PROCEEDINGS

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Constraints on mSUGRA and SUSY particle production at future e^+e^- linear colliders

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Abstract: We perform a complete analysis of the supersymmetric particle spectrum in the Minimal Supergravity (mSUGRA) model. We show that present constraints on the Higgs boson and superparticle masses from collider searches and precision measurements still allow for large regions of the mSUGRA parameter space where some sparticles as well as the heavier Higgs particles, are light enough to be produced at the next generation of e^+e^- linear colliders. An important part of this parameter space remains even when we require that the density of the lightest neutralinos left over from the Big Bang falls in the range favored by current determinations of the Dark Matter density in the Universe.

1. Introduction

Although other viable Supersymmetric (SUSY) of the Standard Model (SM) exist, the Minimal Supergravity (mSUGRA) model has become the most frequently used benchmark scenario for supersymmetry, and has been widely used to analyze the expected SUSY particle spectrum and the properties of SUSY particles, and to compare the predictions to available and/or expected data from collider experiments. Several global or partial analyses of the present theoretical and experimental constraints on the mSUGRA model have been performed in the literature. In a recent paper [1], we have performed an independent analysis of the SUSY particle is this model, taking into account theoretical constraints and all available experimental information: searches for the MSSM Higgs bosons and SUSY particles at the LEP and Tevatron colliders [2], electroweak precision measurements [2], the radiative $b \to s\gamma$ decay, etc. Special attention was devoted to the implications of the measurement of the anomalous magnetic moment of the muon recently performed at Brookhaven [2], and to the $\sim 2\sigma$ evidence for a SM–like Higgs boson with a mass $M_{\text{Higgs}} \sim 115.6$ GeV seen by the LEP collaborations [2]. We have also discussed the implication of requiring thermal relic neutralinos to form the Dark Matter in the Universe

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[2]. We have then discussed prospects for producing SUSY particles and the heavier Higgs bosons of the MSSM at future high–energy e^+e^- linear colliders with c.m. energies around 800 GeV. This talk summarizes the main results of this analysis; for more details and for a complete set of references, we refer the reader to the original work.

2. The mSUGRA model and the calculation of the spectrum

We have performed our analysis in the constrained MSSM or mSUGRA model, where the MSSM soft breaking parameters obey a set of universal boundary conditions at the GUT scale, $M_{\rm GUT} \simeq 2.10^{16}$ GeV, so that the electroweak symmetry is broken radiatively. In this model, where the gauge couplings are unified at $M_{\rm GUT}$, one has only four continuous free parameters, and an unknown sign in addition to to the parameters of the SM:

$$\tan\beta \ , \ m_{1/2} \ , \ m_0 \ , \ A_0 \ , \ \ {\rm sign}(\mu).$$

where $\tan\beta$ is the ratio of the vevs of the MSSM Higgs fields $m_{1/2}, m_0$ and A_0, are respectively, the common soft–SUSY breaking gaugino mass, scalar mass and trilinear couplings at the GUT scale, and μ is the higgsino mass parameter, the absolute value of which is determined by the requirement of a proper electroweak symmetry breaking (EWSB). All the soft SUSY breaking parameters at the weak scale are then obtained through Renormalization Group Equations (RGE).

All results are based on the numerical FORTRAN code `SuSpect` version 2.0 [3], to which we refer for a more detailed description. The algorithm essentially includes:

– RGE of parameters between the low energy scale and the GUT scale. For the gauge and Yukawa couplings and gaugino masses, we use two–loop RGE. All the one–loop SUSY threshold effects are implemented in the RG evolution via step functions in the β functions for each particle threshold.

– Consistent implementation of EWSB. Loop corrections (with all SUSY and Higgs particles) to the effective potential are included using the tadpole method. The SUSY parameters are frozen at the EWSB scale. μ^2 and $B\mu$ are determined from the minimization of the potential at this scale. Since these parameters affect mass of some sparticles, this procedure has to be iterated until stability is reached and a consistent value of μ is obtained.

– Calculation of the physical (pole) masses of the Higgs bosons and the sparticles including all the important ingredients. For instance, we include the dominant radiative correction to the 3d generation fermion masses and to all SUSY particles masses. The Higgs sector is treated in the effective potential approach with RGE improved QCD corrections. In calculating the masses, the procedure is iterated at least twice until stability is reached, in order to take into account the (multi–scale) thresholds and the radiative corrections.

