A New Model-Independent Analysis of $B \to X_s\gamma$ in Supersymmetry [*]

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ABSTRACT: Using the example of the rare decay $B \to X_s\gamma$, we analyse the importance of interference effects for the bounds on the parameters in the squark mass matrices within the unconstrained MSSM. In former model-independent analyses no correlations between the different sources of flavour violation were taken into account. In our new analysis we include the contributions from charged Higgs bosons, charginos and neutralinos and their interference effects and, even more important, the effects that appear when several flavour violating parameters, i.e. several off-diagonal elements in the squark mass matrices, are switched on simultaneously. We derive new bounds on certain off-diagonal elements of the squark-mass matrix which are in general one order of magnitude weaker than the previous bounds.

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1. Introduction

Flavour changing neutral current (FCNC) processes provide crucial guidelines for supersymmetry model building. Besides the Cabibbo–Kobayashi–Maskawa (CKM)-induced contributions, there are generic supersymmetric contributions induced by flavour mixing in the squark mass matrices within the so-called unconstrained minimal supersymmetric standard model (MSSM). The structure of the MSSM does not explain the suppression of FCNC

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processes, which is observed in experiments; this is the crucial point of the well-known supersymmetric flavour problem.

Among neutral flavour transitions involving the third generation, the rare decay $B \to X_s \gamma$ is at present the most important one [1], as it is the only inclusive mode that is already measured [2] and that already provides theoretically clean and rather stringent constraints on the parameter space of various extensions of the SM [3]. Although the theoretical SM prediction, up to next-to-leading logarithmic (NLL) precision [4] for its branching ratio, is in agreement with the experimental data, it is still possible that the rare decay $B \to X_s \gamma$ leads to the first evidence of new physics by a significant deviation from the SM prediction, for example in the observables concerning direct CP violation [5].

The decay $B \to X_s \gamma$ is sensitive to the mechanism of supersymmetry breaking because, in the limit of exact supersymmetry, the decay rate would be just zero, $\mathcal{B}(B \to X_s \gamma) = 0$. Flavour violation thus originates from the interplay between the dynamics of flavour and the mechanism of supersymmetry breaking and FCNC processes may contribute to the question of which mechanism ultimately breaks the supersymmetry.

Former analyses in the unconstrained MSSM neglected QCD corrections and only used the gluino contribution to saturate the experimental bounds. Moreover, no correlations between different sources of flavour violation were taken into account. In this way, one arrived at 'order-of-magnitude bounds' on the soft parameters [6, 7, 8]. In [9], the sensitivity of the bounds on the down squark mass matrix to radiative QCD corrections was analysed, including the SM and the gluino contributions. In the new analysis [10] we present here, we include the contributions from charged Higgs bosons, charginos and neutralinos and their interference effects and, even more important, the effects that result when several flavour violating parameters, i.e. several off-diagonal elements in the squark mass matrices, are switched on simultaneously.

2. Phenomenological Analysis

To understand the sources of flavour violation that may be present in supersymmetric models, in addition to those enclosed in the CKM matrix, one has to consider the contributions to the squark mass matrices

$$\mathcal{M}_f^2 \equiv \begin{pmatrix} m_{f,LL}^2 + F_{f\,LL} + D_{f\,LL} & \left(m_{f,LR}^2 \right) + F_{f\,LR} \\ \left(m_{f,LR}^2 \right)^\dagger + F_{f\,RL} & m_{f,RR}^2 + F_{f\,RR} + D_{f\,RR} \end{pmatrix}, \tag{2.1}$$

where f stands for up- or down-type squarks. We recall that the matrices $m_{u,LL}$ and $m_{d,LL}$ cannot be specified independently; $SU(2)_L$ gauge invariance implies that $m_{u,LL} = K m_{d,LL} K^\dagger$, where K is the CKM matrix. In the super-CKM basis, where the quark mass matrices are diagonal and the squarks are rotated in parallel to their superpartners, the F terms from the superpotential and the D terms turn out to be diagonal 3×3 submatrices of the 6×6 mass matrices $\mathcal{M}_f^2$. This is in general not true for the additional terms m_f^2, originating from the soft supersymmetry breaking potential. Because all neutral gaugino couplings are flavour diagonal in the super CKM basis, the gluino contributions to the decay

$B \to X_s \gamma$ are induced by the off-diagonal elements of the soft terms $m^2_{f,LL}$, $m^2_{f,RR}$, $m^2_{f,RL}$. Since there are different contributions to this decay, with different numerical impact on its rate, some of these flavour violating terms may turn out to be poorly constrained. Thus, given the generality of such a calculation, it is convenient to rely on the mass eigenstate formalism, which remains valid even when some of the intergenerational mixing elements are large, and not to use the approximate mass insertion method (MIA), where the off-diagonal squark mass matrix elements are taken to be small and their higher powers neglected. In the latter approach the reliability of the approximation can be checked only a posteriori.

