

Precision Determination of the Muon $g - 2$ and $\alpha(\text{QED})$ on the Z

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ABSTRACT: We present a summary of the results of two recent precise calculations of the muon anomalous magnetic moment and the electromagnetic coupling on the Z .

1. Introduction

We present a summary of the results of two recent precise calculations of the muon anomalous magnetic moment ($g_\mu - 2$) [1] and the electromagnetic coupling on the Z ($\bar{\alpha}_{\text{Q.E.D.}}(M_Z^2)$) [2]. The main sources of uncertainty are underlined.

2. Calculation of the muon $g_\mu - 2$

The appearance of a new, very precise measurement of the muon magnetic moment [3] has triggered the interest in theoretical calculations of this quantity. Particularly, because the experimental figure (we give the result for the anomaly, averaged with older determinations [4])

$$a_\mu(\text{exp.}) = \frac{g_\mu - 2}{2} = (116\,592\,030 \pm 150) \times 10^{-11} \quad (2.1)$$

lies slightly above theoretical evaluations based on the standard model, as much as 2.6σ in some cases.

It should be noted that all modern theoretical determinations [5, 6, 7] are compatible among themselves within errors (of order 100×10^{-11}) and that, with few exceptions, they are also compatible with the experimental result, at the level of 1.5σ or less. Because of this, it is our feeling that a new, complete evaluation would be welcome since, in fact, there exists as yet no calculation that takes fully into account all theoretical constraints and all

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the new experimental data. These data allow an improved evaluation of the low energy hadronic contributions to a_μ , both directly from e^+e^- annihilations (in the ρ region [8] and around the ω and ϕ resonances [9]) and, indirectly, from τ decays [10] and, also indirectly, from measurements of the pion form factor in the spacelike region [11]. We have also used perturbative QCD in the region $s \geq 2 \text{ GeV}^2$ (away from quark thresholds). In particular, the recent LEP $\tau \rightarrow \nu_\tau + \text{hadrons}$, and BES $e^+e^- \rightarrow \text{hadrons}$ data justify the QCD result for $2 \text{ GeV}^2 \leq s \leq 3^2 \text{ GeV}^2$. Moreover, the BES [12] data, covering e^+e^- annihilations in the vicinity of $\bar{c}c$ threshold, permit a reliable evaluation of the corresponding hadronic pieces. Last but not least, the next order radiative corrections have been taken into account.

In the next sections we concentrate in the two most delicate pieces, from the point of view of the final uncertainty: the pion form factor and the hadronic light-by-light scattering contributions.

2.1 The pion form factor

To obtain $F_\pi(s)$ we fit the recent Novosibirsk data [8] on $e^+e^- \rightarrow \pi^+\pi^-$ and the tau decay data of Aleph and Opal [10]. We take into account, at least partially, isospin breaking effects by allowing different masses and widths for the ρ^0, ρ^+ resonances. Moreover, and to get a good grip in the low energy region where data are inexistent or very poor, we also fit $F_\pi(s)$ at spacelike s [11]. This is possible in our approach because we use an expression for F_π that takes fully into account its analyticity properties. To be precise, we use the Omnès-Muskhelishvili method. We write

$$F_\pi(s) = G(s)J(s).$$

Here J is expressed in terms of the P-wave $\pi\pi$ phase shift, δ_1^1 , as

$$J(s) = e^{1-\delta_1^1(s_0)/\pi} \left(1 - \frac{s}{s_0}\right)^{[1-\delta_1^1(s_0)/\pi]s_0/s} \left(1 - \frac{s}{s_0}\right)^{-1} \exp \left\{ \frac{s}{\pi} \int_{4m_\pi^2}^{s_0} ds' \frac{\delta_1^1(s')}{s'(s'-s)} \right\}.$$

s_0 is the energy at which inelasticity starts becoming important (in practice, above the percent level); we will take $s_0 = 1.1 \text{ GeV}^2$ in actual calculations.

Because of the equality of the phase of $J(s)$ and the phase of $F_\pi(s)$ below $s = s_0$, it follows that $G(s)$ will be an analytic function also for $4m_\pi^2 \leq s \leq s_0$, so in the whole s plane except in a cut from $s = s_0$ to $+\infty$. If we now make the conformal transformation

$$z = \frac{\frac{1}{2}\sqrt{s_0} - \sqrt{s_0 - s}}{\frac{1}{2}\sqrt{s_0} + \sqrt{s_0 - s}}$$

then, as a function of z , G will be analytic in the unit disk and we can thus write a convergent Taylor series for it. Incorporating the condition $G(0) = 1$, that follows from $F_\pi(0) = 1$, and undoing the transformation, we have

$$G(s) = 1 + c_1 \left[\frac{\frac{1}{2}\sqrt{s_0} - \sqrt{s_0 - s}}{\frac{1}{2}\sqrt{s_0} + \sqrt{s_0 - s}} + \frac{1}{3} \right] + c_2 \left[\left(\frac{\frac{1}{2}\sqrt{s_0} - \sqrt{s_0 - s}}{\frac{1}{2}\sqrt{s_0} + \sqrt{s_0 - s}} \right)^2 - \frac{1}{9} \right] + \dots,$$

c_1, c_2, \dots free parameters. Actually, only two terms will be necessary to fit the data.