In the numerical analyses we fix the MSSM parameters $\tan\beta$ given at scale M_Z as well as A_0 and the sign of μ, and then perform a systematic scan over the high energy mSUGRA inputs m_0 and $m_{1/2}$. Given these boundary conditions, all the soft SUSY breaking parameters and couplings are evolved down to the EWSB scale, which we choose to be the geometric mean of the two top squark masses, $M_{\rm EWSB} = (m_{\tilde{t}_1} m_{\tilde{t}_2})^{1/2}$.

The program allowed us to fairly reliably delineate the regions of the mSUGRA parameter space which are still allowed by theoretical constraints [from a proper EWSB breaking, neutralino LSP, non–tachyonic Higgs and SUSY particles, etc..]

3. Constraints on the mSUGRA parameter space

(i) Lower bounds on SUSY particle masses: A wide range of searches for SUSY particles has been performed at LEP2 and at the Tevatron, resulting in limits on the masses of these particles. The most important ones are due to the negative search of charginos, sleptons and third generation squarks at LEP2 and squarks and gluinos at the Tevatron. We therefore impose the following bounds: $m_{\tilde\chi_1^+} \geq 104$ GeV$, m_{\tilde f} \geq 100$ GeV with $\tilde f = \tilde t_1, \tilde b_1, \tilde l^\pm, \tilde\nu$ and $m_{\tilde g} \geq 300$ GeV$, m_{\tilde q_{1,2}} \geq 260$ GeV with $\tilde q = \tilde u, \tilde d, \tilde s, \tilde d$.

(ii) Constraints from the Higgs boson masses: In the SM, a 95% CL lower bound has been set on the Higgs boson mass at LEP2, $M_{H^0} \geq 113.5$ GeV. In the MSSM, this bound is valid in the decoupling regime where the pseudoscalar A boson is very heavy. For small values of M_A, a combined exclusion limit of $M_h \sim M_A \geq 93.5$ GeV has been set. In the intermediate region an interpolation has to be made. We have also studied the implications of the 2.1σ evidence for a SM–like Higgs boson with a mass $M_H = 115.6$ GeV seen by the LEP collaborations. In view of the theoretical and experimental uncertainties, we interpreted this result as favoring the range: $113\,\text{GeV} \leq M_h \leq 117\,\text{GeV}$.

(iii) Constraints from electroweak precision observables: Loops of Higgs and SUSY particles can contribute to electroweak observables which have been precisely measured at LEP, SLC and the Tevatron. The dominant contributions, in particular M_W and the effective angle s_W^2, enter via a deviation from unity of the ρ parameter which measures the breaking of the custodial SU(2) symmetry. In the MSSM, the dominant contributions are due to the 3d generation $(\tilde t, \tilde b)$ and $(\tilde\tau, \tilde\nu)$ weak iso–doublets, which we have required these contributions to stay below the acceptable (2σ) level of $\Delta\rho(\tilde f) \leq 2.2 \cdot 10^{-3}$.

(iv) The $b \to s\gamma$ constraint: Another observable where SUSY particle contributions might be large is the radiative flavor changing decay $b \to s\gamma$, the branching ratio of which has been measured to be $\text{BR}(b \to s\gamma) = (3.37 \pm 0.37 \pm 0.34 \pm 0.24^{+0.35}_{-0.16} \pm 0.38) \cdot 10^{-4}$, including theoretical errors. In our analysis, we will use the most up–to–date determination in the MSSM of the $b \to s\gamma$ decay rate including NLO QCD corrections and allow the branching ratio to vary in the 2σ range: $2.0 \times 10^{-4} \leq \text{BR}(b \to s\gamma) \leq 5.0 \times 10^{-4}$.

(v) The contribution to the muon $g-2$: Recently, the Muon $(g-2)$ Collaboration has reported a new measurement of the anomalous moment of the muon: $(g_\mu - 2) \equiv a_\mu^{\text{exp}} = 11\,659\,202\,(14)(6)\,10^{-10}$, which differs from the predicted SM average value by 2.6σ. We interpret the discrepancy as being a SUSY contribution (chargino–sneutrino and neutralino–smuon loops) of $11 \cdot 10^{-10} \leq a_\mu^{\text{SUSY}} \leq 75 \cdot 10^{-10}$.