As a first step, it is convenient to select *one* possible source of flavour violation in the squark sector at a time and assume that all the remaining ones are vanishing. It should be stressed that one already excludes any kind of interference effects between different sources of flavour violation in this way. Following ref. [6], all diagonal entries in $m^2_{d,LL}$, $m^2_{d,RR}$, and $m^2_{u,RR}$ are set equal and their common value is denoted by $m^2_{\tilde{q}}$. The branching ratio can then be studied as a function of

$$\delta_{LL,ij} = \frac{(m^2_{d,LL})_{ij}}{m^2_{\tilde{q}}}, \qquad \delta_{RR,ij} = \frac{(m^2_{d,RR})_{ij}}{m^2_{\tilde{q}}}, \qquad \delta_{LR,ij} = \frac{(m^2_{d,LR})_{ij}}{m^2_{\tilde{q}}} \quad (i \neq j). \quad (2.2)$$

We find that only those parameters get stringently bounded by $B \to X_s \gamma$, which can generate contributions to the two five-dimensional gluino-induced dipole operators, namely $\delta_{d,LR,23}$ and $\delta_{d,RL,23}$. The two corresponding operators are connected by chirality and are denoted by $\mathcal{O}_{7\tilde{g},\tilde{g}}$ and $\hat{\mathcal{O}}_{7\tilde{g},\tilde{g}}$ in the following. As the gluino yields, intrinsically, the dominant contribution by far, we also find that the bounds on $\delta_{d,LR,23}$ and $\delta_{d,RL,23}$ are only marginally modified by chargino, neutralino and charged Higgs boson contributions.

In the second part of our analysis, we investigate whether the obtained bounds remain stable if *all* off-diagonal elements, which induce the decay $B \to X_s \gamma$, are varied simultaneously. For our analysis we explore various scenarios that are characterized by the values of the parameters $\mu, M_{H^-}, \tan\beta, M_{\text{susy}}, m_{\tilde{g}}$. We regard this as reasonable, because we expect that these input parameters, which are unrelated to flavour physics, will be fixed from flavour conserving observables in the next generations of high energy experiments (provided low energy SUSY exists). We note that the common SUSY scale, M_{susy}, fixes in our scenarios the general soft squark mass scale $m_{\tilde{q}}$ (see (2.2)) and the first diagonal element of the chargino mass matrix M_2. The mass of the charged Higgs boson M_{H^-} are fixed to be $M_{H^-} = 300\,GeV$. We also allow for a non-degeneracy of the diagonal elements in the matrices $m^2_{d,LL}$, $m^2_{d,RR}$, and $m^2_{u,RR}$. To implement this, we define δ-quantities in addition to those in eq. (2.2), which parametrize this non-degeneracy: $\delta_{f,LL,ii} = ((m^2_{f,LL})_{ii} - m^2_{\tilde{q}})/(m^2_{\tilde{q}})$; and analogously for $\delta_{f,RR,ii}$. In our Monte Carlo analysis these diagonal δ-parameters are varied in the interval $[-0.2, 0.2]$. On the other hand, the off-diagonal ones (in eq. (2.2)) are varied in the interval $[-0.5, 0.5]$, by use of a Monte Carlo program. There are, however, two exceptions. First, we do not vary those off-diagonal δ's with an index 1; the latter δ's we set to zero, since they are severely constrained by kaon decays (see for example [6]). Second, also $(m^2_{u,LR})_{33}$ is not varied, but fixed such that the mass of the lightest neutral Higgs boson gets heavy enough
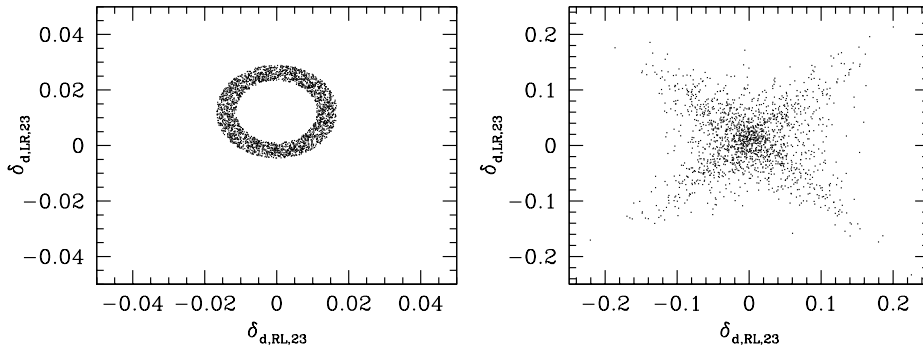
Figure 1: Contours in the $\delta_{d,LR,23}$-$\delta_{d,RL,23}$ plane. In the left frame, $\delta_{d,LR,23}$ and $\delta_{d,RL,23}$ are the only flavour violating parameters. In the right frame, $\delta_{d,LR,23}$, $\delta_{d,RL,23}$, $\delta_{d,LL,23}$ and $\delta_{d,RR,23}$ are flavour violating parameters; $\delta_{d,LR,22}$, $\delta_{d,LR,33}$ are also non-zero. We only consider SM and gluino contributions and the other parameters are $\mu = 300\,GeV$, $\tan\beta = 10$, $M_{\text{susy}} = 500\,GeV$, $x = 1$.

