Asymmetrically Warped Spacetimes*

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ABSTRACT: We investigate spacetimes in which the speed of light along flat 4D sections varies over the extra dimensions due to different warp factors for the space and the time coordinates ("asymmetrically warped" spacetimes). The main property of such spaces is that while the induced metric is flat, implying Lorentz invariant particle physics on a brane, bulk gravitational effects will cause apparent violations of Lorentz invariance and of causality from the brane observer's point of view. An important experimentally verifiable consequence of this is that gravitational waves may travel with a speed different from the speed of light on the brane, and possibly even faster. We find the most general spacetimes of this sort, which are given by certain types of black hole spacetimes characterized by the mass and the charge of the black hole. We show how to satisfy the junction conditions and analyze the properties of these space-times.

1. Introduction

There is a crucial difference between the behavior of 4D and 5D theories in the presence of non-vanishing energy densities [1]. In a 4D theory, the presence of an ordinary 4D energy density will imply that the Universe is expanding according to the FRW equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho. \tag{1.1}$$

However, in a 5D theory, a 4D energy density (a brane tension type object) can be balanced by a 5D energy density (a bulk cosmological constant, since the expansion equation is of the form $H^2 \propto \alpha\rho^2 + \beta\Lambda$, where ρ is the 4D energy density (brane tension), and Λ is the bulk (5D) cosmological constant. Thus, the effective 4D energy density may vanish, if the 4D and 5D sources precisely cancel each other [1]. But then space is necessarily curved, since the sources are not vanishing themselves. This simple observation [1] is the basis of

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much of the recent interest in theories with extra dimensions. A concrete realization of this idea is obtained by considering "warped spacetimes", that is spaces for which the form of the metric is given by

$$ds^2 = a^2(y)(dt^2 - d\vec{x}^2) - dy^2, \tag{1.2}$$

where $a(y)$ is the warp factor, and due to the form of the metric 4D Lorentz invariance is always maintained, since the induced metric is proportional to $\eta_{\mu\nu}$. The best-known example of metrics of this form is provided by the Randall-Sundrum (RS) model [2], where the only sources in the model are given by a brane tension V and a bulk cosmological constant Λ, fine tuned such that the effective 4D cosmological constant vanishes. This model is extremely interesting, since it may localize gravity to the brane with a positive tension, and thus avoid the need for compactification of the extra dimension. A slightly modified version of the model where the fifth dimension ends on a second brane (with negative tension) may solve the hierarchy problem due to the exponential falloff in the strength of gravity. A second interesting example of a metric of the form (1.2) is provided by the "self-tuning" brane models of [3]. In these models, there is an extra bulk scalar field Φ besides the sources of the RS model, which make it possible for a flat solution to exist for any values values of the brane tension (while no other maximally symmetric solutions exist). However, these solutions are nakedly singular at a finite distance from the brane, and the effective cosmological constant vanishes only after fine-tuning is reintroduced into the theory [4].

2. Asymmetric Warping

Here we will consider a slightly modified version of the metric, which takes the form [5]

$$ds^2 = a^2(y)dt^2 - b^2(y)d\vec{x}^2 - c^2(y)dy^2, \tag{2.1}$$

where the warp factor for the space and time components of the metric differ, $a \neq b$. Some of the properties of such space-times were also investigated in [6].

The first thing that one learns from the metric in (2.1) is that the induced metric at any point $y = y_0$ is still flat, however one needs to use a different (y-dependent) rescaling of the time coordinate t to obtain a flat induced metric $\eta_{\mu\nu}$. Thus we expect that in these metrics 4D Lorentz invariance would be locally respected, but broken globally. Therefore, if the standard model particles were to be localized at a fix value of y, then particle physics would appear to be completely Lorentz invariant, still leaving open the possibility that gravitational effects violate Lorentz invariance. What these effects could be is most easily seen by considering the propagation of massless fields in such a space-time. The local speed of light is given by $a(y)/b(y)$, which by definition is varying along the extra dimension. This is analogous to propagation of light in a medium with a changing index of refraction, governed by Fermat's principle. Thus if the local speed of light increases away from the brane, then it might be advantageous for the gravitational waves to bend into the extra dimension, and arrive earlier than electromagnetic waves which are forced to travel with the local speed of light (see Fig. 1). This also implies, that regions that one thought not to

have been in causal contact with each other may have been in contact after all, leading to apparent violation of causality. We have to stress that this is only an apparent violation from the brane observer's point of view, and the full 5D theory is completely causal (there is no propagation backwards in time, that is there are no closed time-like curves). It also implies that these theories may have an unconventional 4D effective theory, which may circumvent the no-go theorem of Weinberg for the adjustment of the cosmological constant.