To obtain J , we use the effective range theory to write

$$\cot \delta_1^1(s) = \frac{s^{1/2}}{2k^3}(m_\rho^2 - s)\hat{\psi}(s), \quad k = \frac{\sqrt{s - 4m_\pi^2}}{2},$$

where we have extracted the zero corresponding to the rho resonance. The effective range function $\hat{\psi}(s)$ is analytic in the full s plane except for a cut for $[-\infty, 0]$ and the inelastic cut $[s_0, +\infty]$. We can profit from this analyticity by making again a conformal transformation into the unit circle, which is now given by

$$w = \frac{\sqrt{s} - \sqrt{s_0 - s}}{\sqrt{s} + \sqrt{s_0 - s}}.$$

We expand $\hat{\psi}$ in a convergent series of powers of w ; undoing the transformation we have

$$\delta_1^1(s) = \text{Arc cot} \left\{ \frac{s^{1/2}}{2k^3}(m_\rho^2 - s) \left[b_0 + b_1 \frac{\sqrt{s} - \sqrt{s_0 - s}}{\sqrt{s} + \sqrt{s_0 - s}} + \dots \right] \right\}.$$

For the actual fits, only b_0 , b_1 , and m_ρ are needed as parameters.

We give results both using only F_π from e^+e^- annihilations, or involving also the decay $\tau^+ \rightarrow \bar{\nu}_\tau \pi^+ \pi^0$, which last we consider to be our best estimates:

$$10^{11} \times a_\mu(2\pi; t \leq 0.8 \text{ GeV}^2) = \begin{cases} 4774 \pm 31, & (\text{TY1} : e^+e^- + \tau + \text{spacelike}). \\ 4754 \pm 55, & (\text{TY2} : e^+e^- + \text{spacelike}). \end{cases} \quad (2.2)$$

As a byproduct of the calculation we can also give precise values for the ρ^0 , ρ^+ masses and widths or the pion electromagnetic radius; see [1].

2.2 Hadronic light-by-light contributions

A contribution to a_μ in a class by itself is the hadronic light-by-light scattering. It can be evaluated only using models. One can make a chiral model calculation, in the Nambu–Jona-Lasinio version or the chiral perturbation theory variety, with a cut-off, or one can use a constituent quark model. The result depends on the cut-off (for the chiral calculation) or on the constituent mass chosen for the quarks. What is worse, for the chiral model, a recent calculation [13] has challenged even its sign. The variation from one model to the other is too large for comfort. We take as a typical value for the chiral calculation [14],

$$10^{11} \times a_\mu(\text{Hadronic light-by-light}) = -86 \pm 25 \quad (\text{Chiral calculation; BPP, HKS}).$$

In the calculation by [13] one simply reverses the sign.

For the constituent quark model we use the results of Laporta and Remiddi [15]. The contribution to a_μ of light-by-light scattering, with a loop with a fermion of charge Q_i , and mass m_i larger than the muon mass, is

$$10^{11} \times a_\mu(\text{Hadronic light-by-light}) = +92 \pm 20 \quad (\text{Quark const. model})$$

and the error is estimated by varying $m_{u,d}$ by 10%. This is essentially the same result following the chiral model calculation by [13].

Besides the sign problem, the variations are unfortunate; one expects the chiral calculation to be valid for small values of the virtual photon momenta, and the constituent model to hold for large values of the same. Thus, almost half of the contribution to $a_\mu(\text{Hadronic light-by-light})$ in the chiral calculation comes from a region of momenta above 0.5 GeV, where the chiral perturbation theory starts to fail, while for this range of energies, and at least for the imaginary part of (diagonal) light-by-light scattering, the quark model reproduces reasonably well the experimental data. Because of the uncertainties, we will give results with the extreme choices.

2.3 Results

Our results for the hadronic part of the anomaly depend on which model one believes for the hadronic light-by-light contribution, discussed in the previous section. So we write,

$$10^{11} \times a_\mu(\text{Hadronic}) = \begin{cases} 6993 \pm 69 & (\text{TY1; Q.c.m.}) \\ 6815 \pm 71 & (\text{TY1; Ch.m.(BPP, HKS)}) \\ 6973 \pm 99 & (\text{TY2; Q.c.m.}) \\ 6795 \pm 100 & (\text{TY2; Ch.m.(BPP, HKS)}) \end{cases} \quad (2.3)$$

Note that in $a_\mu(\text{Hadronic})$ we include all hadronic contributions, $O(\alpha^3)$ as well as $O(\alpha^2)$. In eqn. (2.3) ‘‘Q.c.m.’’ means that the light-by-light hadronic contribution was calculated with the quark constituent model, and ‘‘Ch.m.’’ that a chiral model was used. The errors include the statistical errors, as well as the estimated systematic and theoretical ones. This is to be compared with the value deduced from eqn. (2.1) and electroweak corrections

$$10^{11} \times a_\mu^{\text{exp.}}(\text{Hadronic}) = 7174 \pm 150,$$

from which eqn. (2.3) differs by something between 1.1σ and 2.1σ . Our numbers are compared to other evaluations in Fig. 1. Whether these results should be interpreted as providing further experimental validation of the standard model, or one can consider them as ‘‘harbingers of new physics’’, we will leave for the reader to decide.