to be compatible with experimental bounds [11]. We plot those events, corresponding to $2.0 \times 10^{-4} \le BR(B \to X_s\gamma) \le 4.5 \times 10^{-4}$, which is the range allowed by the CLEO measurement. We start with the following parameter set: $\mu = 300\,GeV$, $M_{H^-} = 300\,GeV$, $\tan\beta = 10$, $M_{\text{susy}} = 500\,GeV$, $x = m_{\tilde{g}}^2\,/\,M_{\text{susy}}^2 = 1$ and $X_t = 750\,GeV$. In fig. 1, we only consider SM and gluino contributions. In the left frame we present the constraints on $\delta_{d,LR,23}$ and $\delta_{d,RL,23}$ when these are the only flavour-violating soft parameters; the diagonal δ-parameters defined above are also switched off. As expected, stringent constraints are obtained. The hole inside the dotted area represents values of $\delta_{d,LR,23}$ and $\delta_{d,RL,23}$ for which the branching ratio is too small to be compatible with the measurements. In the right frame we now investigate interference effects from different sources of flavour violation, where we allow for non-zero $\delta_{d,LR,23}$, $\delta_{d,RL,23}$, $\delta_{d,LL,23}$, $\delta_{d,RR,23}$, $\delta_{d,LR,22}$, and $\delta_{d,LR,33}$. All these parameters are varied between ± 0.5. As can be seen, the bounds on $\delta_{d,LR,23}$ and $\delta_{d,RL,23}$ get destroyed dramatically (note the different scale in the two plots). The reason for this is that there are now new contributions to the five-dimensional dipole operators. As an example, the combined effect of $\delta_{d,LR,33}$ and $\delta_{d,LL,23}$ leads to a contribution to the Wilson coefficient of the gluino-induced magnetic dipole operator $\mathcal{O}_{7\tilde{g},\tilde{g}}$. The sign of this contribution can be different from the one generated by $\delta_{d,LR,23}$. As a consequence, the bound on $\delta_{d,LR,23}$ gets weakened. To illustrate this more quantitatively, we assume for the moment that there are only these two sources that can generate $\mathcal{O}_{7\tilde{g},\tilde{g}}$, i.e. we switch off the other δ-quantities. If $\delta_{d,LR,23}$ is larger than the individual bound from the first part of the analysis, it is necessary that the product of $\delta_{d,LR,33}$ and $\delta_{d,LL,23}$ also be relatively large; only in this case can the two sources lead to a branching ratio compatible with experiment. This feature is illustrated in fig. 2; only values of $\delta_{d,LR,23}$ and values of $\delta_{d,LR,33} \cdot \delta_{d,LL,23}$ that are strongly correlated lead to an acceptable branching ratio. As clearly visible from fig. 2, the correlation between the two sources for $\mathcal{O}_{7\tilde{g},\tilde{g}}$ is essentially linear. This implies that the linear combination $com := \delta_{d,LR,23} + f\,\delta_{d,LR,33} \cdot \delta_{d,LL,23}$ gets constrained if f is chosen appropriately. Stated differently, the Wilson coefficient of the five-dimensional magnetic dipole operator is essentially proportional to the combination com defined above. This implies in turn, that for the values of the parameters we are using at the moment, the Wilson coefficient is well approximated by its double mass insertion expression. Thus,
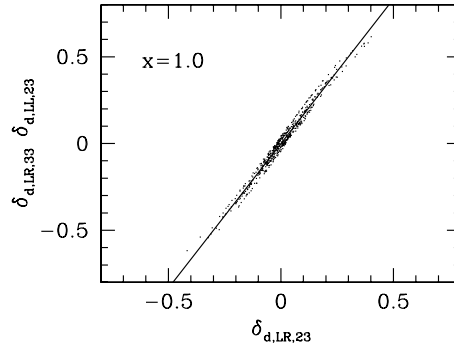
Figure 2: The parameters $\delta_{d,LR,23}$ and $\delta_{d,LR,33} \cdot \delta_{d,LL,23}$, which are compatible with the data on $B \to X_s\gamma$, are shown by dots. Values lying on the solid line lead to a vanishing contribution of the five-dimensional magnetic dipole operator $\mathcal{O}_{7\tilde{g},\tilde{g}}$ in the MIA. See text.