(vi) Cosmological constraints: We have analyzed the contribution of the χ_1^0 particles, which are the lightest SUSY particles, to the (normalized) overall matter density of the Universe $\Omega_{\tilde\chi_1^0} h^2$ ($h \sim 0.5$ is the Hubble constant). The χ_1^0, is neutral, weakly interacting, massive and absolutely stable since R-parity is conserved in mSUGRA, and is therefore a good candidate for the cold Dark Matter. Recent evidence suggests that $\Omega_{\tilde\chi_1^0} h^2 \simeq 0.2$ and we define $0.1(0.025) \leq \Omega_{\tilde\chi_1^0} h^2 \leq 0.3(0.5)$ as the (conservative) cosmologically favored region. The calculation of the relic density is made using standard assumptions and includes all the annihilation channels, with a proper treatment of the s–channel poles, as well as co–annihilation with gauginos, sleptons and top squarks.

The main outcome of this part our analysis can be summarized as follows:

– There are large areas of the $(m_{1/2}, m_0)$ parameter space which are still allowed by present experimental constraints. In particular, for large enough values of $\tan\beta$, the bound on the lightest h boson mass, $M_h \gtrsim 113$ GeV, does not place too severe constraints. If $\mu > 0$, which is favored by the $(g_\mu - 2)$ anomaly, the constraint from the radiative decay $b \to s\gamma$ are not severe even for large values of $\tan\beta$. In fact, if $A_0 = 0$ it is always superseded by the Higgs boson mass constraint, but for $A_0 = -1$ TeV the $b \to s\gamma$ constraint can be more severe. Precision electroweak measurements are easily accommodated.

– For $\tan\beta \gtrsim 10$ and small values of the coupling A_0, the requirement of 113 GeV $\lesssim M_h \lesssim 117$ GeV favors moderate values of the gaugino mass, $m_{1/2} \lesssim 500$ GeV, leading to relatively light chargino and neutralino states, $m_{\tilde{\chi}_1^\pm} \sim m_{\tilde{\chi}_2^0} \sim 2m_{\tilde{\chi}_1^0} \lesssim 400$ GeV. For large (and negative) values of A_0, which lead to a strong mixing in the stop sector, M_h in this range can be accommodated in large regions of the parameter space even for rather small $\tan\beta(\sim 5)$ values. In this case $\tilde{t}_1$ can be rather light, if the parameters m_0 and $m_{1/2}$ are not too high. The range of $m_{1/2}$ favored by the LEP Higgs evidence strongly depends on the exact value of M_t, calling for a more precise determination of this parameter.

– The $(g_\mu - 2)$ excess, which can be accommodated in the MSSM only if $\mu > 0$, typically gives a stronger upper bound on m_0 than the requirement $M_H = 115 \pm 2$ GeV. For $\tan\beta \sim 40$, m_0 and $m_{1/2}$ values below ~ 600 GeV [and slightly above ~ 300 GeV] are needed, implying again relatively light electroweak gaugino and slepton states. However, the value of this upper bound increases roughly proportional to $\tan\beta$, so that at $\tan\beta = 60$, m_0 as large as 1.0 (1.6) TeV can be accommodated at the 1 (2) σ level.

– For small and moderate $\tan\beta(\lesssim 40)$ the requirement that the density of the lightest neutralinos accounts for the Dark Matter density in the Universe is very constraining indeed. In this case most of the region where $\Omega_{\tilde{\chi}_1^0} h^2$ is "naturally" in the interesting range is excluded by the Higgs mass constraints, which requires SUSY breaking masses above those preferred by Dark Matter calculations. Only a small band in the region with a relatively light bino–like neutralino and relatively light $\tilde{\tau}$ survives. In addition, there are "exceptional" regions: a narrow strip in the $\tilde{\tau}_1 \tilde{\chi}_1^0$ co–annihilation region near the boundary where the $\tilde{\tau}_1$ slepton is the LSP, and a strip in the focus point region at large m_0 and small $m_{1/2}$ values where neutralinos and charginos are relatively light and have large higgsino components. Requiring in addition $M_h = 115 \pm 2$ GeV and a SUSY interpretation for the $(g_\mu - 2)$ anomaly removes most of these "exceptional" regions with acceptable relic density. On the other hand, for large values of $\tan\beta(\gtrsim 50)$, the area of the $(m_0, m_{1/2})$ parameter space favored by cosmology extends significantly due to the opening of the pseudoscalar A–boson pole. This allows to fit all the requirements [M_h, $(g_\mu - 2)$ and the DM constraint] in a somewhat larger area of the $(m_0, m_{1/2})$ parameter space.

– In spite of the strong constraints on mSUGRA obtained by taking seriously all the positive indications for SUSY it is still not possible to give tight limits on any one single parameter. We found overlap regions with $5 \leq \tan\beta \leq 60$, 0.1 TeV $\lesssim m_0 \lesssim$ 1.5 TeV, 160 GeV $\lesssim m_{1/2} \lesssim$ 550 GeV. Furthermore, allowing for a large negative A_0 plays an important role in extending the allowed region to smaller values of $\tan\beta$. In fact, the allowed $(m_{1/2}, m_0)$ region plane could be further extended by considering more A_0 choices.

