The Width Difference of B_d Mesons

Amol Dighe

Max-Planck-Institute for Physics, Föhringer Ring 6, D-80805 Munich, Germany

Tobias Hurth*

CERN, Theory Division, CH–1211 Geneva 23, Switzerland

Choong Sun Kim

Department of Physics and IPAP, Yonsei University, Seoul 120-749, Korea

Tadashi Yoshikawa

Department of Physics, University of North Carolina, Chapel Hill, NC 27599-3255, USA

ABSTRACT:
We estimate $\Delta\Gamma_d/\Gamma_d$, including $1/m_b$ contributions and part of the next-to-leading order QCD corrections. We find that adding the latter corrections decreases the value of $\Delta\Gamma_d/\Gamma_d$ computed at the leading order by a factor of almost 2. We also show that under certain conditions an upper bound on the value of $\Delta\Gamma_d/\Gamma_d$ in the presence of new physics can be derived. With the high statistics and accurate time resolution of the upcoming LHC experiment, the measurement of $\Delta\Gamma_d$ seems to be possible. This measurement would be important for an accurate measurement of $\sin(2\beta)$ at the LHC. In addition, we point out the possibility that the measurement of width difference leads to a clear signal for new physics.

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1. Introduction

The two mass eigenstates of the neutral B_d system have slightly different lifetimes. Within the standard model (SM), the difference in the decay widths, however, is CKM-suppressed with respect to that in the B_s system. A rough estimate leads to $\frac{\Delta\Gamma_d}{\Gamma_d} \sim \frac{\Delta\Gamma_s}{\Gamma_s} \cdot \lambda^2 \approx 0.5\%$, where $\lambda = 0.225$ is the sine of the Cabibbo angle, and we have taken $\Delta\Gamma_s/\Gamma_s \approx 15\%$ [1]

*Speaker.

(see also [2, 3]). Here $\Gamma_{d(s)} = (\Gamma_L + \Gamma_H)/2$ is the average decay width of the light and heavy $B_{d(s)}$ mesons (B_L and B_H respectively). We denote these decay widths by Γ_L, Γ_H respectively, and define $\Delta\Gamma_{d(s)} \equiv \Gamma_L - \Gamma_H$.

At the present accuracy of measurements, this lifetime difference $\Delta\Gamma_d$ can well be ignored. As a result, the measurement and the phenomenology of $\Delta\Gamma_d$ have been neglected so far, as compared with the lifetime difference in the B_s system for example. However, with the possibility of experiments with high time resolution and high statistics, such as at the LHC, this quantity is becoming more and more relevant.

Taking the effect of $\Delta\Gamma_d$ into account is important in two aspects. There is the interlinked nature of the accurate measurements of β and $\Delta\Gamma_d/\Gamma_d$ through the conventional gold-plated decay. In the future experiments that aim to measure β to an accuracy of 0.005 or better, the corrections due to $\Delta\Gamma_d$ will form an important part of the systematic error. On the other hand, the measurement of $\Delta\Gamma_d$ allows for the possibility to detect a clear signal for new physics beyond the SM.

It is known that, if $(\Gamma_{21})_s$ is unaffected by new physics, the value of $\Delta\Gamma_s$ in the B_s system is bounded from above by its value as calculated in the SM. In the B_d system, this statement does not strictly hold true. However, if $(\Gamma_{21})_d$ is unaffected by new physics and the unitarity of the 3×3 CKM matrix holds, an upper bound on the value of $\Delta\Gamma_d$ may then be found.

With the possibility of experiments with high time resolution and high statistics, it is worthwhile to have a look at this quantity and make a realistic estimate of the possibility of its measurement (see also [4]).

2. Measurability of $\Delta\Gamma_d$

At LHCb, the proper time resolution is expected to be as good as $\Delta\tau \approx 0.03$ ps. This indeed is a very small fraction of the B_d lifetime ($\tau_{B_d} \approx 1.5$ ps [5]), so the time resolution is not a limiting factor in the accuracy of the measurement, and the statistical error plays the dominant role. Taking into account the estimated number of B_d produced — for example the number of reconstructed $B_d \to J/\psi\, K_S$ events at the LHC is expected to be 5×10^5 ([6] table 3) — the measurement of the lifetime difference does not look too hard at first glance. One may infer that if the number of relevant events with the proper time of decay measured with the precision $\Delta\tau$ is N, then the value of $\Delta\Gamma_d/\Gamma_d$ is measured with an accuracy of $1/\sqrt{N}$. With a sufficiently large number of events N, it should be possible to reach the accuracy of 0.5% or better.

However, the time measurements of the decay of an untagged B_d to a single final state can only be sensitive to quadratic terms in $\Delta\Gamma_d/\Gamma_d$. This would imply that, for determining $\Delta\Gamma_d/\Gamma_d$ using only one final state, the accuracy of the measurement needs to be $(\Delta\Gamma_d/\Gamma_d)^2 \sim 10^{-5}$. In [4] we gave an explicit derivation of that general statement, pointing out the exact conditions under which the above statement is valid. Ways of getting around these conditions lead us to the decay modes that can provide measurements sensitive linearly to $\Delta\Gamma_d/\Gamma_d$. This discussion indicates the necessity of combining measurements from two different final states in order to be sensitive to a quantity that is linear in $\Delta\Gamma_d/\Gamma_d$.