the coefficient f can be fixed analytically (see [10]); the numerical values of $f(x)$ for some values of x read 0.74 for $x = 0.3$, 0.68 for $x = 0.5$, 0.60 for $x = 1.0$ and 0.52 for $x = 2.0$, respectively. The solid line in fig. 2 represents pairs $(\delta_{d,LR,23}, \delta_{d,LR,33} \cdot \delta_{d,LL,23})$ for which the combination *com* is zero. The points scattered around this line therefore represent Monte Carlo events for which this combination is small. We now turn back to the scenario of fig. 1, in which all the parameters $\delta_{d,LR,23}$, $\delta_{d,RL,23}$, $\delta_{d,LL,23}$, $\delta_{d,RR,23}$, $\delta_{d,LR,22}$, $\delta_{d,LR,33}$ are varied simultaneously. In this case, the linear combinations

$$LC_1 = \delta_{d,RL,23} + f(x)\delta_{d,RR,23} \cdot \delta_{d,RL,33} + f(x)\delta_{d,RL,22} \cdot \delta_{d,LL,23},$$
$$LC_2 = \delta_{d,LR,23} + f(x)\delta_{d,LR,22} \cdot \delta_{d,RR,23} + f(x)\delta_{d,LL,23} \cdot \delta_{d,LR,33}, \tag{2.3}$$

are expected to get constrained. In fig. 3 we show the allowed region for LC_1 and LC_2. There, we allow all non-diagonal δ-parameters to vary between ± 0.5. In addition, we also allow for non-equal diagonal soft entries, by varying the parameters $\delta_{f,LL,ii}$ and $\delta_{f,RR,ii}$ between ± 0.2. With the latter choice we still guarantee the hierarchy between diagonal and off-diagonal entries, but we get rid of the unnatural assumption of degenerate diagonal entries. In the left frame, we include only the SM and gluino contributions. We find that the linear combinations LC_1 and LC_2 indeed get stringently bounded. In the right frame of fig. 3 we test the resistance of these bounds when the additional contributions (i.e., those from charginos, charged Higgs bosons and neutralinos) are turned on. In this case also $\delta_{u,LR,23}$, $\delta_{u,RL,23}$, $\delta_{u,LL,23}$, $\delta_{u,RR,23}$ and $\delta_{u,LR,22}$ are varied in the range ± 0.5. We find that the bound on LC_1 remains unchanged, while the one on LC_2 gets somewhat weakened. This feature is expected, because charginos and charged Higgs bosons contribute to the unprimed operator.

At this point we should stress that these plots were obtained by choosing the renormalization scale $\mu_b = 4.8\,GeV$ and by requiring all squark masses to be larger than $150\,GeV$. We checked that the bounds on LC_1 and LC_2 remain practically unchanged when the renormalization scale is varied between $2.4\,GeV$ and $9.6\,GeV$; they are also insensitive to the value of the required minimal squark mass, as we found by changing $m_{\text{squark min}}$ from $150\,GeV$ to $100\,GeV$ or $250\,GeV$. Moreover, we also checked whether the restriction to the
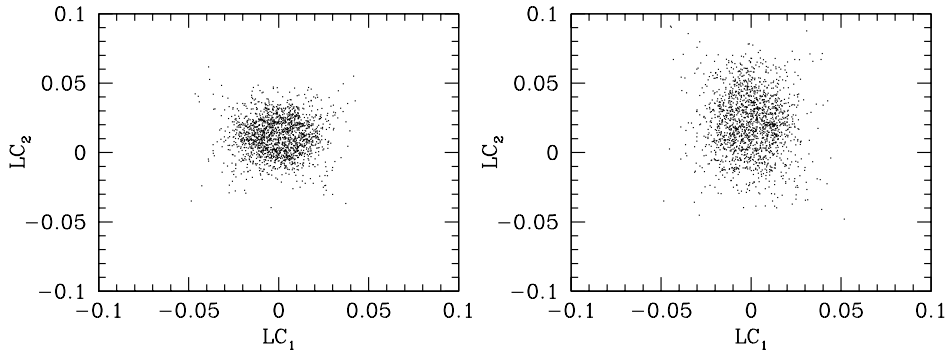
Figure 3: Contours in the LC_1–LC_2 plane with $\delta_{d,LR,23}$, $\delta_{d,RL,23}$, $\delta_{d,LL,23}$, $\delta_{d,RR,23}$, $\delta_{d,LR,22}$, $\delta_{d,LR,33}$, $\delta_{u,LR,23}$, $\delta_{u,RL,23}$, $\delta_{u,LL,23}$, $\delta_{u,RR,23}$, and $\delta_{u,LR,22}$ all non-vanishing. In the left frame, we consider only SM and gluino contributions whereas in the right frame we also include chargino, charged Higgs boson and neutralino contributions.