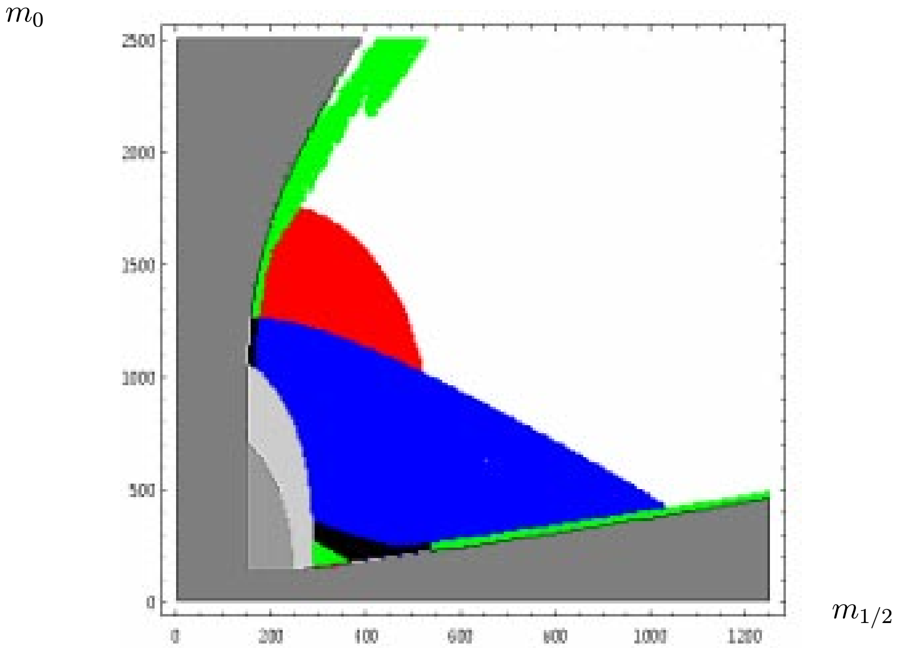

Figure 1: Constraints on the $(m_{1/2}, m_0)$ mSUGRA plane for $\tan\beta = 40$, $A_0 = 0$ and sign$(\mu) > 0$. The grey areas are those excluded by the requirement of EWSB and limits on SUSY particle masses (darker grey), BR$(b \to s\gamma)$ (medium grey) and $M_h > 113$ GeV (light and dark grey). The colors are for the "evidence" for the h boson (red), the $(g_\mu - 2)$ (blue) and Dark Matter (green).

4. Sparticle and Higgs production in e^+e^- Collisions

We have then analyzed the prospects for producing SUSY particles and heavy Higgs bosons at high–energy and high–luminosity e^+e^- colliders, requiring a sample of 50 events per year to establish discovery; this should be sufficient in the clean environment provided by e^+e^- colliders. At c.m. energies $\sqrt{s} \sim 800$ GeV, typical of the TESLA machine [2], we have shown that charginos, neutralinos and sleptons [in particular $\tilde{\tau}$ and $\tilde{\nu}$] are accessible in rather large regions of the parameter space. In particular, already at $\sqrt{s} = 800$ GeV associated $\tilde{\chi}_1^0\tilde{\chi}_2^0$ production is accessible in the entire overlap region described above. Almost all of this region can also be probed through $\tilde{\chi}_1^\pm$ pair production, and in much of it $\tilde{\tau}_1$ pair production can also be studied. In some areas, top squarks and even bottom squarks can be produced. In the large $\tan\beta$ regime, where the present indications for SUSY can be accommodated in a larger fraction of the $(m_{1/2}, m_0)$ plane, there is a large region where the heavier MSSM Higgs bosons H, A and $H^\pm$ are kinematically accessible.