3. Calculation of $\bar{\alpha}_{\text{Q.E.D.}}$ on the Z

With a simple change of integration kernel the previous analysis can be extended to evaluate the hadronic contribution to the QED running coupling, $\bar{\alpha}_{\text{Q.E.D.}}(s)$, in particular for $s = M_Z^2$; an important quantity that enters into precision evaluations of electroweak observables.

By using a dispersion relation one can write this hadronic contribution at energy squared s as

$$\Delta_{\text{had}}\alpha(s) = -\frac{s\alpha}{3\pi} \int_{4m_\pi^2}^{\infty} dt \frac{R(t)}{t(t-s)} \quad (3.1)$$

with

$$R(t) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons}; t)}{\sigma^{(0)}(e^+e^- \rightarrow \mu^+\mu^-; t)}, \quad \sigma^{(0)}(e^+e^- \rightarrow \mu^+\mu^-; t) \equiv \frac{4\pi\alpha^2}{3t}$$

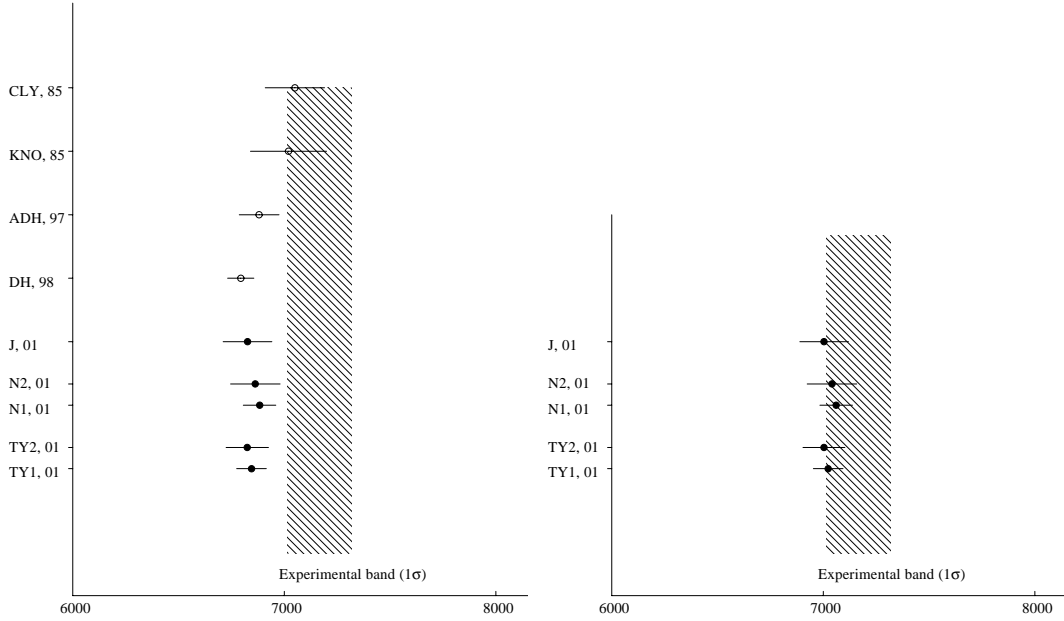


Figure 1: (Left) Theoretical results on $a_\mu(\text{Hadronic}) \times 10^{-11}$, and experiment. The chiral calculation result used for the hadronic light-by-light contribution. (Right) The same, but with the quark model result for the hadronic light-by-light contribution. Labels as in references [5, 6, 7]. TY1 and TY2 refer to the results in eqns. (2.2) and (2.3).

and the integral in eqn. (3.1) has to be understood as a principal part integral. This is similar to the Brodsky–de Rafael expression for the hadronic contribution to the muon magnetic moment anomaly,

$$a_\mu(\text{h.v.p.}) = \int_{4m_\pi^2}^{\infty} dt K(t)R(t).$$

Therefore, we can carry over all the work from the previous section with the simple replacement

$$K(t) \rightarrow -\frac{s\alpha}{3\pi} \frac{1}{t(t-s)}.$$

We find to next to leading order in α ,

$$10^5 \times \Delta_{\text{had}}\alpha(M_Z^2) = 2740 \pm 12$$

or, excluding the top quark contribution,

$$10^5 \times \Delta_{\text{had}}\alpha^{(5)}(M_Z^2) = 2747 \pm 12.$$

Other recent determinations can be found in [7, 16].

4. Conclusions

In conclusion, we have performed a detailed evaluation of the hadronic contributions to the muon $g_\mu - 2$ [1], and to the running electromagnetic coupling [2].

In the case of the muon $g_\mu - 2$ an important limitation of the attainable accuracy is the light-by-light scattering contribution; in particular, a third chiral model calculation is required to clear the sign controversy.

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