$\mu = +300\,GeV$ scenario is too severe: we redid the complete analysis for $\mu = -300\,GeV$ and confirmed that there are no differences between the results with these two choices.

There is a remark in order. If we got rid of the hierarchy of diagonal and off-diagonal entries in the squark mass matrices, stringent bounds on the simple combinations LC_1 and LC_2 certainly would no longer exist, simply because there would then be more contributions to the five-dimensional operators of similar magnitude. In this case, however, the *full* Wilson coefficients of the five-dimensional operators would still be stringently constrained by the experimental data on $B \to X_s\gamma$. Unfortunately, in this case not much information can be extracted for the individual soft parameters or simple combinations thereof.

Finally, we extend our analysis to other values of the input parameters. We analyse the bounds on the soft parameters within the following parameter sets: $M_{\text{susy}} = 300\,GeV$, $500\,GeV$, $1000\,GeV$. For $\tan\beta$ we explore the values: $\tan\beta = 2, 10, 30, 50$. Furthermore, the gluino mass $m_{\tilde{g}}$ is varied over the values $x = m_{\tilde{g}}^2\,/\,M_{\text{susy}}^2 = 0.3, 0.5, 1, 2$. Surprisingly, we find that the constraints on LC_1 and LC_2 are completely stable over large parts of the parameter space. However, the bounds get weakened when $\tan\beta$ values as large as 50 are chosen. This effect gets enhanced when the general mass scale $m_{\tilde{q}}$ in the squark mass matrices decreases with the parameter M_{susy}. There are two main reasons why the bounds get weakened in these scenarios. First, in the large $\tan\beta$ regime, the term $(F_{d,LR})_{33}$ gets strongly enhanced because it is proportional to $\tan\beta$. Particularly, for $\tan\beta = 50$ and $M_{\text{susy}} = 300\,GeV$, the term $(F_{d,LR})_{33}$ is of the same magnitude as the diagonal entries of the squark mass matrix. Thus, the contributions to the Wilson coefficients of the five-dimensional gluino operators (induced by $(F_{d,LR})_{33}$ in combination with $\delta_{d,LL,23}$ or $\delta_{d,RR,23}$) become important enough to weaken the bounds on LC_1 and LC_2 significantly. The relative importance of this F term is of course increased if the general soft squark mass scale M_{susy} is decreased. Second, within the large $\tan\beta$ regime the contributions from charginos get enhanced and therefore also weaken the bounds on LC_2. Because the parameter $(F_{d,LR})_{33}$ is actually proportional to the product of $\tan\beta$ and μ, we conclude from the above findings that the bound on LC_1 is unchanged if we increase the value of μ and decrease the value of $\tan\beta$ so that the product of both parameters is constant; the bound on LC_2 is then even stronger, because the chargino contribution is smaller for increasing μ.

3. Conclusions

Our new model-independent analysis of the rare decay $B \to X_s\gamma$, based on a systematic leading logarithmic QCD analysis, mainly explored the interplay between the various sources of flavour violation and the interference effects of SM, gluino, chargino, neutralino, and charged Higgs boson contributions. In former analyses, no correlations between the different sources of flavour violation were taken into account. Unlike previous work, which used the mass insertion approximation, we used in our analysis the mass eigenstate formalism, which remains valid even when some of the intergenerational mixing elements are large. We singled out two simple combinations of elements of the soft parts of the down squark mass matrices, which stay stringently bounded over large parts of the supersymmetric parameter space, excluding the large $\tan\beta$ and the large μ regime. These new bounds are in general one order of magnitude weaker than the bound on the single off-diagonal element $\delta_{d,LR,23}$, which was derived in previous work [6, 12] by neglecting any kind of interference effects. It seems that the flavour problem is less severe in the B system than often stated. Thus, it would be interesting to explore also the consequences of natural interference effects between different sources of flavour changes within the kaon sector.

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