A viable option, perhaps the most efficient among the ones considered in [4], is to compare the measurements of the untagged lifetimes of the semileptonic decay mode τ_{SL} and of the CP-specific decay modes $\tau_{CP_\pm}$. The ratio between the two lifetimes $\tau_{CP\pm}$ and τ_{SL} is

$$\frac{\tau_{SL}}{\tau_{CP\pm}} = 1 \pm \frac{\cos(2\beta)}{2} \frac{\Delta\Gamma_d}{\Gamma_d} + \mathcal{O}\left[(\Delta\Gamma_d/\Gamma_d)^2\right] \quad . \tag{2.1}$$

The measurement of these two lifetimes should be able to give us a value of $|\Delta\Gamma_d|$, since $|\cos(2\beta)|$ will already be known to a good accuracy by that time.

Since the CP-specific decay modes of B_d (e.g. $J/\psi K_{S(L)}, D^+D^-$) have smaller branching ratios than the semileptonic modes, and the semileptonic data sample may be enhanced by including the self-tagging decay modes (e.g. $D_s^{(*)+}D^{(*)-}$) which also have large branching ratios, we expect that the most useful combination will be the measurement of τ_{SL} through all self-tagging decays and that of τ_{CP_+} through the decay $B_d \to J/\psi K_S$. After 5 years of LHC running, we should have about 5×10^5 events of $J/\psi K_S$, whereas the number of semileptonic decays, at LHCb alone, that will be directly useful in the lifetime measurements is expected to be more than 10^6 per year, even with conservative estimates of efficiencies.

3. Estimation of $\Delta\Gamma_d$

In [4] we estimated $\Delta\Gamma_d/\Gamma_d$ including $1/m_b$ contributions and part of the next-to-leading order QCD corrections. We find that adding the latter corrections decreases the value of $\Delta\Gamma_d/\Gamma_d$ computed at the leading order by a factor of almost 2. The final result is

$$\Delta\Gamma_d/\Gamma_d = (2.6^{+1.2}_{-1.6}) \times 10^{-3} \, . \tag{3.1}$$

Using another expansion of the partial NLO QCD corrections proposed in [7], we get

$$\Delta\Gamma_d/\Gamma_d = (3.0^{+0.9}_{-1.4}) \times 10^{-3} \, , \tag{3.2}$$

where we have used the preliminary values for the bag factors from the JLQCD collaboration [8]. In the error estimation, the errors are the uncertainties on the values of the CKM parameters, of the bag parameters, of the mass of the b quark, and of the measured value of x_d. Further sources of error are the assumption of naive factorization made for the $1/m_b$ matrix elements, the scale dependence and the missing terms in the NLO contribution. Although the latter error is decreased in the second estimate by smallness of CKM factors, a complete NLO calculation is definitely desirable for the result to be more reliable.

4. Interlinked Nature of $\sin(2\beta)$ and $\Delta\Gamma_d$

The time-dependent CP asymmetry measured through the "gold-plated" mode $B_d \to J/\psi K_S$ is

$$\mathcal{A}_{CP} = \frac{\Gamma[\bar{B}_d(t) \to J/\psi K_S] - \Gamma[B_d(t) \to J/\psi K_S]}{\Gamma[\bar{B}_d(t) \to J/\psi K_S] + \Gamma[B_d(t) \to J/\psi K_S]} \approx \sin(\Delta m_d t)\sin(2\beta) \quad , \tag{4.1}$$

which is valid when the lifetime difference, the direct CP violation, and the mixing in the neutral K mesons are neglected. As the accuracy of this measurement increases, the corrections due to these factors will have to be taken into account. Keeping only linear terms in small quantities, we obtain

$$\mathcal{A}_{CP} = \sin(\Delta mt)\sin(2\beta)\left[1 - \sinh\left(\frac{\Delta\Gamma_d t}{2}\right)\cos(2\beta)\right] \tag{4.2}$$

$$+2\mathrm{Re}(\bar{\epsilon})\left[-1 + \sin^2(2\beta)\sin^2(\Delta mt) - \cos(\Delta mt)\right] \tag{4.3}$$

$$+2\mathrm{Im}(\bar{\epsilon})\cos(2\beta)\sin(\Delta mt) \quad . \tag{4.4}$$

The first term in (4.2) represents the standard approximation used (4.1) and the correction due to the lifetime difference $\Delta\Gamma_d$. The rest of the terms [(4.3) and (4.4)] are corrections due to the CP violation in B–$\bar{B}$ and K–$\bar{K}$ mixings. Note that $\bar{\epsilon}$ is an effective parameter that absorbs several small uncertainties and equals a few $\times 10^{-3}$ (see [4]).

