Supersymmetric warped brane worlds

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Abstract: We present a supersymmetric extension of the Randall-Sundrum model. The basic set-up is 5d gauged supergravity on $M_4 \times S_1/Z_2$ with the gauge charge which changes sign at the orbifold fixed points.

Brane worlds with warped geometries offer new perspectives in understanding the hierarchy of mass scales in field theory models [1]. The basic set-up is five-dimensional gravity with a negative cosmological constant, where the fifth dimension is the orbifold S_1/Z_2. At the Z_2 fixed points reside four-dimensional hypersurfaces (branes) hosting familiar gauge and charged matter field. The basic drawback of the scenario presented in [1] is the unnatural correlation between the bulk cosmological constant and the brane tensions. This situation has prompted the proposal [2, 3, 4, 6], that it is a version of brane-bulk supersymmetry that may be able to explain the apparent fine-tunings. Indeed, the brane-bulk supersymmetry turns out to correlate in the right way the brane tensions and bulk cosmological constant in the supersymmetric Randall–Sundrum model.

Let us begin with a brief review of the original RS model [1]. The action is that of 5d gravity on $M_4 \times S_1/Z_2$, with negative cosmological constant:

$$S = M^3 \int d^5x\sqrt{-g}(\tfrac{1}{2}R + 6k^2) + \int d^5x\sqrt{-g_i}(-\lambda_1\delta(x^5) - \lambda_2\delta(x^5 - \pi\rho)). \tag{1}$$

Three-branes of non-zero tension are located at Z_2 fixed points. The ansatz for vacuum solution preserving 4d Poincare invariance has the warped product form:

$$ds^2 = a^2(x^5)\eta_{\mu\nu}dx^\mu dx^\nu + R_0^2(dx^5)^2. \tag{2}$$

The size of the fifth dimension is parametrized by R_0. The Einstein equations determine the solution of for the warp factor $a(x^5)$ is:

$$a(x^5) = \exp(-R_0 k|x^5|). \tag{3}$$

It has an exponential form, which can generate large hierarchy of scales between the branes. Matching delta functions in the equations of motion requires fine-tuning of the brane tensions:

$$\lambda_1 = -\lambda_2 = 6k. \tag{4}$$

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With the choice (4) the matching conditions are satisfied for arbitrary R_0, so the fifth dimension is not stabilized in the original RS model. Thus R_0 enters the 4d effective theory as a massless scalar (radion), which couples to gravity in the manner of a Brans–Dicke scalar. This is at odds with the precision tests of general relativity, so any realistic model should contain a potential for the radion field.

The Randall–Sundrum model can be extended to a locally supersymmetric model [2, 3, 4]. The basic set-up consists of 5d $N = 2$ gauged supergravity [7, 8], which includes the gravity multiplet $(e^m_\alpha, \psi^A_\alpha, \mathcal{A}_\alpha)$, that is the metric (vielbein), a pair of symplectic Majorana gravitinos, and a vector field called the graviphoton. The 5d SUGRA action is

$$S = M^3 \int d^5x e_5 \left(\tfrac{1}{2}R - \tfrac{1}{2}\overline{\psi_\alpha}^A \gamma^{\alpha\beta\gamma} D_\beta \psi_{A\gamma} - \tfrac{3}{4}\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta} + \dots \right) \qquad (5)$$

and the supersymmetry transformations are given by

$$\begin{aligned} \delta e^m_\alpha &= \tfrac{1}{2}\bar{\epsilon}^A \gamma^m \psi_{A\alpha} \\ \delta\psi^A_\alpha &= D_\alpha \epsilon^A + \dots \\ \delta\mathcal{A}_\alpha &= -\tfrac{i}{2\sqrt{2}}\overline{\psi_\alpha}^A \epsilon_A. \end{aligned}$$

Let us now add the brane tension at the brane located at $x^5 = 0$, $S_1 = \int d^5x e_4(-6k)\delta(x^5)$, and perform the supersymmetry transformation on the determinant of the induced vierbein. This produces a delta-type variation in the action: $\delta e \Rightarrow 3\delta(x^5)e_4 k(\bar{\psi^1}_\mu \gamma^\mu \epsilon^1 + (1 \leftrightarrow 2))$. It is straightforward to notice that this can be cancelled through the variation of the term $\bar{\psi^A}_\mu \gamma^{\mu 5 \rho} D_5 \psi^A_\rho$ upon introducing new terms in the transformations of gravitini: $\delta\psi^1_\alpha = +\frac{k}{2}\epsilon(x^5)\gamma_\alpha\epsilon^1$, $\delta\psi^2_\alpha = -\frac{k}{2}\epsilon(x^5)\gamma_\alpha\epsilon^2$. These corrections introduce further variations in the bulk Lagrangian, which require further new terms in the bulk Lagrangian: $\mathcal{L}_{\psi^2} = +\frac{3e_5}{4}k\epsilon(x^5)(\bar{\psi^1}_\alpha\gamma^{\alpha\beta}\psi^1_\beta - \bar{\psi^2}_\alpha\gamma^{\alpha\beta}\psi^2_\beta)$ and $\mathcal{L}_{cc} = 6e_5k^2$, which is precisely the bulk potential needed in the RS model. The continuation through $x^5 = \pi\rho$ gives on the second brane the tension term $+\delta(x^5 - \pi\rho)e_4 6k$. Thus the fine-tuning present in the original RS model can be explained by the requirement of local supersymmetry [4].

The resulting locally supersymmetric Lagrangian is in fact that of a gauged supergravity. The symmetry that is gauged is the $U(1)$ subgroup of the R-symmetry, $\psi^A_\alpha \to e^{i\phi}\psi^A_\alpha$, the gauge field being $\mathcal{A}^R_\alpha = -\frac{1}{2\sqrt{2}}\mathcal{A}_\alpha$. Gauging of the $U(1)_R$ symmetry means that we replace the derivative acting on the gravitino with the $U(1)_R$ covariant derivative:

$$D_\alpha\psi^A_\beta \to D_\alpha\psi^A_\beta - \tfrac{3}{\sqrt{2}}(\sigma^3)^A{}_B k\epsilon(x^5)\mathcal{A}_\alpha\psi^B_\beta,$$

where D_α denotes the ordinary space-time covariant derivative. The coefficient of the coupling $\mathcal{A}_\alpha\psi^B_\beta$ defines the prepotential $\mathcal{P} = \frac{i}{4}\sigma^3$ and the Z_2-odd gauge coupling $g = \frac{6k\epsilon(x^5)}{\sqrt{2}}$.

New bosonic and fermionic fields do not affect the vacuum solution, so that the equations of motion for the warp factor are the same as in the original, non-supersymmetric RS model. In consequence, the warp factor has the exponential profile $a(x_5) = e^{-kR_0|x_5|}$. Morevoer, the RS solution satisfies the BPS conditions and preserves one half of the supercharges, which corresponds to unbroken $N = 1$ supersymmetry in four dimensions.

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