3. Solutions with Asymmetric Warping

Here we find the solutions of the form (2.1) corresponding to the simplest possible physical sources in the bulk and on a brane, assumed to have a Z_2 symmetry (for details of this symmetry and more details of the solution see [5]). We assume that the metric is homogeneous and isotropic, thus taking the form

$$ds^2 = -n^2(t,r)\, dt^2 + a^2(t,r) d\Sigma_k^2 + b^2(t,r)\, dr^2, \tag{3.1}$$

where $d\Sigma_k^2 = d\sigma^2/(1 - kL^{-2}\sigma^2) + \sigma^2 d\Omega_2^2$ is the metric of the spatial 3-sections, with curvature parameter k, L being a parameter with dimension of length that will be set to the length scale given by the cosmological constant in the bulk. Using Birkhoff's theorem, this solution can always be transformed into the following simple form:

$$ds^2 = -h(r)\, dt^2 + l^{-2} r^2\, d\Sigma_k^2 + h(r)^{-1}\, dr^2, \tag{3.2}$$

which describes a black hole space-time. Depending on the sources in the problem this may be an ordinary black hole, an AdS black hole, a Reissner-Nordstrom (RN) black-hole, an AdS-RN black hole, etc. The most important cases for us are the AdS black hole solution for which

$$h(r) = k + \frac{r^2}{l^2} - \frac{\mu}{r^2}, \quad l^{-2} = -\tfrac{1}{6}\kappa_5^2 \Lambda_{bk}, \tag{3.3}$$

and the AdS-RN solution in which case

$$h(r) = k + \frac{r^2}{l^2} - \frac{\mu}{r^2} + \frac{Q^2}{r^4}, \tag{3.4}$$

where μ is the mass and (for the RN case) Q the charge of the black hole. Thus we know what the form of the solutions in the bulk can be, but in order to find a viable metric, a brane has to be introduced and the Israel junction conditions at the brane have to be satisfied. The details of the matching procedure are described in [5], here we summarize the most important features. For the case of and AdS BH solution, there is only one new parameter, the mass of the black hole μ, and the junction conditions will still require a fine tuning between the energy density on the brane and the bulk cosmological constant. The black hole singularity is naked if in the equation of state on the brane $p = \omega\rho$ the parameter $\omega \geq -1$. For the case of the AdS RN BH solution there are two new parameters μ and Q, and if the Z_2 parities are chosen appropriately no fine tuning will be needed to satisfy the junction conditions. The naked BH singularity is again only avoidable for $\omega \leq -1$.

Once we obtain a solution with a horizon, one can show that the space-time can be cut at the horizon without reintroducing fine-tuning into the model. One can also show [5], that except for generic values of the parameters of the theory the only maximally symmetric solution on the brane is the flat solution, thus there is no AdS and dS solution on the brane for this setup.

4. Lorentz Violations and Experimental Signatures

Once the possible solutions with asymmetrically warped spaces are found, we can ask whether any of the phenomena involving violations of causality would occur in these solutions. For the general space-time of the form (3.2) the local speed of light is given by $c^2(r) = h(r)l^2/r^2$. One would need a growing $c(r)$ away from the brane. Examining the full solutions that also incorporate the junction conditions we find that this indeed may happen for a significant fraction of the parameter space. An example for such a case is illustrated in Fig. 1.