The BaBar collaboration gives the bound on the coefficient of $\cos(\Delta mt)$ in (4.3), while neglecting the other correction terms [11]. When the measurements are accurate enough to measure the $\cos(\Delta mt)$ term, the complete expression for $\mathcal{A}_{CP}$ above (4.2–4.4) needs to be used. In the future experiments that aim to measure β to an accuracy of 0.005 [6]. The corrections due to $\bar{\epsilon}$ and $\Delta\Gamma_d$ will form a major part of the systematic error, which can be taken care of by a simultaneous fit to $\sin(2\beta), \Delta\Gamma_d$ and $\bar{\epsilon}$.

5. New Physics

The calculations of the lifetime difference in B_d and in the B_s system (as in [1]) run along similar lines. However, there are some subtle differences involved, due to the values of the different CKM elements involved, which have significant consequences.

In particular, whereas the upper bound on the value of $\Delta\Gamma_s$ (including the effects of new physics) is the value of $\Delta\Gamma_s(\mathrm{SM})$ [9], the upper bound on $\Delta\Gamma_d$ involves a multiplicative factor in addition to $\Delta\Gamma_d(\mathrm{SM})$: using the definitions $\Theta_q \equiv \mathrm{Arg}(\Gamma_{21})_q, \Phi_q \equiv \mathrm{Arg}(M_{21})_q$, where $q \in \{d, s\}$, we can write

$$\Delta\Gamma_q = -2|\Gamma_{21}|_q \cos(\Theta_q - \Phi_q) \quad . \tag{5.1}$$

Since the contribution to Γ_{21} comes only from tree diagrams, we expect the effect of new physics on this quantity to be very small. We therefore take $|\Gamma_{21}|_q$ and Θ_q to be unaffected by new physics. On the other hand, the mixing phase Φ_q appears from loop diagrams and can therefore be very sensitive to new physics. Based on these assumptions, one derives an upper bound on new physics within the B_s system [9]:

$$\Delta\Gamma_s \leq \frac{\Delta\Gamma_s(\mathrm{SM})}{\cos(2\Delta\gamma)} \approx \Delta\Gamma_s(\mathrm{SM}) \quad , \tag{5.2}$$

with $2\Delta\gamma \approx -0.03$. Thus, the value of $\Delta\Gamma_s$ can only decrease in the presence of new physics.

In the B_d system, an upper bound for $\Delta\Gamma_d$, based on the additional assumption of three-generation unitarity, can be derived:

$$\Delta\Gamma_d \leq \frac{\Delta\Gamma_d(\mathrm{SM})}{\cos[\mathrm{Arg}(1+\delta f)]} \quad . \tag{5.3}$$

We can calculate the bound (5.3) in terms of the extent of the higher order NLO corrections. In [4], we got $|\mathrm{Arg}(1+\delta f)| < 0.6$, so that we have the bound $\Delta\Gamma_d < 1.2\ \Delta\Gamma_d(\mathrm{SM})$. A complete NLO calculation will be able to give a stronger bound.

We have seen that the ratio of two effective lifetimes can enable us to measure the quantity $\Delta\Gamma_{obs(d)} \equiv \cos(2\beta)\Delta\Gamma_d/\Gamma_d$. In the presence of new physics, this quantity is in fact (see eq. (5.1)) $\Delta\Gamma_{obs(d)} = -2(|\Gamma_{21}|_d/\Gamma_d)\cos(\Phi_d)\cos(\Theta_d - \Phi_d)$. In SM, we get

$$\Delta\Gamma_{obs(d)}(\mathrm{SM}) = 2(|\Gamma_{21}|_d/\Gamma_d)\cos(2\beta)\cos[\mathrm{Arg}(1+\delta f)] \quad . \tag{5.4}$$

If $|\delta f| < 1.0$, we have $\cos[\mathrm{Arg}(1+\delta f)] > 0$ (in fact, from the fit in [10] and our error estimates, we have $\cos[\mathrm{Arg}(1+\delta f)] > 0.8$). Then $\Delta\Gamma_{obs(d)}(\mathrm{SM})$ is predicted to be positive. New physics is not expected to affect Θ_d, but it may affect Φ_d in such a way as to make the combination $\cos(\Phi_d)\cos(\Theta_d - \Phi_d)$ change sign. A negative sign of $\Delta\Gamma_{obs(d)}$ would therefore be a clear signal of such new physics.

It is well known, that the B_d–$\bar{B}_d$ mixing phase Φ_d is efficiently measured through the decay modes $J/\psi K_s$ and $J/\psi K_L$. If we take the new physics effects into account, the time-dependent asymmetry is $\mathcal{A}_{CP} = -\sin(\Delta M_d t)\sin(\Phi_d)$; in the SM, we have $\Phi_d = -2\beta$. The measurement of $\sin(\Phi_d)$ still allows for a discrete ambiguity $\Phi_d \leftrightarrow \pi - \Phi_d$. It is clear that, if Θ_d can be determined independently of the mixing in the B_d system, then measuring $\Delta\Gamma_{obs(d)}$, which is proportional to $\cos(\Phi_d)\cos(\Theta_d - \Phi_d)$, resolves the discrete ambiguity in principle. We note that these features are unique to the B_d system.

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