Even for lower c.m. energies, $\sqrt{s} \sim 500$ GeV, charginos, neutralinos and charged (τ) sleptons can be produced in a significant region of parameter space not excluded by the present constraints. However, discovery of sparticles can then no longer be guaranteed [in the framework of mSUGRA] even if all positive indications for SUSY hold up to further scrutiny. On the other hand, if the c.m. energy of the collider is increased to $\sqrt{s} = 1.2$ TeV, the mSUGRA parameter space where SUSY and Higgs particles are kinematically accessible and have sufficiently large cross sections to be detected becomes very wide. The e^+e^- collider will then have a search potential of SUSY particles that is comparable to the range probed at the LHC. This is largely due to the fact that, thanks to the high

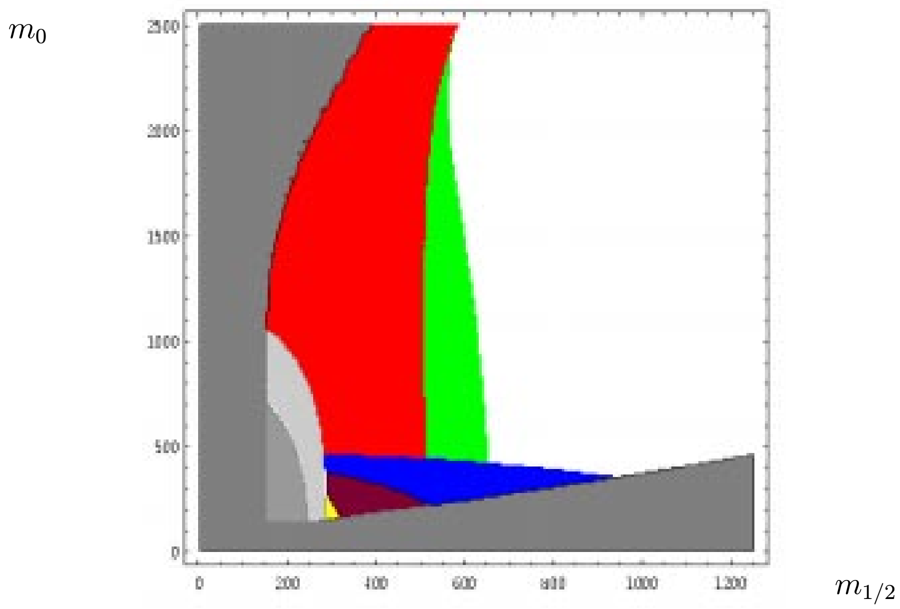

Figure 2: The $(m_{1/2}, m_0)$ mSUGRA plane with $\tan\beta = 40, A_0 = 0$ and sign$(\mu) > 0$ where SUSY and Higgs particles can be produced at and e^+e^- collider with a c.m. energy $\sqrt{s} = 800$ GeV. The grey areas are those excluded by theoretical and experimental constraints. The colored regions are those where then cross sections are large enough for the particles to be produced: $\tilde{\chi}_1^0\tilde{\chi}_2^0$ (green), $\tilde{\chi}_1^+\tilde{\chi}_1^-$ (red), $\tilde{l}^+\tilde{l}^-$ (blue), $\tilde{\nu}\tilde{\nu}^*$ (purple), $\tilde{t}_1\tilde{t}_1^*$ (dark blue), $\tilde{b}_1\tilde{t}_1^*$ (dark blue) and the heavy MSSM $H, A, H^\pm$ bosons (yellow). Note that some of these regions are overlapping.

luminosities expected at future e^+e^- colliders, the process $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ can probe large values of the parameter $m_{1/2}$: only from kinematical arguments, values $m_{1/2} \sim 1$ TeV can be probed at $\sqrt{s} = 1.2$ TeV, corresponding to a gluino mass of the order of 2 TeV. Heavy Higgs particles can be searched if their masses are smaller than the beam energy. For large values of $\tan\beta$, this occurs in a large region of the mSUGRA parameter space.

Once these particles are found, precision measurements at an e^+e^- collider could reveal a great deal about the MSSM spectrum. In particular, threshold scans allow the measurement of some sparticle masses at the permile level. Making use of the ability to vary the beam polarization at will, various couplings appearing in the production cross sections of SUSY and Higgs particles can be measured with a high precision. Additional couplings can be determined through the careful measurement of decay branching ratios. By combining the information on sleptons and electroweak gauginos that one can obtain at e^+e^- colliders with the information on squark and gluino production obtained at the LHC would allow very stringent tests of the model.

References

[1] For a detailed analysis and a complete set of references see the original paper, A. Djouadi, M. Drees and J.L. Kneur, JHEP 0108:055,2001 [hep-ph/0107316].

[2] See the talks given in these proceedings: F. Gianotti (Higgs and SUSY), W. Venus and D. Charlton (LEP and SM Physics), S. Sarkar (cosmology) and A. Wagner (TESLA).

[3] A. Djouadi, J.L. Kneur and G. Moultaka, hep–ph/9901246 [new version to appear].