Finally we address the issue whether any of these effects could be experimentally observable. The simplest evidence for asymmetrically warped spaces would be to measure the speed of gravitational waves and find that it is larger than the speed of light in the vacuum. The LIGO experiment may be able to detect gravitational waves from type II supernovae up to a distance of about 20 Mpc ($\sim 6 \cdot 10^7$ ly). For objects of such a distance even a tiny difference in the speeds of gravitational and electromagnetic waves would cause a huge time difference, and thus the possible values of μ and Q could be severly constrained. In fact, the limitations of such measurements are likely not to lie in the time resolution of the gravitational and the light signal, but rather the opposite problem: if there is an appreciable difference in the propagation speeds then due to the huge distance to the expected sources the arrival time differences could turn out to be way too big to be able to identify the fact that the source for the gravitational wave and the light was the same. For a supernova 20 Mpc away from us, and very conservatively assuming that the arrival time difference should be less than 5 years, in order to be able to actually detect the different arrival times one needs to have the difference in the speeds to be less than $\frac{\delta c}{c} \leq 10^{-7}$. Otherwise the gravitational wave experiments will simply not be able to identify the source for the observed gravitational waves. Type I supernovae could likely be detected by LIGO only if they happen within our galaxy. These are very rare, however assuming the best possible scenario one could see a supernova a few hundred thousand light years from us. In this case (again assuming a very conservative time difference of 5 years) the maximum value of $\frac{\delta c}{c}$ that could be tested is of the order of 10^{-3}.

5. Conclusions

We have considered space-times where the local speed of light varies along the extra dimensions. Such asymmetrically warped spaces could have interesting physical properties, since gravitational waves might propagate faster than electromagnetic waves. This could be verified experimentally by the upcoming gravitational wave experiments. We have found a
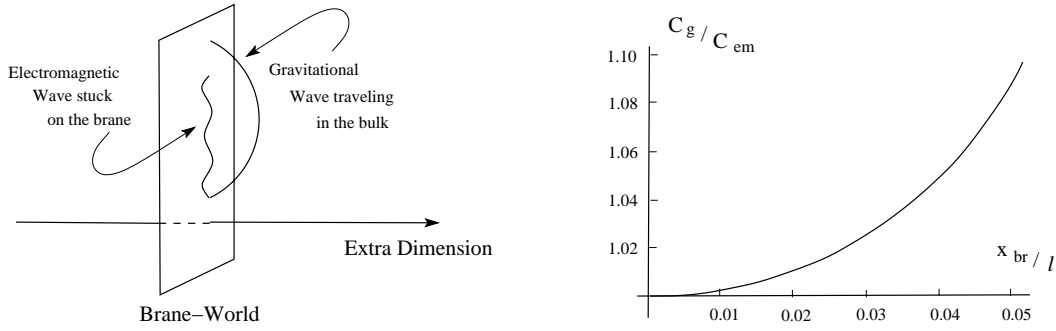
Figure 1: A graviton emitted on the brane will travel along a geodesic in the bulk before returning to the brane. A photon emitted at the same time can propagate only along the brane and may wander a shorter distance along the brane than the graviton in the same time. The 4D effective propagation speed of gravity is distance dependent (x_{br} is the distance traveled along the brane and l characterizes the curvature of the bulk).

large class of such solutions in forms of various black hole spacetimes characterized by the mass and the charge of the black hole, showed how the junction conditions can be satisfied, and analyzed the physical properties of these solutions.

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References

[1] V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. **125B**, 139 (1983).

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [`hep-ph/9905221`]; Phys. Rev. Lett. **83**, 4690 (1999) [`hep-th/9906064`].

[3] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, Phys. Lett. **B480**, 193 (2000) [`hep-th/0001197`]; S. Kachru, M. Schulz and E. Silverstein, Phys. Rev. D **62**, 045021 (2000) [`hep-th/0001206`]; C. Csáki, J. Erlich, C. Grojean and T. Hollowood, Nucl. Phys. **B584**, 359 (2000) [`hep-th/0004133`].

[4] S. Förste, Z. Lalak, S. Lavignac and H. P. Nilles, Phys. Lett. **B481**, 360 (2000) [`hep-th/0002164`]; D. Youm, Nucl. Phys. **B589**, 315 (2000) [`hep-th/0002147`].

[5] C. Csáki, J. Erlich and C. Grojean, Nucl. Phys. B **604**, 312 (2001) [`hep-th/0012143`]; `gr-qc/0105114`.

[6] G. Kälbermann and H. Halevi, `gr-qc/9810083`; G. Kälbermann, Int. J. Mod. Phys. **A15**, 3197 (2000) [`gr-qc/9910063`]; D. J. Chung and K. Freese, Phys. Rev. **D62**, 063513 (2000) [`hep-ph/9910235`]; H. Ishihara, Phys. Rev. Lett. **86**, 381 (2001) [`gr-qc/0007070`]; D. J. Chung, E. W. Kolb and A. Riotto, `hep-ph/0008126`;