prediction (1.2) is in good agreement with the measured experimental value (1.1).

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